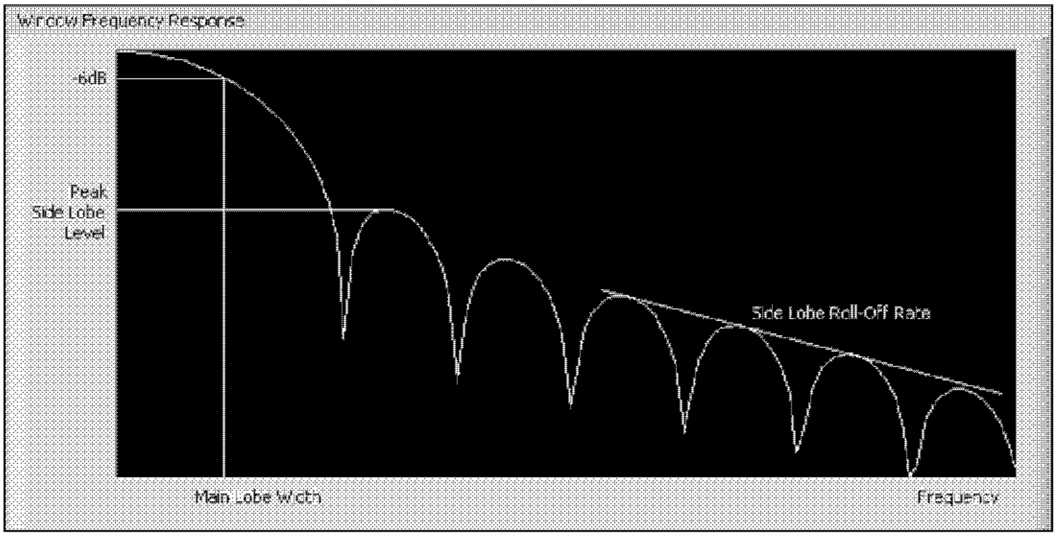
Characteristics of Different Smoothing Windows

To simplify choosing a smoothing window, you need to define various characteristics so that you can make comparisons between smoothing windows. An actual plot of a smoothing window shows that the frequency characteristic of the smoothing window is a continuous spectrum with a main lobe and several side lobes. The following front panel shows the spectrum of a typical smoothing window.



Main Lobe

The center of the main lobe of a smoothing window occurs at each frequency component of the time-domain signal. By convention, to characterize the shape of the main lobe, the widths of the main lobe at –3 dB and –6 dB below the main lobe peak describe the width of the main lobe. The unit of measure for the main lobe width is FFT bins or frequency lines.

The width of the main lobe of the smoothing window spectrum limits the frequency resolution of the windowed signal. Therefore, the ability to distinguish two closely spaced frequency components increases as the main lobe of the smoothing window narrows. As the main lobe narrows and spectral resolution improves, the window energy spreads into its side lobes, increasing spectral leakage and decreasing amplitude accuracy. A trade-off occurs between amplitude accuracy and spectral resolution.

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Side Lobes

Side lobes occur on each side of the main lobe and approach zero at multiples of f_s/N from the main lobe. The side lobe characteristics of the smoothing window directly affect the extent to which adjacent frequency components leak into adjacent frequency bins. The side lobe response of a strong sinusoidal signal can overpower the main lobe response of a nearby weak sinusoidal signal.

Maximum side lobe level and side lobe roll-off rate characterize the side lobes of a smoothing window. The maximum side lobe level is the largest side lobe level in decibels relative to the main lobe peak gain. The side lobe roll-off rate is the asymptotic decay rate in decibels per decade of frequency of the peaks of the side lobes. The following table lists the characteristics of several smoothing windows.

Smoothing Window	–3 dB Main Lobe Width (bins)	–6 dB Main Lobe Width (bins)	Maximum Side Lobe Level (dB)	Side Lobe Roll-Off Rate (dB/decade)
Rectangular (none)	0.89	1.21	–13	20
Hanning	1.44	2.00	–31	60
Hamming	1.30	1.82	–43	20
Blackman-Harris	1.62	2.27	–71	20
Exact Blackman	1.61	2.25	–68	20
Blackman	1.64	2.30	–58	60
Flat Top	3.72	4.58	–93	20

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Rectangular (None)

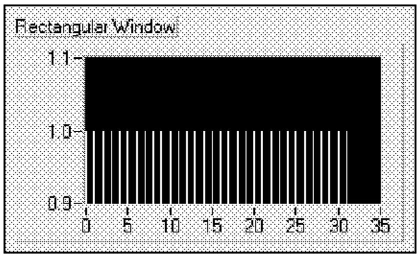
The rectangular window has a value of one over its length. The following equation defines the rectangular window.

$w(n) = 1.0$ for $n = 0, 1, 2, \ldots, N - 1$ **(A)**

where N is the length of the window and w is the window value.

Applying a rectangular window is equivalent to not using any window because the rectangular function just truncates the signal to within a finite time interval. The rectangular window has the highest amount of spectral leakage.

The following front panel shows the rectangular window for $N = 32$.



The rectangular window is useful for analyzing transients that have a duration shorter than that of the window. Transients are signals that exist only for a short time duration. The rectangular window also is used in order tracking, where the effective sampling rate is proportional to the speed of the shaft in rotating machines. In order tracking, the rectangular window detects the main mode of vibration of the machine and its harmonics.

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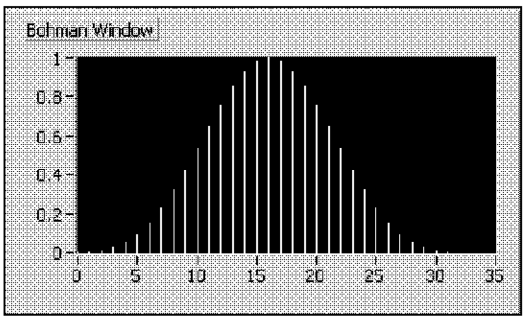
Bohman

The following equation defines the Bohman window.

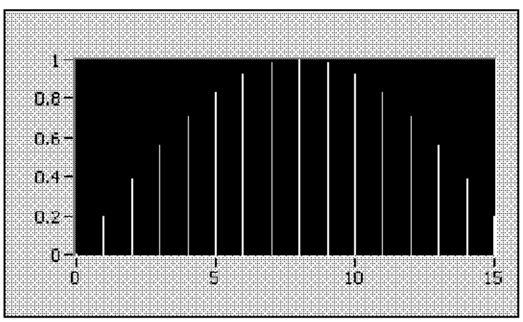
$$w(n) = \left(1 - \frac{|n - N/2|}{N/2}\right)\cos\left(\pi\frac{|n - N/2|}{N/2}\right) + \frac{1}{\pi}\sin\left(\pi\frac{|n - N/2|}{N/2}\right) \quad \textbf{(B)}$$

for $n = 0, 1, 2, \ldots, N - 1$, where N is the length of the window.

The following front panel shows a Bohman window with $N = 32$.



LabVIEW convolutes two sine lobes of half length to obtain the Bohman window. The following front panel shows the sine lobe.



Top

Hanning

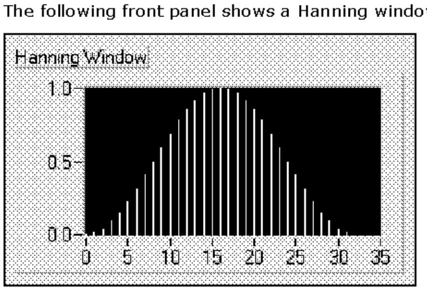
The Hanning window has a shape similar to that of half a cycle of a cosine wave. The following equation defines the Hanning window.

$$w(n) = 0.5 - 0.5\cos\frac{2\pi n}{N} \quad \textbf{(C)}$$

for $n = 0, 1, 2, \ldots, N - 1$

where N is the length of the window and w is the window value.

The following front panel shows a Hanning window with $N = 32$.



The Hanning window is useful for analyzing transients longer than the time duration of the window and for general-purpose applications.

Hamming

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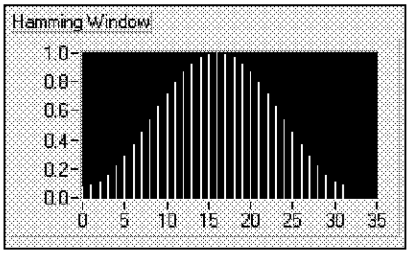
The Hamming window is a modified version of the Hanning window. The shape of the Hamming window is similar to that of a cosine wave. The following equation defines the Hamming window.

$$w(n) = 0.54 - 0.46\cos\frac{2\pi n}{N} \quad \textbf{(D)}$$

for $n = 0, 1, 2, \ldots, N - 1$

where N is the length of the window and w is the window value.

The following front panel shows a Hamming window with $N = 32$.



The Hanning and Hamming windows are similar, as shown in the previous two front panels. However, in the time domain, the Hamming window does not get as close to zero near the edges as does the Hanning window.

Kaiser-Bessel

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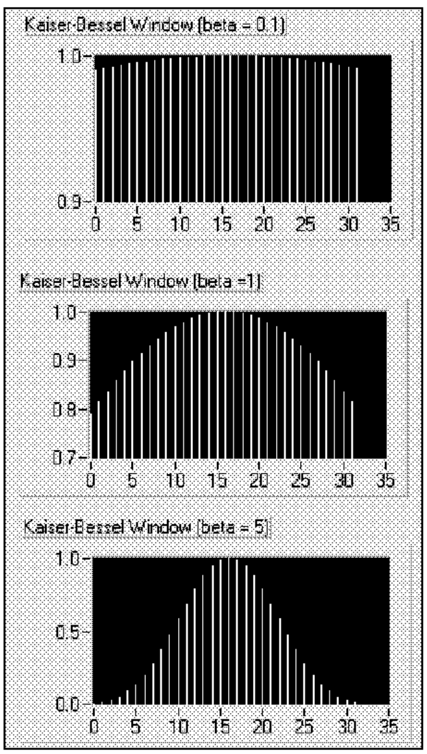
The Kaiser-Bessel window is a flexible smoothing window whose shape you can modify by adjusting the **beta** input. Thus, depending on the application, you can change the shape of the window to control the amount of spectral leakage.

The following front panel shows the Kaiser-Bessel window for different values of **beta**.



For small values of **beta**, the shape is close to that of a rectangular window. Actually, for **beta** = 0.0, you do get a rectangular window. As you increase **beta**, the window tapers off more to the sides.

The Kaiser-Bessel window is useful for detecting two signals of almost the same frequency but with significantly different amplitudes.

Low Sidelobe

Top

The Low Sidelobe window reduces the level of the side lobe at the cost of broadening the main lobe. The following equation defines the Low Sidelobe window.

$$w(n) = \sum_{k=0}^{4} (-1)^k a_k \cos(k\omega) \quad \textbf{(E)}$$

for $n = 0, 1, \ldots, N - 1$

where N is the length of the window

$$\omega = \frac{2\pi n}{N}$$

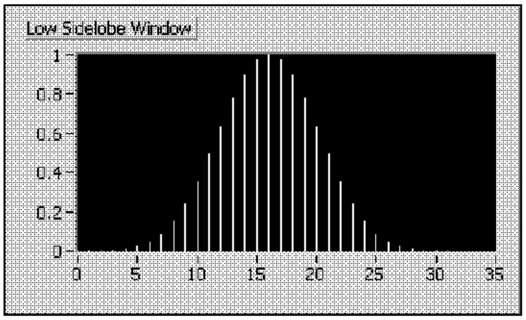
$a_0 = 0.471492057$

$a_2 = 0.17553428$

$a_3 = 0.028497078$

$a_4 = 0.001261367$

The following front panel shows a Low Sidelobe window with N = 32.



Modified Bartlett-Hanning

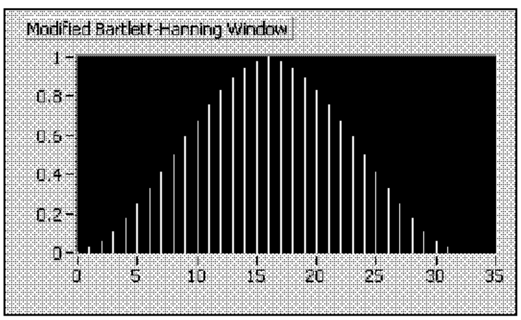
Top

The modified Bartlett-Hanning window is an integration of the Bartlett and Hanning windows. The following equation defines the modified Bartlett-Hanning window.

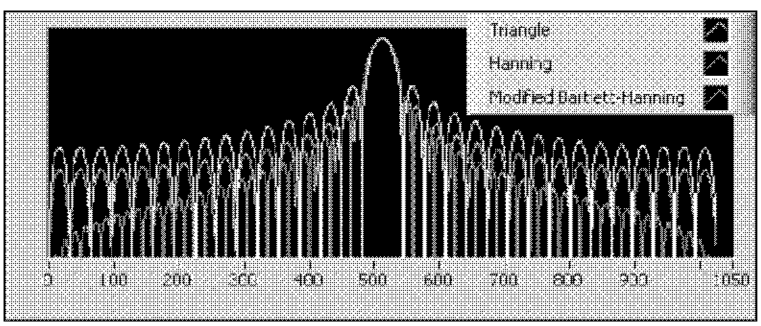
$$w(n) = 0.62 - 0.48\left|\frac{n}{N} - 0.5\right| + 0.38\cos\left(2\pi\left(\frac{n}{N} - 0.5\right)\right) \quad \textbf{(F)}$$

for n = 0, 1, 2, ..., N − 1, where N is the length of the window.

The following front panel shows a modified Bartlett-Hanning window with N = 32.



The following front panel shows the difference between the triangle, Hanning, and modified Bartlett-Hanning windows.



Parzen

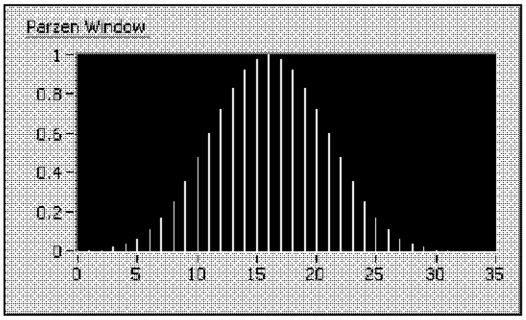
Top

The Parzen window is a piecewise cubic curve window obtained by the convolution of two triangles of half length or four rectangles of one-fourth length. The following equation defines the Parzen window.

$$w(n) = \begin{cases} 1 - 6\left(\frac{n - N/2}{N/2}\right)^2 + 6\left(\frac{|n - N/2|}{N/2}\right)^3 & 0 \le \left|n - \frac{N}{2}\right| \le \frac{N}{4} \\ 2\left(1 - \frac{|n - N/2|}{N/2}\right)^3 & \frac{N}{4} < \left|n - \frac{N}{2}\right| \le \frac{N}{2} \end{cases} \quad \textbf{(G)}$$

for $n = 0, 1, 2, ..., N - 1$, where N is the length of the window.

The following front panel shows a Parzen window with $N = 32$.



Top

Triangle

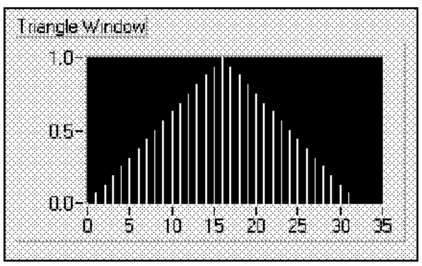
The shape of the triangle window (Bartlett Window) is that of a triangle. The following equation defines the triangle window.

$$w(n) = 1 - \left|\frac{2n - N}{N}\right| \quad \textbf{(H)}$$

for $n = 0, 1, 2, ..., N - 1$

where N is the length of the window and w is the window value.

The following front panel shows a triangle window for $N = 32$.



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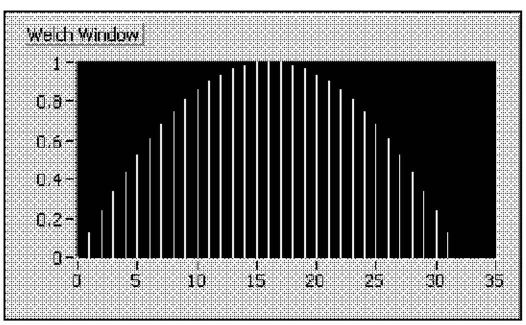
Welch

The Welch window is a continuous polynomial window. The following equation defines the Welch window.

$$w(n) = 1 - \left(\frac{n - N/2}{N/2}\right)^2 \quad \textbf{(I)}$$

for $n = 0, 1, 2, ..., N - 1$, where N is the length of the window.

The following front panel shows a Welch window with $N = 32$.



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Flat Top

The flat top window has the best amplitude accuracy of all the smoothing windows at ±0.02 dB for signals exactly between integral cycles. Because the flat top window has a wide main lobe, it has poor frequency resolution. The following equation defines the flat top window.

$$w(n) = \sum_{k=0}^{4} (-1)^k a_k \cos(k\omega) \quad \textbf{(J)}$$

where $\omega = \frac{2\pi n}{N}$

$a_0 = 0.215578948$

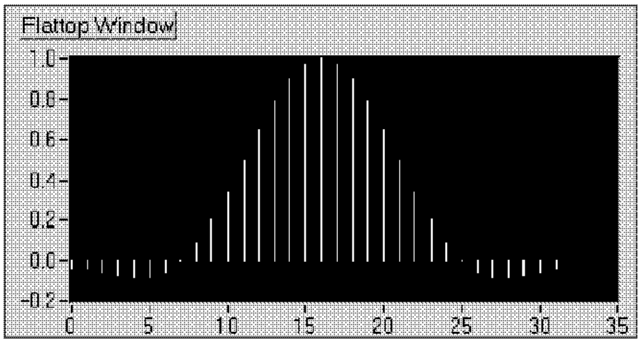
$a_1 = 0.416631580$

a_2 = 0.277263158

a_3 = 0.083578947

a_4 = 0.006947368

The following front panel shows a flat top window.



The flat top window is most useful in accurately measuring the amplitude of single frequency components with little nearby spectral energy in the signal.

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Exponential

The shape of the exponential window is that of a decaying exponential. The following equation defines the exponential window.

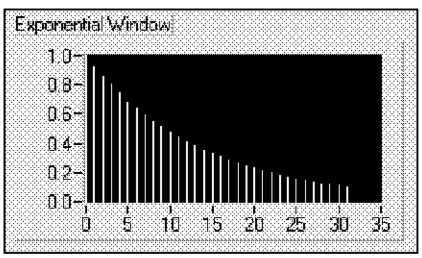
$$w(n) = e^{\left(\frac{n \ln(f)}{N-1}\right)} = f^{\left(\frac{n}{N-1}\right)} \quad \textbf{(K)}$$

for n = 0, 1, 2, ..., N – 1

where N is the length of the window, w is the window value, and f is the final value.

The initial value of the window is one and gradually decays toward zero. You can adjust the final value of the exponential window to between 0 and 1.

The following front panel shows the exponential window for N = 32, with the final value specified as 0.1.



The exponential window is useful for analyzing transient response signals whose duration is longer than the length of the window. The exponential window damps the end of the signal, ensuring that the signal fully decays by the end of the sample block. You can apply the exponential window to signals that decay exponentially, such as the response of structures with light damping that are excited by an impact, such as the impact of a hammer.

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Exact Blackman

The following equation defines the Exact Blackman window.

$$w(n) = a_0 - a_1\cos(\omega) + a_2\cos(2\omega) \quad \textbf{(L)}$$

for n = 0, 1, 2, ..., N – 1 and $\omega = \frac{2\pi n}{N}$

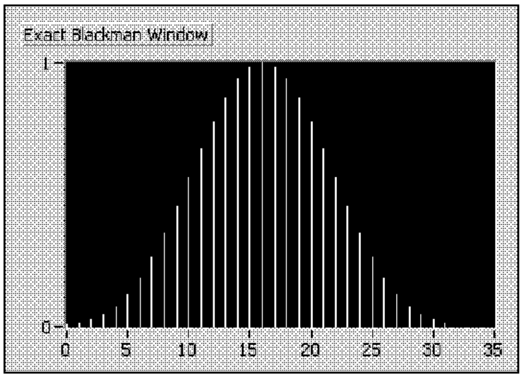
where N is the length of the window

a_0 = 7938/18608

a_1 = 9240/18608

a_2 = 1430/18608.

The following front panel shows the Exact Blackman window for N = 32.



The Exact Blackman window is useful for single tone measurement. The Exact Blackman window has a lower main lobe width and a lower maximum side lobe level than the Blackman window. However, the Blackman window has a higher side lobe roll-off rate than the Exact Blackman window.

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Blackman

The Blackman window is a modified version of the Exact Blackman window. The following equation defines the Blackman window.

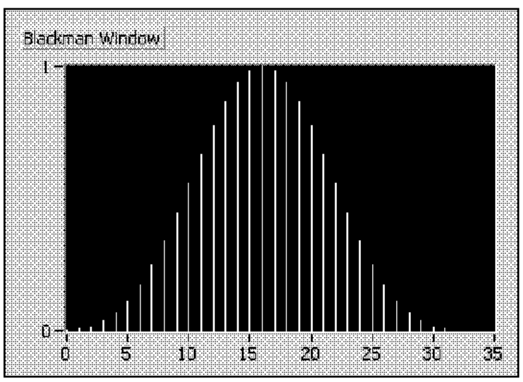
$w(n) = 0.42 - 0.50\cos(\omega) + 0.08\cos(2\omega)$ **(M)**

for $n = 0, 1, 2, \ldots, N - 1$

where N is the length of the window and

$\omega = \frac{2\pi n}{N}$.

The following front panel shows the Blackman window for $N = 32$.



The Blackman window is useful for single tone measurement because it has a low maximum side lobe level and a high side lobe roll-off rate.

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Blackman-Harris

The Blackman-Harris window is a modified version of the Exact Blackman window. The following equation defines the Blackman-Harris window.

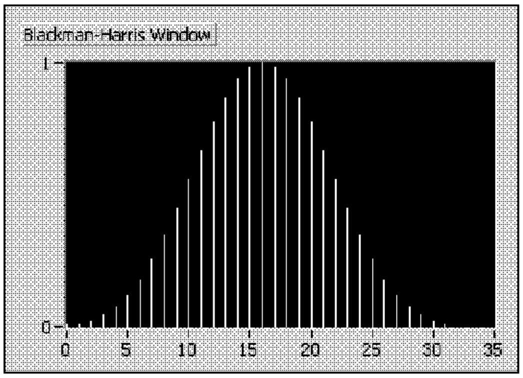
$w(n) = 0.422323 - 0.49755\cos(\omega) + 0.07922\cos(2\omega)$ **(N)**

for $n = 0, 1, 2, \ldots, N - 1$

where N is the length of the window and

$\omega = \frac{2\pi n}{N}$.

The following front panel shows the Blackman-Harris window for $N = 32$.



The Blackman-Harris window is useful for single tone measurement. The Blackman-Harris window has a wider main lobe and a lower maximum side lobe level than the Exact Blackman window.

Blackman-Nuttall

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The Blackman-Nuttall window is a modified version of the Exact Blackman window. The following equation defines the Blackman-Nuttall window.

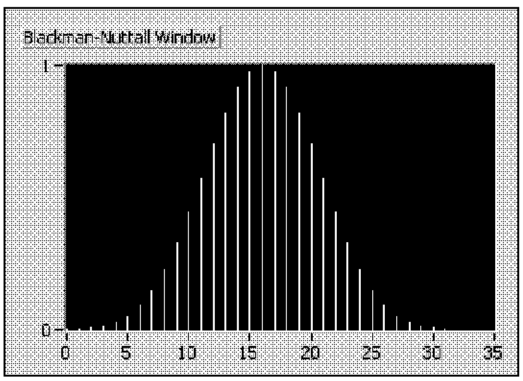
$$w(n) = 0.3635819 - 0.4891775\cos(\omega) + 0.1365995\cos(2\omega) - 0.0106411\cos(3\omega) \quad \textbf{(O)}$$

for $n = 0, 1, 2, \ldots, N - 1$

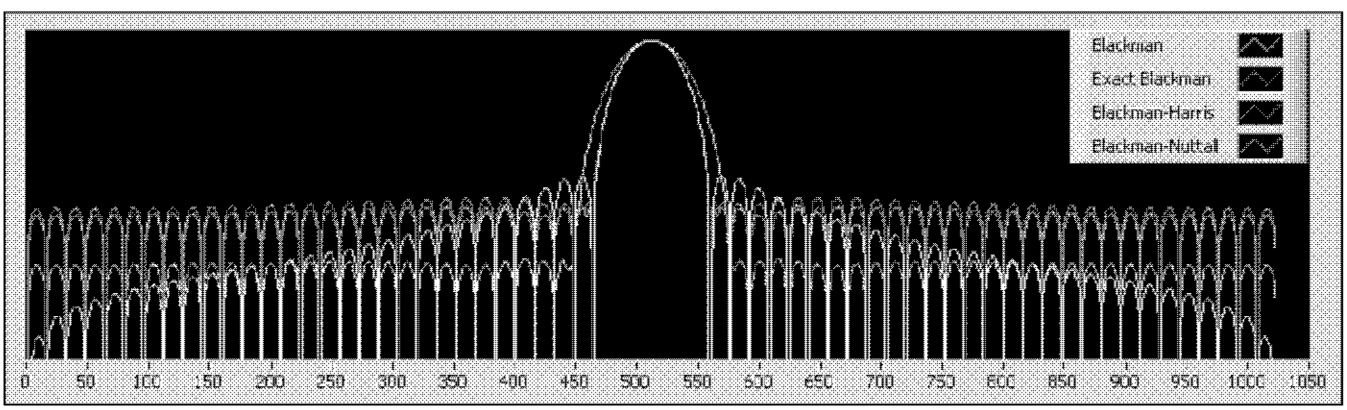
where N is the length of the window and

$$\omega = \frac{2\pi n}{N}.$$

The following front panel shows the Blackman-Nuttall window for $N = 32$.



The Blackman-Nuttall window is useful for single tone measurement. Among the Blackman, Exact Blackman, Blackman-Harris, and Blackman-Nuttall windows, the Blackman-Nuttall window has the widest main lobe and the lowest maximum side lobe level. The following front panel shows the frequency spectrums of the Blackman, Exact Blackman, Blackman-Harris, and Blackman-Nuttall windows.



General Cosine

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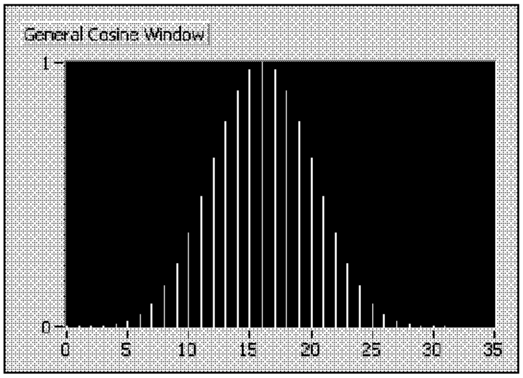
The following equation defines the general cosine window.

$$w(n) = \sum_{k=0}^{m-1} (-1)^k a_k \cos(k\omega) \quad \textbf{(P)}$$

for $n = 0, 1, 2, \ldots, N-1$ and $\omega = \frac{2\pi n}{N}$

where N is the length of the window and m is the number of coefficients that define the general cosine window.

The following front panel shows the general cosine window for $N = 32$ with coefficients 0.323215218, 0.471492057, 0.175534280, 0.028497078, and 0.001261367.



The Hanning, Hamming, Flat Top window, and Blackman windows are special cases of the general cosine window.

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Cosine Tapered

The following equation defines the cosine tapered window.

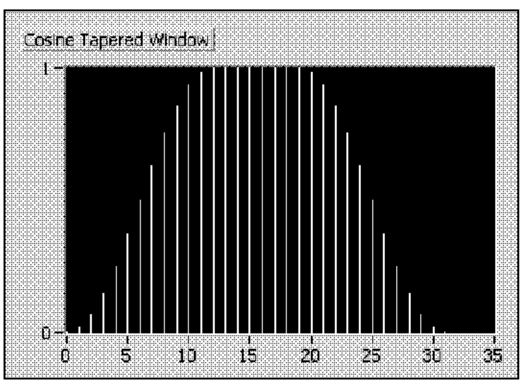
$$w(n) = \begin{bmatrix} 0.5\left(1-\cos\left(\frac{2\pi n}{2m}\right)\right) & \text{where } i = 0, 1, 2, \ldots, m-1 \\ 0.5\left(1-\cos\left(\frac{2\pi(N-n-1)}{2m}\right)\right) & \text{where } i = n-m, \ldots, n-1 \end{bmatrix} \quad \textbf{(Q)}$$

where $m = \left\lfloor \frac{Nr}{2.0} \right\rfloor$

N is the length of the window

r is the ratio of the total length of the tapered section to the whole signal length.

The following front panel shows the cosine tapered window for $N = 32$ and $r = 0.8$.



The cosine tapered window smoothly sets the data to zero at the boundaries without reducing significantly the processing gain of the windowed transform.

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Gaussian

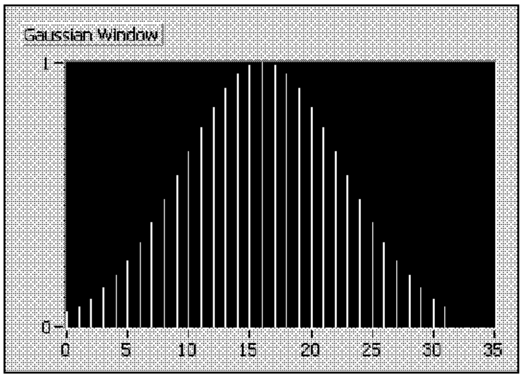
The following equation defines the Gaussian window.

$$w(n) = e^{-(n-m)^2 / 2(\sigma N)^2} \quad \textbf{(R)}$$

for $n = 0, 1, 2, \ldots, N-1$

where N is the length of the window, $m = (N-1)/2.0$, and σ is the standard deviation of the Gaussian window.

The following front panel shows the Gaussian window for $N = 32$ and $\sigma = 0.2$.



The Gaussian window is useful for time-frequency analysis because the Fourier transform and the derivative of a Gaussian window both are Gaussian functions. For example, a Short-Time Fourier Transform with a Gaussian window is the Gabor transform.

Dolph-Chebyshev

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The following equation defines the Dolph-Chebyshev window.

$$w(n) = \frac{1}{N}\left[s + 2\sum_{k=1}^{\lfloor (N-1)/2 \rfloor} C_{N-1}\left(t_0 \cos\left(\frac{k\pi}{N}\right)\right)\cos\left(\frac{2k\pi(n-(N-1)/2.0)}{N}\right)\right] \quad \textbf{(S)}$$

for $n = 0, 1, 2, \ldots, N-1$

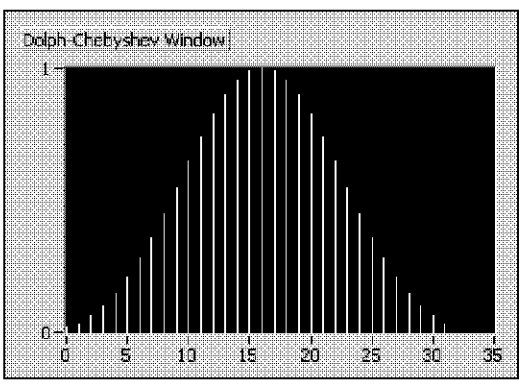
where N is the length of the window

s is the height ratio of the main lobe to the side lobe in dB

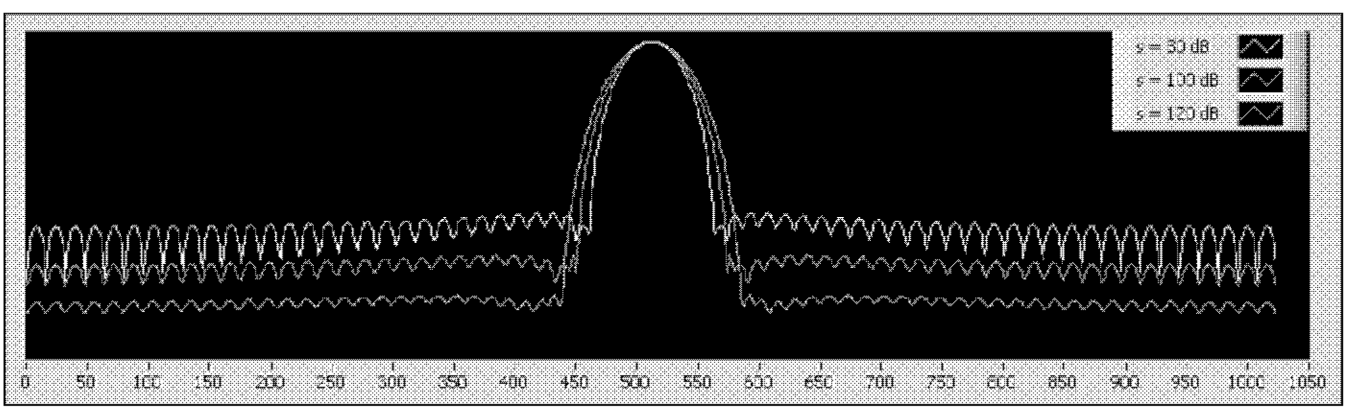
$$C_m(x) = \begin{cases} \cos\left[m\cos^{-1}(x)\right], & |x| \le 1 \\ \cosh\left[m\cosh^{-1}(x)\right], & |x| > 1 \end{cases}$$ is the m-th order Chebyshev polynomial

$$t_0 = \cosh\left[\frac{1}{N-1}\cosh^{-1}(s)\right].$$

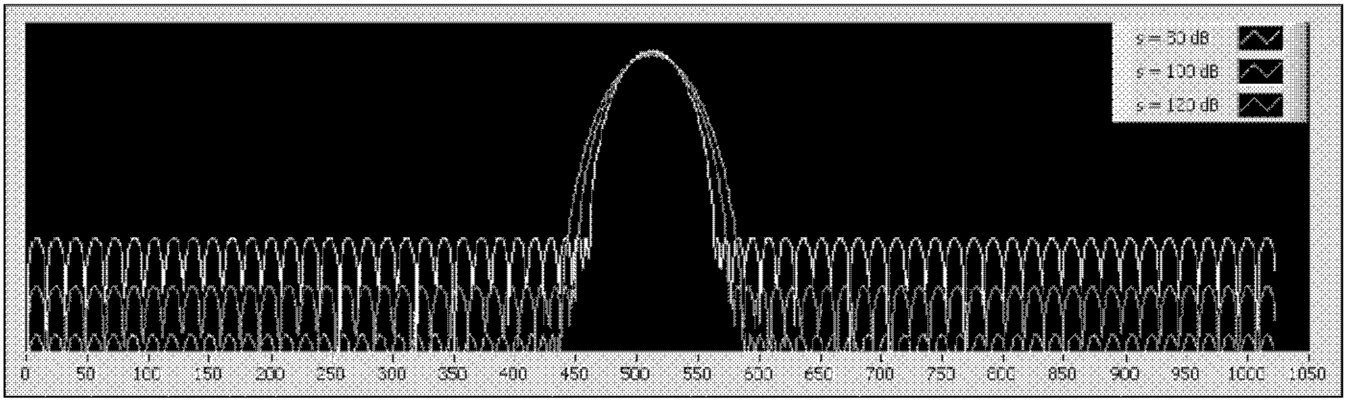
The following front panel shows the Dolph-Chebyshev window for N = 32 and lobe ratio 60.



The s parameter adjusts the side lobe level of the Dolph-Chebyshev window. The lower the side lobe level, the wider the main lobe. The following front panel shows the fast Fourier transforms of Dolph-Chebyshev windows with s = 80, 100 and 120 dB, respectively.



All side lobe levels of the symmetric Dolph-Chebyshev window have the same height, as shown in the following front panel.



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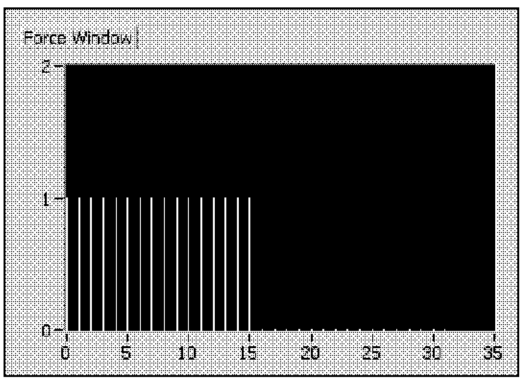
Force

The following equation defines the force window.

$$w(n) = \begin{cases} 1 & \text{if } 0 \le n \le d \\ 0 & \text{elsewhere} \end{cases} \quad \textbf{(T)}$$

$d = (0.01)(N)(duty\ cycle)$, where N is the length of the window and *duty cycle* is the percentage of time the signal remains high versus low over one period.

The following front panel shows the force window for $N = 32$ and *duty cycle* 50.



You can use a force window to analyze transients.

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