

The Generalized Discontinuous PWM Scheme for Three-Phase Voltage Source Inverters

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Abstract—This paper presents analytical techniques for the determination of the expressions for the modulation signals used in the carrier-based sinusoidal and generalized discontinuous pulse-width modulation schemes for two-level, three-phase voltage source inverters. The proposed modulation schemes are applicable to inverters generating balanced or unbalanced phase voltages. Some results presented in this paper analytically generalize the several expressions for the modulation signals already reported in the literature and new ones are set forth for generating unbalanced three-phase voltages. Confirmatory experimental and simulation results are provided to illustrate the analyses.

Index Terms—Balanced and unbalanced voltages, generalized discontinuous pulse-width modulation (PWM), space-vector modulation (SVM), three-phase converters.

I. INTRODUCTION

THREE-PHASE dc/ac voltage source inverters (VSIs) schematically shown in Fig. 1(a) are now used extensively in motor drives, active filters, unified power flow controllers in power systems, and uninterrupted power supplies to generate controllable frequency and ac voltage magnitudes using various pulsewidth modulation (PWM) strategies. Of the possible PWM methods, the carrier-based PWM is very popular due to its simplicity of implementation, known harmonic waveform characteristics, and low harmonic distortion. In the traditional sine-triangle intersection PWM (SPWM) technique, three reference modulation signals are compared with a triangular carrier signal, and the intersections define the switching instants of the controllable switching devices [1]. It has been shown that the properties of the three-phase voltage source inverters feeding three-phase star-connected loads can be improved by augmenting the modulation signals with an appropriate zero-sequence or nonsinusoidal waveform. The absence of neutral wires in star-connected three-phase loads provides this degree of freedom in modulation methodology since the voltage between the neutral of the load and the reference of the dc source V_{pn} can take any value. This zero-sequence waveform is used to alter the duty cycle of the inverter switches. Adding the same zero-sequence waveform

to each of the three reference phase voltages does not change the inverter output line-line voltage per carrier cycle average value; however, if the waveform is properly selected, one can achieve any of the following advantages: switching losses can be drastically reduced, the waveform quality may be improved, the linear modulation range can be extended, and common mode voltage of motor drives fed from the converters can also be drastically diminished. These potentials have been explored leading to investigations into and determination of various zero-sequence waveforms, resulting in a large number of published carrier-based PWM methods [2]–[6] and references therein. When the augmenting zero-sequence waveform is continuous, the continuous PWM (CPWM) scheme is produced; however, when the zero-sequence signal is discontinuous with a potential for the modulator to have phase segments clamped either to the positive or negative dc rails, the scheme is called the discontinuous PWM (DPWM). The nonsinusoidal PWM may operate as either a CPWM or a DPWM. A distinction between CPWM and DPWM has been clarified in [2]. A carrier-based PWM scheme with generalized phase-shift dependent discontinuous modulation waveforms which captures all of the known discontinuous modulation signals has been discovered and coined as the generalized discontinuous PWM (GDPWM) [2], [3].

The degree of freedom occasioned by the nonconnection of the star-point of the three-phase loads appears as the partitioning of the total time the switching devices in the two zero states are used in the space-vector modulation (SVM) methodology [3]. It has been conclusively shown that changing the ratio by which the two zero states are applied in the space-vector implementation produces the same effect on the inverter performance as the classical sine-triangle intersection technique in which the modulation signals are augmented with an appropriate zero-sequence signal [7]–[9]. It therefore follows that the same ac voltages with similar characteristics may be synthesized by either of the carrier-based or SVM techniques. Due to the simplicity of the sine-triangle algorithm, its analog or digital/microprocessor implementation is presumed to be easier to develop when compared to the direct digital (space-vector PWM) implementation, which is more intense both from computational and complexity view points [5], [8]. This advantage recommends a greater focus on carrier-based PWM modulation techniques.

The GDPWM algorithm for three-phase inverters feeding a star-connected load and based on the “magnitude tests” has been set forth and confirmed with some experimental results. The advantages it has over the traditional sine-triangle PWM

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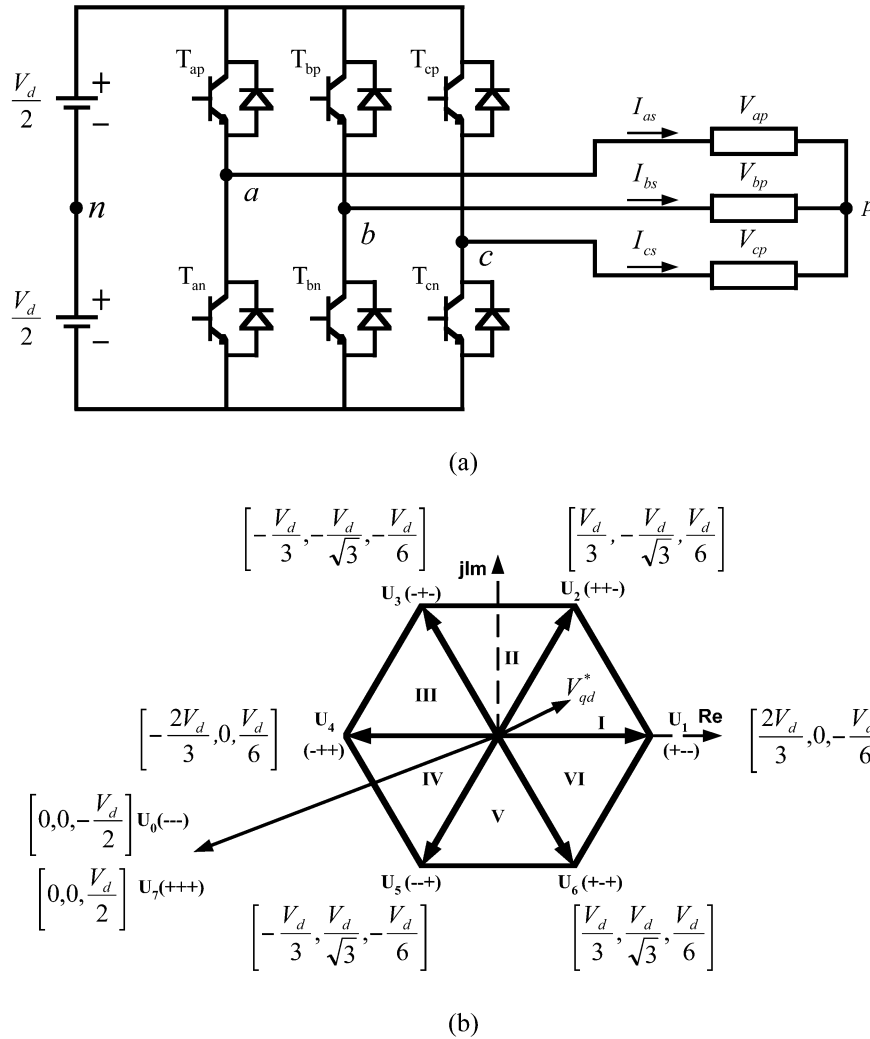


Fig. 1. Two-level PWM voltage-source inverters. (a) Three-phase with six switching devices. (b) Voltage space-vector diagram including zero-sequence voltages.

(without zero-sequence waveform) in terms of improved waveform quality, lower switching losses, extended linear modulation range, and increased voltage gain especially at high modulation index [2], [3] have been demonstrated. Inspired by the seminal contributions in [2], [3], and [9] on three-phase inverters, this paper develops a comprehensive methodology for the determination of the sinusoidal and generalized discontinuous PWM modulation signals for three-phase inverters which may be used to generate either balanced or unbalanced three-phase voltage set. In the first case, an analytical expression for the generalized/nonsinusoidal neutral voltage waveform is determined based on the average of the neutral voltages of the two active and two null vector of the space vector. This result is similar to the neutral voltage expression given in [10] and [11]. Our second significant and novel contribution is the development of the expressions for the continuous modulating signals for the switching devices of the voltage source inverter required to generate unbalanced three-phase voltages. Then, for the generation of unbalanced phase voltages, a generalized discontinuous PWM scheme in which the modulation signals of the six devices are inherently discontinuous is proposed. The variation of a parameter (α) representing the zero state partitioning within the

expression for the modulation signals yields an infinite number of modulation possibilities. It will be shown presently that when the zero partitioning parameter is either zero or unity and for any depth of modulation magnitude, each switch ceases to switch for a total of 120° per fundamental cycle. Under this condition, the switching losses and effective converter switching frequency are significantly reduced.

This paper has a strong educational value in that the modulation techniques are derived in a transparent and easy-to-understand fashion. Significantly, the methodology for arriving at the expressions for the discontinuous modulation signals for the three-phase inverter can be extended for the determination of the discontinuous modulation signals for four-leg dc/ac inverters, three-phase dc/ac current source converters, and various types of multilevel and minimalist converters. The proposed carrier-based generalized discontinuous modulation schemes are experimentally implemented with an Analog ADMC401 DSP and used to modulate a three-phase inverter feeding a three-phase induction machine. Experimental waveforms and simulation results are shown to highlight the nature of the generalized discontinuous modulation for three-phase, two-level converters generating either balanced or unbalanced three-phase voltages.

TABLE I
SWITCHING MODES OF THE THREE-PHASE VOLTAGE SOURCE INVERTER AND CORRESPONDING STATIONARY REFERENCE FRAME $qdpn$ VOLTAGES

Mode	S_{ap}	S_{bp}	S_{cp}	V_{qs}	V_{ds}	V_{os}
U_0	0	0	0	0	0	$-V_d/2$
U_5	0	0	1	$-V_d/3$	$V_d/\sqrt{3}$	$-V_d/6$
U_3	0	1	0	$-V_d/3$	$-V_d/\sqrt{3}$	$-V_d/6$
U_4	0	1	1	$-2V_d/3$	0	$V_d/6$
U_1	1	0	0	$2V_d/3$	0	$-V_d/6$
U_6	1	0	1	$V_d/3$	$V_d/\sqrt{3}$	$V_d/6$
U_2	1	1	0	$V_d/3$	$-V_d/\sqrt{3}$	$V_d/6$
U_7	1	1	1	0	0	$V_d/2$

II. GENERALIZED DISCONTINUOUS PWM

The turn-on and turnoff sequences of any of the switching transistors of the three-phase voltage source inverter shown in Fig. 1(a) are represented by an existence function which has a value of unity when it is turned on and becomes zero when it is turned off. In general, an existence function of a two-level converter is represented by S_{ij} , $i = a, b, c$, and $j = p, n$, where i represents the load phase to which the device is connected, and j signifies the top (p) and bottom (n) device of an inverter leg. Hence, S_{ap} , S_{an} , which take values of zero or unity, are, respectively, the existence functions of the top device (T_{ap}) and bottom device (T_{an}) of the inverter leg which are connected to phase "a" load [12]. In order to prevent short-circuiting the inverter dc source and thereby inviolate the Kirchoff's voltage law, T_{ip} and T_{in} cannot be turned on at the same time. Kirchoff's voltage law constrains the existence functions such that $S_{ip} + S_{in} = 1$, hence,

$$\begin{aligned} 0.5 V_d (2S_{ap} - 1) &= V_{ap} + V_{pn} \cong 0.5 V_d M_{ap} \\ 0.5 V_d (2S_{bp} - 1) &= V_{bp} + V_{pn} \cong 0.5 V_d M_{bp} \\ 0.5 V_d (2S_{cp} - 1) &= V_{cp} + V_{pn} \cong 0.5 V_d M_{cp}. \end{aligned} \quad (1)$$

In (1), V_{ap} , V_{bp} , and V_{cp} are the phase voltages of the load while the voltage of the load neutral to inverter reference is V_{pn} . The voltage equations expressed in terms of the modulation signals in (1) are facilitated by the Fourier series approximation of the existence functions, which are approximated as [12], [13]

$$\begin{aligned} S_{ap} &\cong 0.5(1 + M_{ap}) \\ S_{bp} &\cong 0.5(1 + M_{bp}) \\ S_{cp} &\cong 0.5(1 + M_{cp}) \end{aligned} \quad (2)$$

where M_{ap} , M_{bp} , and M_{cp} , which range between -1 and 1 (for the linear modulation range), are the carrier-based modulation waveforms comprising of fundamental frequency components. In general, the actual existence functions are usually

generated by comparing the high-frequency triangle waveform, which ranges between -1 and 1 with the modulation waveforms (M_{ap} , M_{bp} , M_{cp}). The equations for the modulating signals of the top devices from (1) are expressed as

$$M_{ip} = \frac{2V_{ip}}{V_d} + \frac{2V_{pn}}{V_d}, \quad i = a, b, c. \quad (3)$$

The average neutral voltage V_{pn} is determined using the method of SVM. The eight feasible switching modes for the three-phase voltage source inverter are enumerated in Table I.

The stationary reference frame qd and neutral voltages of the switching modes given in Fig. 1(b) are expressed in the complex variable form as ($a = e^{j\zeta}$, $\zeta = 120^\circ$)

$$\begin{aligned} V_{qdp} &= \frac{2}{3}(V_{ap} + a V_{bp} + a^2 V_{cp}) \\ V_{qp} &= \frac{V_d}{3}(2S_{ap} - S_{bp} - S_{cp}), \quad \sqrt{3}V_{dp} = S_{cp} - S_{bp} \\ V_{pn} &= V_{op} + \frac{V_d}{3}(S_{ap} + S_{bp} + S_{cp}) - \frac{V_d}{2} \\ V_{op} &= \frac{1}{3}(V_{ap} + V_{bp} + V_{cp}). \end{aligned} \quad (4)$$

Note that V_{op} is equal to zero only when the reference voltage set is balanced.

In the space-vector PWM method, the complex plane stationary reference frame qdp output voltage vector of the three-phase voltage source inverter is used to calculate the turn-on times of the inverter switching devices required to synthesize a reference three-phase balanced voltage set. In general, the three-phase voltages expressed in the stationary reference frame ($V_{qdpn}^* \Rightarrow V_{qp}^*$, V_{dp}^* , V_{pn}^*), situated in the appropriate sector in Fig. 1(b), are approximated by the time-average over a sampling period (converter switching period T_s) of the two adjacent active voltage inverter vectors and the two zero states U_0 and U_7 . The switching turn-on times of the two active and two null states are utilized to determine the duty cycle information to program the

TABLE II
DEVICE SWITCHING TIMES EXPRESSED IN TERMS OF REFERENCE LINE-LINE VOLTAGES

Sector	I	II	III	IV	V	VI
$V_d t_a$	V_{ab}	V_{ac}	V_{bc}	V_{ba}	V_{ca}	V_{cb}
$V_d t_b$	V_{bc}	V_{ba}	V_{ca}	V_{cb}	V_{ab}	V_{ac}
Max voltage	V_{ap}	V_{bp}	V_{bp}	V_{cp}	V_{cp}	V_{ap}
Min voltage	V_{cp}	V_{cp}	V_{ap}	V_{ap}	V_{bp}	V_{bp}

active switch gate signals [7]. When the inverter is operating in the linear modulation region, the sum of the times the two active switching modes are utilized is less than the switching period, in which case the remaining time is occupied by using the two null vectors U_0 and U_7 . If the four voltage vectors V_{qdpna} , V_{qdpnb} , V_{qdpn0} , V_{qdpn7} are called into play for times t_a , t_b , t_0 , t_7 (normalized with respect to modulator sampling time or converter switching period T_s), respectively, then the qp and dp components of the reference voltage V_{qdp}^* are approximated as

$$V_{qdp}^* = V_{qp}^* + jV_{dp}^* = V_{qdpn}t_a + V_{qdpn}t_b + V_{qdpn}t_0 + V_{qdpn}t_7$$

$$t_c = t_0 + t_7 = 1 - t_a - t_b. \quad (5)$$

When separated into real and imaginary parts, (5) gives the expressions for t_a and t_b as

$$t_a = \nabla [V_{qp}^* V_{dpb} - V_{dp}^* V_{qpb}]$$

$$t_b = \nabla [V_{dp}^* V_{qpa} - V_{qp}^* V_{dpa}]$$

$$\nabla = [V_{dpb} V_{qpa} - V_{qpb} V_{dpa}]^{-1}. \quad (6)$$

It is observed that both V_{qdp0} and V_{qdp7} do not influence the values of t_a and t_b . The times t_a and t_b expressed in terms of the instantaneous line-line reference voltages are given in Table II for the six sectors.

The neutral voltage V_{pn} averaged over the switching period T_s is given as

$$\langle V_{pn} \rangle = V_{pna}t_a + V_{pnb}t_b + V_{pn0}t_0 + V_{pn7}t_7. \quad (7)$$

It should be noted that t_c is partitioned into dwell times for the two null voltage vector— $t_c\alpha$ for U_0 and $t_c(1 - \alpha)$ for U_7 . The averaged neutral voltages for reference voltages in the voltage sectors calculated using (7) are shown in Table III.

Substituting the equations for the dwell times given in Table II into the expressions for the averaged neutral voltages in Table III, the generalized expression for the averaged neutral voltages given in terms of the instantaneous maximum and minimum voltages in all the six sectors is given in (8). Substituting (8) into (3), the expressions for the modulation signals are shown in (9)

$$V_{pn} = 0.5V_d(1 - 2\alpha) - \alpha V_{\min} + V_{\max}(\alpha - 1) \quad (8)$$

TABLE III
AVERAGE ZERO SEQUENCE VOLTAGE FOR THE SECTORS

Sectors	I, III, V	II, IV, VI
$\langle V_{pn} \rangle$	$V_d(t_b - t_a)/6 + V_d(1 - 2\alpha)t_c/2$	$V_d(t_a - t_b)/6 + V_d(1 - 2\alpha)t_c/2$

TABLE IV
EXPRESSIONS FOR THE NEUTRAL VOLTAGE FOR THE SIX SECTORS

Sector	Neutral Voltage $\langle V_{pn} \rangle$
I	$0.5V_{bp} + 0.5V_{op} + 0.5V_d(1 - 2\alpha) + 0.5(1 - 2\alpha)[V_{cp} - V_{ap}]$
II	$0.5V_{ap} + 0.5V_{op} + 0.5V_d(1 - 2\alpha) + 0.5(1 - 2\alpha)[V_{cp} - V_{bp}]$
III	$0.5V_{cp} + 0.5V_{op} + 0.5V_d(1 - 2\alpha) + 0.5(1 - 2\alpha)[V_{ap} - V_{bp}]$
IV	$0.5V_{bp} + 0.5V_{op} + 0.5V_d(1 - 2\alpha) + 0.5(1 - 2\alpha)[V_{ap} - V_{cp}]$
V	$0.5V_{ap} + 0.5V_{op} + 0.5V_d(1 - 2\alpha) + 0.5(1 - 2\alpha)[V_{bp} - V_{cp}]$
VI	$0.5V_{cp} + 0.5V_{op} + 0.5V_d(1 - 2\alpha) + 0.5(1 - 2\alpha)[V_{bp} - V_{ap}]$

$$M_{ip} = \frac{2V_{ip}}{V_d} + (1 - 2\alpha) - \frac{2\alpha V_{\min}}{V_d} + \frac{2V_{\max}(\alpha - 1)}{V_d},$$

$$i = a, b, c. \quad (9)$$

In (9) V_{\max} and V_{\min} are the instantaneous maximum and minimum magnitudes of the three reference balanced phase voltages, respectively. Table II gives the instantaneous maximum and minimum voltages for the six sectors. Equation (9), which first appeared in [9] and determined using a different method, is what has given rise to what is known as the hybrid PWM (HPWM) and later described as a nonsinusoidal PWM scheme [10], [11]. In the implementation of this modulation scheme, α can take any form (constant or time-varying) ranging between zero and unity [9]–[11]. The choice of parameter α affects the average neutral voltage between the load star-point and the inverter reference. In the conventional SVM, α is generally taken as 0.5. However, other functions may be used, which improve the performance of the modulator. The selection of α gives rise to an infinite number of carrier-based PWM modulations, some of which have been discovered [5]–[11]. The generalized discontinuous modulation signal is obtained when $\alpha = 1 - 0.5[1 + \text{Sgn}[\text{Cos } 3(\omega t + \delta)]]$, where $\text{Sgn}(X)$ is 1, 0, 0, and -1 when X is positive, zero, and negative, respectively. By

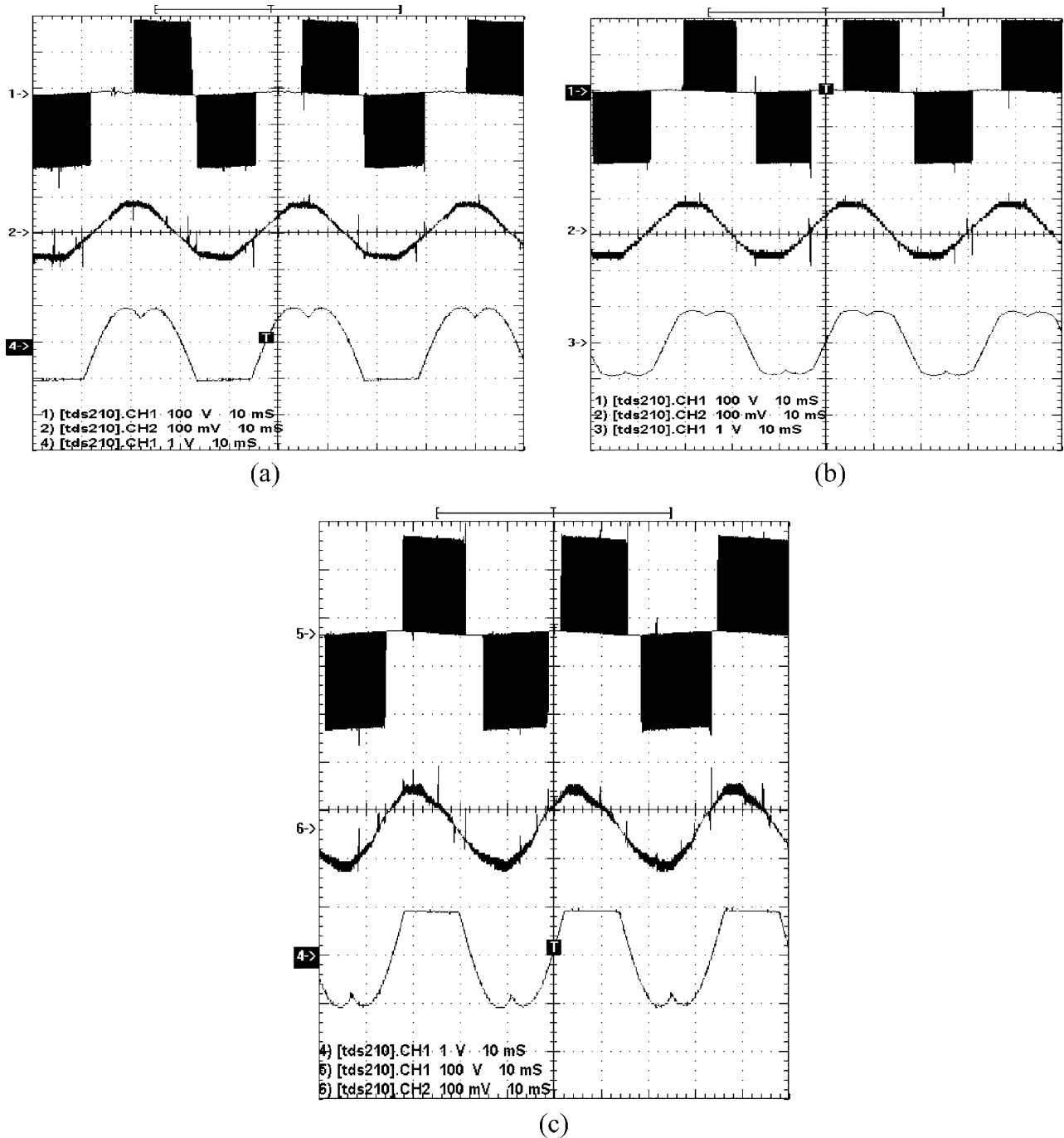


Fig. 2. Experimental results for three-phase inverter under GDPWM modulation feeding an induction motor on no-load. $V_d = 200$ V, frequency = 30 Hz, modulation magnitude = 0.9. From top: (1) motor line-line voltage, (2) motor phase current, (3) modulation signal. (a) $\alpha = 1.0$, DPWMMAX. (b) $\alpha = 0.5$, SVPWM. (c) $\alpha = 0.0$, DPWMMIN.

varying the modulation angle δ , various discontinuous modulation signals are generated.

III. CONTINUOUS PWM FOR UNBALANCED VOLTAGES

There are situations in which it is desirable to impress an unbalanced three-phase voltage set to an unbalanced three-phase load in order to ensure a balanced three-phase load current or to use unbalanced three-phase voltage set for voltage or current compensation in active filters in power distribution lines.

In general, four-leg inverters are used in such applications since the phase currents are not constrained when the load is star-connected with an accessible neutral point. However, when the impressed unbalanced three-phase voltage set is constrained such that the load currents add to zero in star-connected loads, a three-leg inverter can be used. Under such conditions, the expressions for the three modulation signals M_{ip} from (1) must be determined given the phase voltages which are not balanced in general. Since there are three linear independent equations (1) to be solved to determine expressions for three unknown modu-

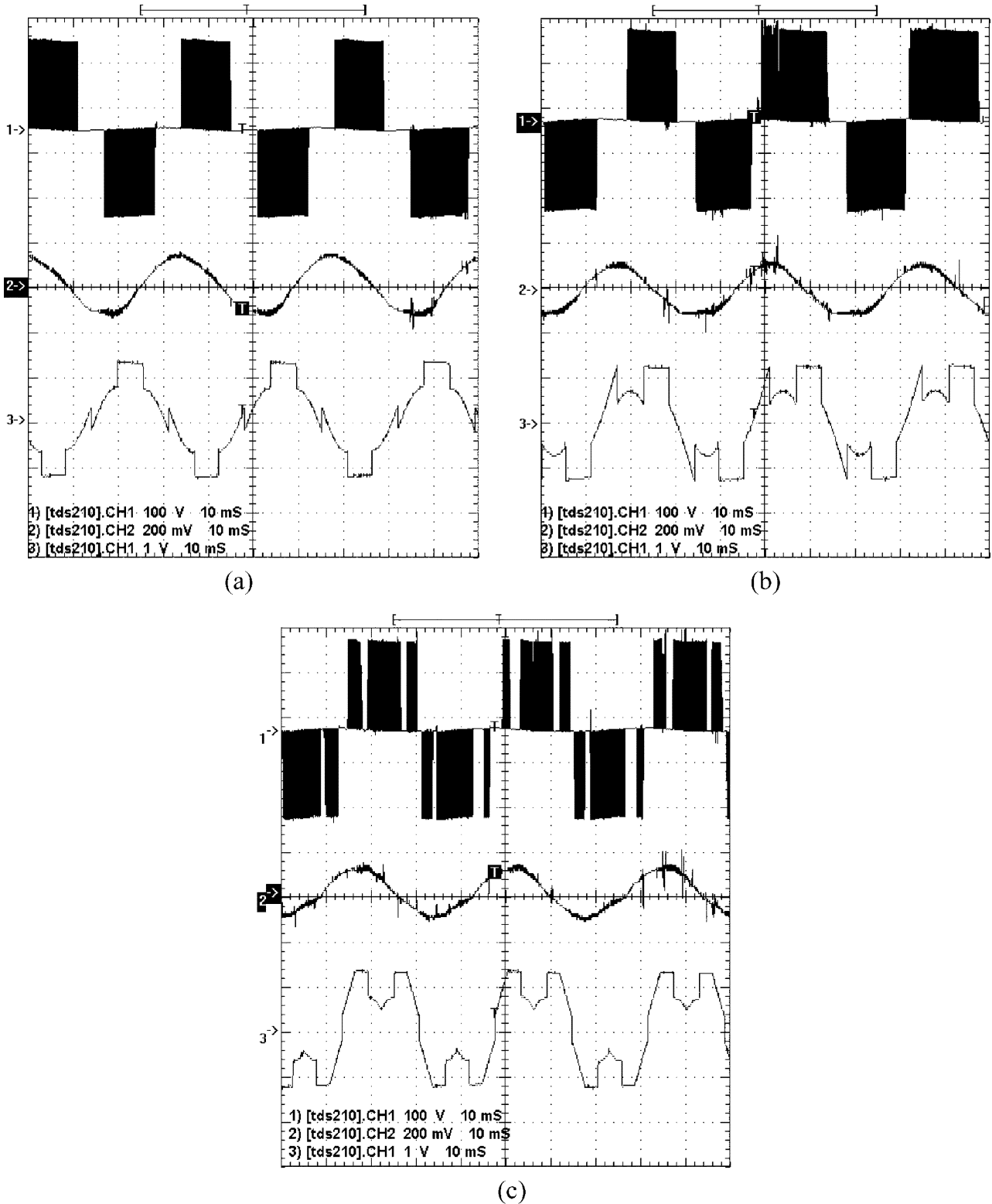


Fig. 3. Experimental results for three-phase inverter under GDPWM modulation feeding an induction motor on no-load. $V_d = 200$ V, frequency= 30 Hz, modulation magnitude = 0.9. From top: (1) motor line-line voltage, (2) motor phase current, (3) modulation signal. $\alpha = 1 - 0.5[1 + \text{Sgn}[\text{Cos } 3(\omega t + \delta)]]$. (a) $\delta = 0$, DPWM1. (b) $\delta = -30^\circ$, DPWM2. (c) $\delta = -60^\circ$, DPWM3.

lation signals and V_{pn} , these equations are underdetermined. In view of this indeterminacy, there is an infinite number of solutions that are obtained by various optimizing performance functions defined in terms of the modulation functions. For a set of linear indeterminate equations expressed as $AX = Y$, a so-

lution that minimizes the sum of squares of the variable X is obtained using the Moore–Penrose inverse [14]. The solution is given as $X = A^T[AA^T]^{-1}Y$. The solution is for the minimization of the sum of the squares of the equally weighted three modulation signals and the square of the normalized neutral voltage

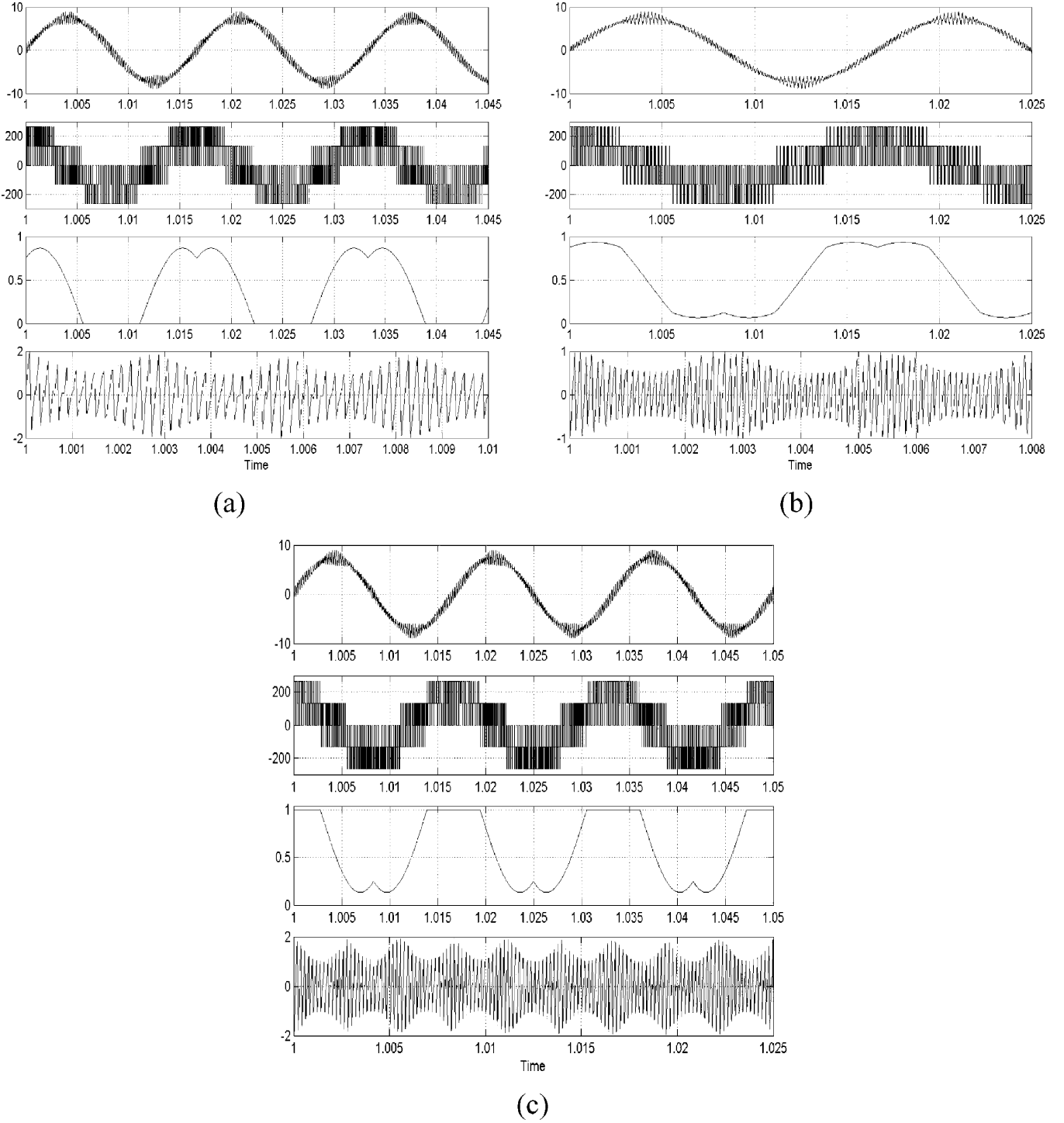


Fig. 4. Simulation results for three-phase inverter under GDPWM modulation feeding an induction motor on no-load. $V_d = 400$ V, frequency = 60 Hz, modulation magnitude = 1.0. (1) Motor phase current, (2) motor phase voltage, (3) phase “a” modulation signal, (4) electromagnetic torque. (a) $\alpha = 1.0$. (b) $\alpha = 0.5$. (c) $\alpha = 0$.

($V_{pn}^* = 2V_{pn}/V_d$). Equivalently, this is the maximization of the inverter output–input voltage gain, i.e., $M_{ap}^2 + M_{bp}^2 + M_{cp}^2 + V_{pn}^{*2}$ subject to the constraints in (1). The resulting expressions for the modulation signals are given as

$$\begin{aligned} M_{ap} &= \frac{1}{4}(3V_{ann} - V_{bnn} - V_{cnn}) \\ M_{bp} &= \frac{1}{4}(-V_{ann} + 3V_{bnn} - V_{cnn}) \\ M_{cp} &= \frac{1}{4}(-V_{ann} - V_{bnn} + 3V_{cnn}) \end{aligned}$$

$$\begin{aligned} V_{pn}^* &= \frac{1}{4}(-V_{ann} - V_{bnn} - V_{cnn}) \\ V_{ann} &= \frac{2V_{an}}{V_d}, \quad V_{bnn} = \frac{2V_{bn}}{V_d}, \quad V_{cnn} = \frac{2V_{cn}}{V_d}. \end{aligned} \quad (10)$$

IV. DISCONTINUOUS PWM FOR UNBALANCED VOLTAGES

An alternative carrier-based discontinuous modulation scheme is obtained by using the space-vector methodology to determine

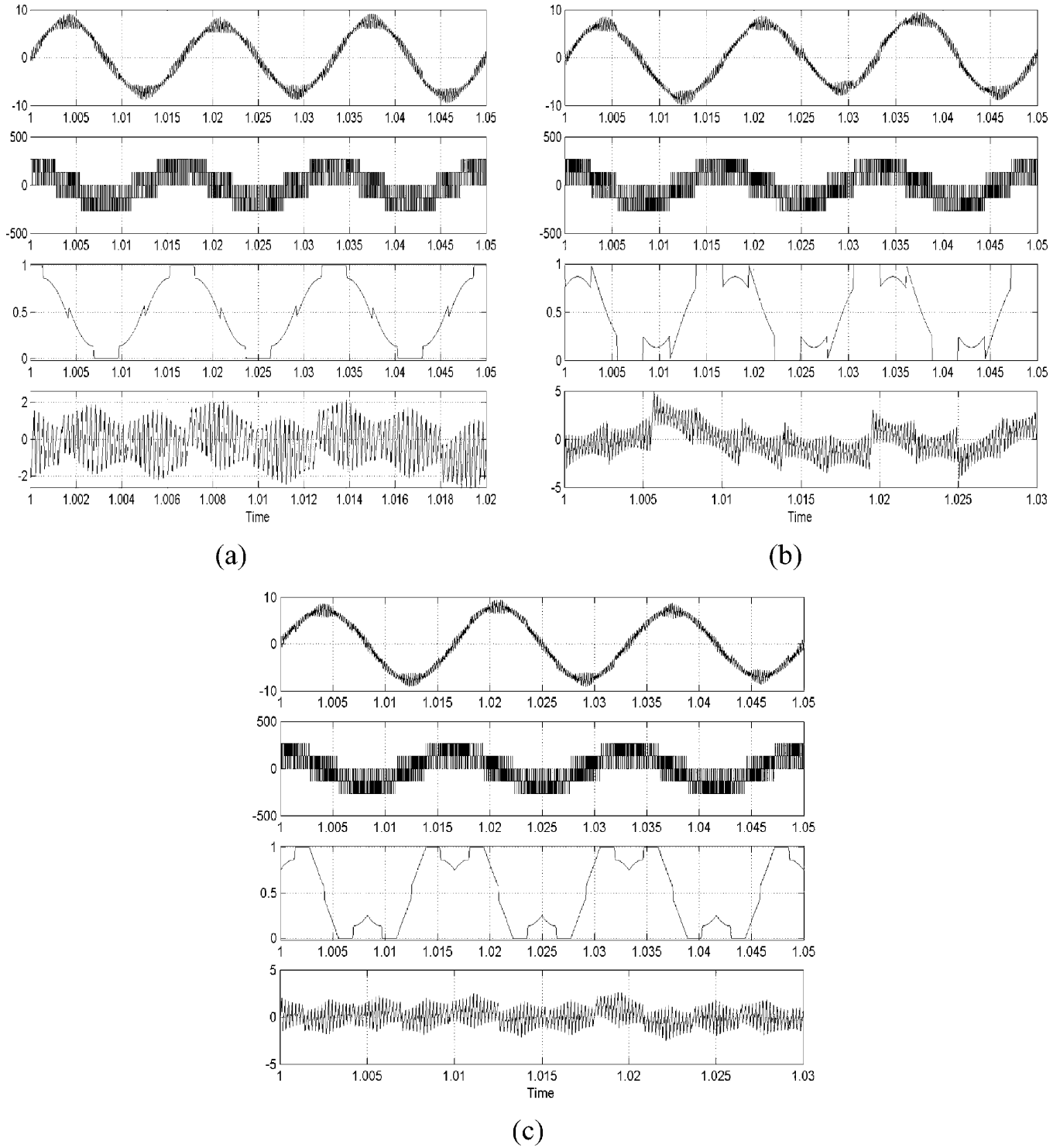


Fig. 5. Simulation results for three-phase inverter under GDPWM modulation feeding an induction motor on no-load. $V_d = 400$ V, frequency = 60 Hz, modulation magnitude = 1.0. (1) Motor phase current, (2) motor phase voltage, (3) phase “a” modulation signal, (4) electromagnetic torque. $\alpha = 1 - 0.5[1 + \text{Sgn}[\cos 3(\omega t + \delta)]]$. (a) $\delta = 0^\circ$. (b) $\delta = -30^\circ$. (c) $\delta = -60^\circ$.

the expression for V_{pn} in (3). For the unbalanced voltage set, the reference three-phase voltages mapped onto the stationary reference frame (V_{qdpn}) has in addition to the qp and dp voltage components the nonzero, zero-sequence voltage V_{op} . This reference voltage is approximated by the time-average over a sampling period (converter switching period T_s) of the two adjacent active states (V_{qdpna} , V_{qdpnb}) and the two zero states (V_{qdpn0} , V_{qdpn7}).

The expressions for the reference average neutral voltage V_{pn} are also approximated by time-averaging the neutral voltages of the two active and two null modes as expressed in (7). Using the expressions for t_a and t_b given in Table II (which also apply for the case with unbalanced three-phase reference voltages), the expressions for the averaged neutral voltage $\langle V_{pn} \rangle$ for the six sectors of the space vector are shown in Table IV.

Inspection of Table IV, assuming balanced three-phase reference voltage set, yields (8). Hence the partitioning variable α in Table IV has the same function as in (8) and (9). Hence, given the unbalanced voltage set at any instant, V_{qdp}^* in the stationary reference frame is found and the sector (Table IV) in which it is located is determined. The corresponding expression for V_{pn} is then selected from Table IV and is subsequently used in (3) to determine the modulation signals for the three top devices. When the zero-sequence component voltage (V_{op}) is removed from each of the three-phase unbalanced fundamental voltages, the resulting set of three-phase voltage signals add up to zero. Hence, if the unbalanced phase voltages are instantaneously diminished by the value of the zero-sequence voltage, the modified phase voltages can be used to generalize neutral voltage expressions given in Table IV and it is the same as in (8) with the following provisions:

$$\begin{aligned} V_{\min} &= \min(V'_{ap}, V'_{bp}, V'_{cp}) \\ V_{\max} &= \max(V'_{ap}, V'_{bp}, V'_{cp}) \\ V'_{ip} &= V_{ip} - V_{op}, \quad i = a, b, c. \end{aligned}$$

V. EXPERIMENTAL AND SIMULATION RESULTS

Illustrative experimental results are given in Figs. 2 and 3 showing the nature of the discontinuous modulation waveforms and the corresponding voltage and current waveforms when an unloaded induction machine is fed with a balanced three-phase voltages from a voltage source inverter. The various generalized discontinuous modulation signals are generated using (9). In Fig. 2(a) and (c) the switching devices are clamped to the dc bus for $1/3$ of the cycle while Fig. 2(b) is the scheme corresponding to the classical space vector when $\alpha = 0.5$. Fig. 3 shows how the change in the values of the so-called modulation angle δ in the expression for the partition quotient α influences the modulation waveforms with the significant implication that the modulation signal of a phase has at least one segment that is clamped to either the positive or negative dc rail for a total of 120° . Figs. 4 and 5 are simulation results of an inverter feeding an unloaded induction machine under various modulation schemes especially showing how the developed electromagnetic torque and phase currents are influenced by the modulation schemes. It appears that $\alpha = 0.5$ corresponding to the SVPWM in Fig. 4(b) gives the least pulsating torque, while $\delta = -30^\circ$ corresponding to DPWM2 gives the largest torque pulsation among the various schemes considered. However, GDPWM modulation schemes have reduced switching losses and reduced effective converter switching frequency when compared to SVPWM.

Fig. 6 shows the simulation results of an inverter feeding an unbalanced star-connected load with the desired objective of balancing the load current. The resistances of the R - L load used are $r_a = 0.5 \Omega$, $r_b = 0.5 \Omega$, $r_c = 0.5 \Omega$ and the corresponding load inductances are $L_a = 0.025 H$, $L_b = 0.02 H$, $L_c = 0.0125 H$, respectively. With given reference three-phase currents, the corresponding phase voltages are determined and used in (10) to realize the continuous modulation signals for the switching devices. The same balanced current set can be generated by using the generalized discontinuous modulation scheme

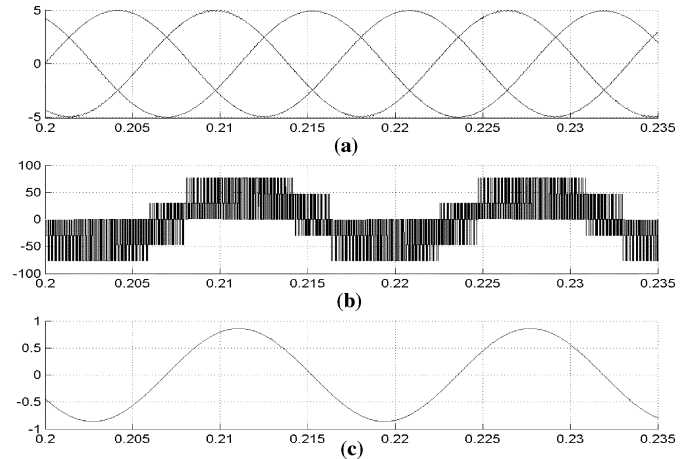


Fig. 6. Continuous PWM: balanced current in an unbalanced load. Reference peak current is 5 A. (a) Balanced three-phase actual currents. (b) Phase a voltage. (c) Modulating signal based on continuous PWM.

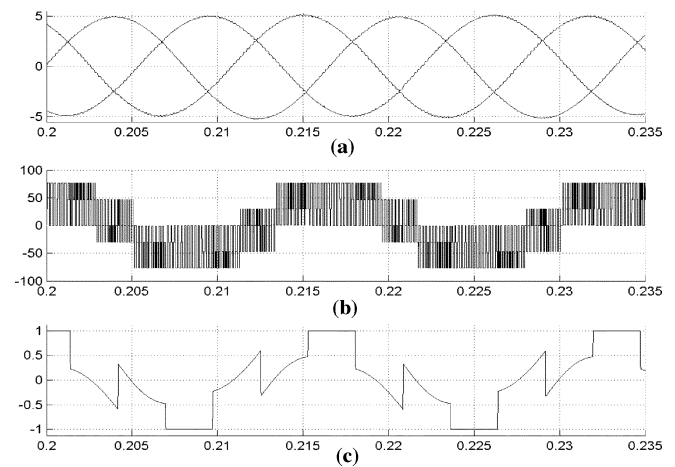


Fig. 7. Simulation results for unbalanced three-phase voltage under GDPWM. $\alpha = 1 - 0.5[1 + \text{Sgn}[\cos 3(\omega t + \delta)]]$, $\delta = 0$, DPWM1. (a) Balanced three-phase current. (b) Phase "a" voltage V_a . (c) Phase "a" discontinuous modulation signal.

based on Table IV and (3). Figs. 7–9 are generated for values of modulation angles of $\delta = 0^\circ$, -30° , and -60° , respectively. It is observed that devices are clamped to the positive or negative rail for less than 120° (unlike when the reference phase voltages are balanced) while ensuring that balanced phase currents flow through the unbalanced star-connected load with inaccessible neutral connection.

VI. CONCLUSION

This paper has presented an analytical methodology for the determination of the expressions for the modulation signals required in carrier-based generalized discontinuous PWM modulation schemes in three-phase inverters required to generate either balanced or unbalanced three-phase voltage set. We have generalized and clarified the methods for arriving at the modulation schemes of converters in the process of which several known modulation schemes have been brought under a unifying and easy-to-understand theory. This development is made possible with the extension of the classical space-vector technique

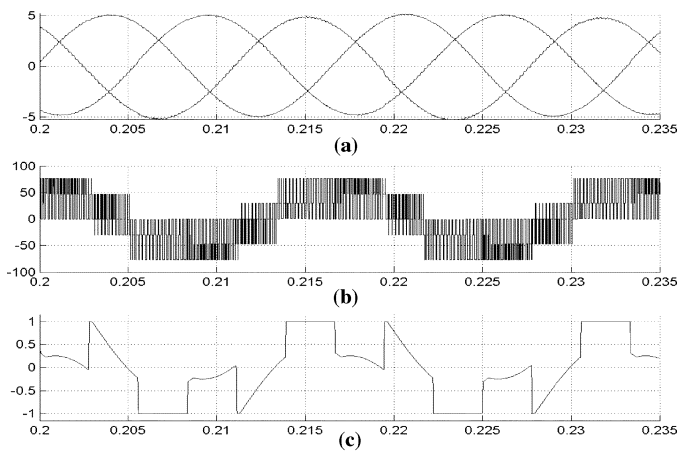


Fig. 8. Simulation results for unbalanced three-phase voltage under GDPWM. $\alpha = 1 - 0.5[1 + \text{Sgn}[\text{Cos } 3(\omega t + \delta)]]$, $\delta = -30^\circ$, DPWM2. (a) Balanced three-phase current. (b) Phase "a" voltage V_a s. (c) Phase "a" discontinuous modulation signal.

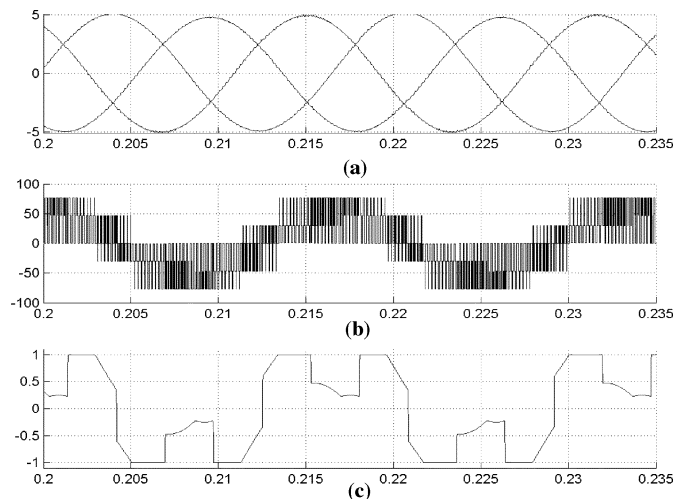


Fig. 9. Simulation results for unbalanced three-phase voltage under GDPWM. $\alpha = 1 - 0.5[1 + \text{Sgn}[\text{Cos } 3(\omega t + \delta)]]$, $\delta = -60^\circ$, DPWM3. (a) Balanced three-phase current. (b) Phase "a" voltage V_a s. (c) Phase "a" discontinuous modulation signal.

and the appropriate definition of the distribution or partition ratio of the times the zero (null) switching modes which are used in the synthesis of a reference voltage. Some confirmatory experiments results have been provided to illustrate some of the generalized carrier-based modulation signals possible. How unbalanced phase voltages can be generated to obtain balanced phase currents with star-connected three-phase loads is demonstrated by simulation results.

This paper contributes to a deeper understanding of the carrier-based modulation methodology for converters; especially extending existing known generalized carrier-based discontin-

uous PWM method [2], [3], [9]–[11] to situations where unbalanced three-phase voltage set is required to be generated. The methodology is extendable to the determination of generalized discontinuous modulation schemes of other converters such as four-leg, multilevel, and ac–ac and other variants.

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REFERENCES

- [1] A. Schonug and H. Stemmler, "Static frequency changers with subharmonic control in conjunction with reversible variable speed AC drives," *Brown Boveri Rev.*, pp. 555–577, Sept. 1964.
- [2] A. Hava, R. J. Kerkman, and T. A. Lipo, "Simple analytical and graphical methods for carrier based PWM VSI drives," *IEEE Trans. Power Electron.*, vol. 14, pp. 49–61, Jan. 1999.
- [3] —, "A high performance generalized discontinuous PWM algorithm," *IEEE Trans. Ind. Applicat.*, vol. 34, pp. 1059–1071, Sept./Oct. 1998.
- [4] B. K. Bose, *Power Electronics and Variable Frequency Drives, Technology and Applications*. New York: IEEE Press, 1997.
- [5] J. Holtz, "Pulse-width modulation for electronic power conversion," *Proc. IEEE*, vol. 8, pp. 1194–1214, Aug. 1994.
- [6] B. K. Bose, *Modern Power Electronics and AC Drives*. Upper Saddle River, NJ: Prentice-Hall, 2002.
- [7] H. Van Der Broeck, H. Skudelny, and G. Stanke, "Analysis and realization of a pulse width modulator based on voltage space vectors," *IEEE Trans. Ind. Applicat.*, vol. 24, pp. 142–150, Jan./Feb. 1988.
- [8] S. Ogasawa, H. Akagi, and A. Nabae, "A novel PWM scheme of voltage source inverter based on space vector theory," in *Proc. Eur. Power Electronics Conf.*, Aachen, Germany, Oct. 1989, pp. 1197–1202.
- [9] V. Blasko, "Analysis of a hybrid PWM based on modified space vector and triangle comparison methods," *IEEE Trans. Ind. Applicat.*, vol. 33, pp. 756–764, May/June 1997.
- [10] C. B. Jacobina, A. M. Lima, and E. Da Silva, "Digital scalar pulse-width modulation: A simple approach to introduce nonsinusoidal modulating waveforms," *IEEE Trans. Power Electron.*, vol. 16, pp. 351–359, May 2001.
- [11] K. Zhou and D. Wang, "Relationship between space vector modulation and three-phase carrier-based PWM: A comprehensive analysis," *IEEE Trans. Ind. Applicat.*, vol. 49, pp. 186–196, Jan./Feb. 2002.
- [12] P. Wood, *Switching Power Converters*. New York: Van Nostrand Reinhold, 1981.
- [13] R. Erickson and D. Maksimovic, *Fundamentals of Power Electronics*. Norwell, MA: Kluwer, 2001.
- [14] R. Stengel, *Stochastic Optimal Control*. New York: Wiley, 1986.



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