Input Filter Design for Switching Power Supplies

Michele Sclocchi
Application Engineer
National Semiconductor

The design of a switching power supply has always been considered a kind of magic and art, for all the engineers that design one for the first time. Fortunately, today the market offers different tools such as powerful online WEBENCH® Power Designer tool that help designers design and simulate switching power supply systems. New ultra-fast MOSFETs and synchronous high switching frequency PWM controllers allow the realization of highly efficient and smaller switching power supply. All these advantages can be lost if the input filter is not properly designed. An oversized input filter can unnecessarily add cost, volume and compromise the final performance of the system.

This document explains how to choose and design the optimal input filter for switching power supply applications. Starting from your design requirements (Vin, Vout, Load), WEBENCH Power Designer can be used to generate a components list for a power supply design, and provide calculated and simulated evaluation of the design. The component values, plus additional details about your power source, can then be used as input to the method and Mathcad applications described below, to design and evaluate an optimized input filter.

The input filter on a switching power supply has two primary functions. One is to prevent electromagnetic interference, generated by the switching source from reaching the power line and affecting other equipment. The second purpose of the input filter is to prevent high frequency voltage on the power line from passing through the output of the power supply.

A passive L-C filter solution has the characteristic to achieve both filtering requirements. The goal for the input filter design should be to achieve the best compromise between total performance of the filter with small size and cost.

UNDAMPED L-C FILTER

The first simple passive filter solution is the undamped L-C passive filter shown in figure (1).

Ideally a second order filter provides 12dB per octave of attenuation after the cutoff frequency f_0, it has no gain before f_0, and presents a peaking at the resonant frequency f_0.

$$f_0 := \frac{1}{2\cdot\pi\cdot\sqrt{L\cdot C}}$$ Cutoff frequency [Hz] (resonance frequency

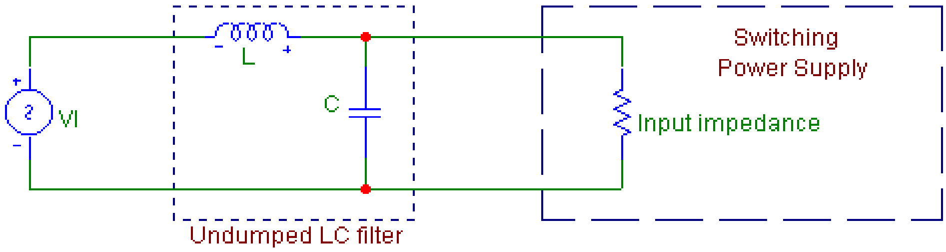


Figure 1: Undamped LC filter

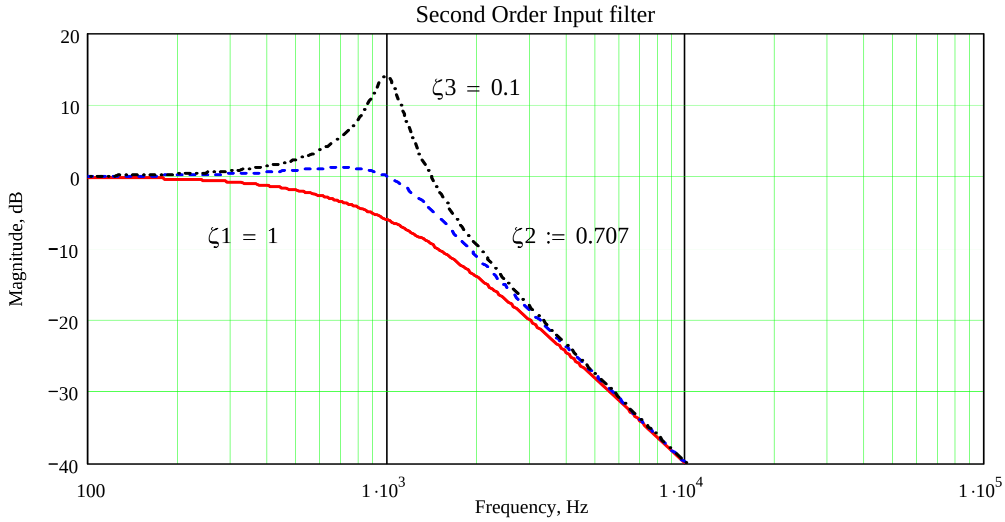


Figure 2 : Transfer Function of L-C Filter for differents damping factors

One of the critical factors involved in designing a second order filter is the attenuation characteristics at the corner frequency f_0. The gain near the cutoff frequency could be very large, and amplify the noise at that frequency.

To have a better understanding of the nature of the problem it is necessary to analyze the transfer function of the filter:

$$F_{filter1}(s) := \frac{Vout_{filter}(s)}{Vin_{filter}(s)} = \frac{1}{1 + s\cdot\frac{L}{R_{load}} + L\cdot C\cdot s^2}$$

The transfer function can be rewritten with the frequency expressed in radians:

$$F_{filter1}(\omega) := \frac{1}{1 - L \cdot C \cdot \omega^2 + j \cdot \omega \cdot \frac{L}{R_{load}}} = \frac{1}{1 + j \cdot 2 \cdot \zeta \cdot \frac{\omega}{\omega_0} - \frac{\omega^2}{\omega_0^2}}$$

$$s := j \cdot \omega$$

$$\omega_0 := \frac{1}{\sqrt{L \cdot C}}$$ Cutoff frequency in radiant

$$\zeta := \frac{L}{2 \cdot R \cdot \sqrt{L \cdot C}}$$ Damping factor (zeta)

The transfer function presents two negative poles at: $-\zeta \cdot \omega_0 + -\sqrt{\zeta - 1}$

The damping factor ζ describes the gain at the corner frequency.
For $\zeta>1$ the two poles are complex, and the imaginary part gives the peak behavior at the resonant frequency.

As the damping factor becomes smaller, the gain at the corner frequency becomes larger, the ideal limit for zero damping would be infinite gain, but the internal resistance of the real components limits the maximum gain. With a damping factor equal to one the imaginary component is null and there is no peaking. A poor damping factor on the input filter design could have other side effects on the final performance of the system. It can influence the transfer function of the feedback control loop, and cause some oscillations at the output of the power supply.
The Middlebrook's extra element theorem (paper [2]), explains that the input filter does not significantly modify the converter loop gain if the output impedance curve of the input filter is far below the input impedance curve of the converter. In other words to avoid oscillations it is important to keep the peak output impedance of the filter below the input impedance of the converter. (See figure 3)

From a design point of view, a good compromise between size of the filter and performance is obtained with a minimum damping factor of $1/\sqrt{2}$, which provides a 3 dB attenuation at the corner frequency and a favorable control over the stability of the final control system.

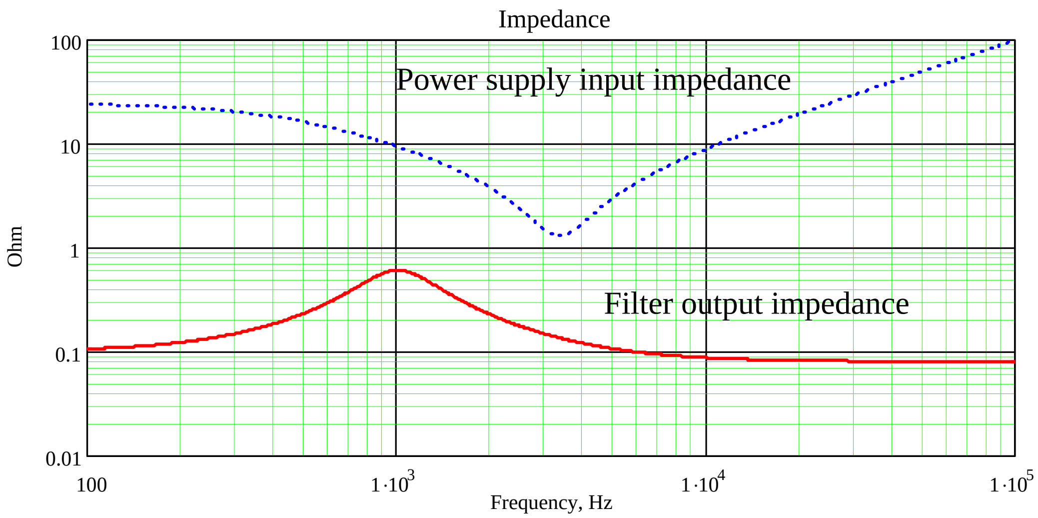


Figure 3 : Output impedance of the input filter, and input impedance of the switching power supply: the two curves should be well separated.

PARALLEL DAMPED FILTER

In most of the cases an undamped second order filter like that shown in fig. 1 does not easily meet the damping requirements, thus, a damped version is preferred:

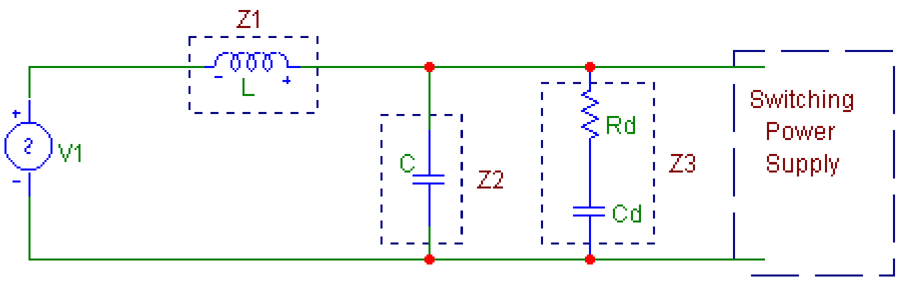


Figure 4 : Parallel damped filter

Figure 4 shows a damped filter made with a resistor Rd in series with a capacitor C_d, all connected in parallel with the filter's capacitor C_f.

The purpose of resistor Rd is to reduce the output peak impedance of the filter at the cutoff frequency. The capacitor Cd blocks the dc component of the input voltage and avoids the power dissipation on Rd.

The capacitor Cd should have lower impedance than Rd at the resonant frequency and be a bigger value than the filter capacitor in order not to affect the cutoff point of the main R-L filter.

The output impedance of the filter can be calculated from the parallel of the three block impedancesZ_1, Z_2, and Z_3:

$$Z_{filter2}(s) := \frac{1}{\frac{1}{Z_1(s)} + \frac{1}{Z_2(s)} + \frac{1}{Z_3(s)}} = \frac{s \cdot L \cdot \left(1 + R_d \cdot C_d \cdot s\right)}{s^3 \cdot L \cdot C \cdot C_d \cdot R_d + s^2 \cdot L \cdot \left(C + C_d\right) + s \cdot R_d \cdot C_d + 1}$$

The transfer function is:

$$F_{filter2}(s) := \frac{Z_{eq2.3}}{Z_1 + Z_{eq2.3}} = \frac{1 + R_d \cdot C_d \cdot s}{s^3 \cdot L \cdot C \cdot C_d \cdot R_d + s^2 \cdot L \cdot \left(C + C_d\right) + R_d \cdot C_d \cdot s + 1}$$

Where $Z_{eq2.3}$ is Z_2 parallel with Z_3.

The transfer function presents a zero and three poles, where the zero and the first pole fall close to each other at frequency $\omega \approx 1/R_dC_d$. The other two dominant poles fall at the cutoff frequency, $\omega_o = 1/\sqrt{LC}$. Without compromising the results, the first pole and the zero can be ignored and the formula can be approximated to a second order one:

$$F_{filter2}(s) := \frac{1}{1 + \frac{L \cdot \left(C + C_d\right) \cdot s^2}{\left(1 + R_d \cdot C_d \cdot s\right)} + \frac{L \cdot C \cdot C_d \cdot R_d \cdot s^3}{\left(1 + R_d \cdot C_d \cdot s\right)}} = \frac{1}{1 + \frac{L \cdot C(n+1) \cdot s^2}{R_d \cdot C \cdot n \cdot s} + \frac{L \cdot C \cdot C_d \cdot R_d \cdot s^3}{R_d \cdot C_d \cdot s}}$$

$$= \frac{1}{1 + \frac{L}{R_d} \cdot \frac{(n+1)}{n} \cdot s + L \cdot C \cdot s} \qquad \text{Where} \qquad C_d := n \cdot C$$

(for frequencies higher than $\omega \approx 1/RdCd$, the term $(1+RdCd\ s) \approx RdCd\ s$)

The approximated formula for the parallel damped filter is identical to the transfer function of the undamped filter; the only difference being the damping factor ζ is calculated with the Rd resistance.

$$\zeta_2 := \frac{n+1}{n} \cdot \frac{L}{2 \cdot R_d \cdot \sqrt{L \cdot C}}$$

It is demonstrated that for a parallel damped filter the peaking is minimized with a damping factor equal to:

$$\zeta_{2opt} := \sqrt{\frac{(2+n) \cdot (4 + 3 \cdot n)}{2 \cdot n^2 \cdot (4+n)}}$$

Combining the last two equations, the optimum damping resistance value Rd is equal to:

$$Rd_{opt} := \sqrt{\frac{L}{C}} \cdot \frac{n+1}{2 \cdot n} \cdot \sqrt{\frac{2 \cdot n^2 \cdot (4+n)}{(2+n) \cdot (4 + 3 \cdot n)}} = \sqrt{\frac{L}{C}} \qquad \text{with } n = 4$$

$$C_d := 4 \cdot C$$

With the blocking capacitor Cd equal to four times the filter capacitor C.
Figures 5 and 6 show the output impedance and the transfer function of the parallel damped filter respectively.

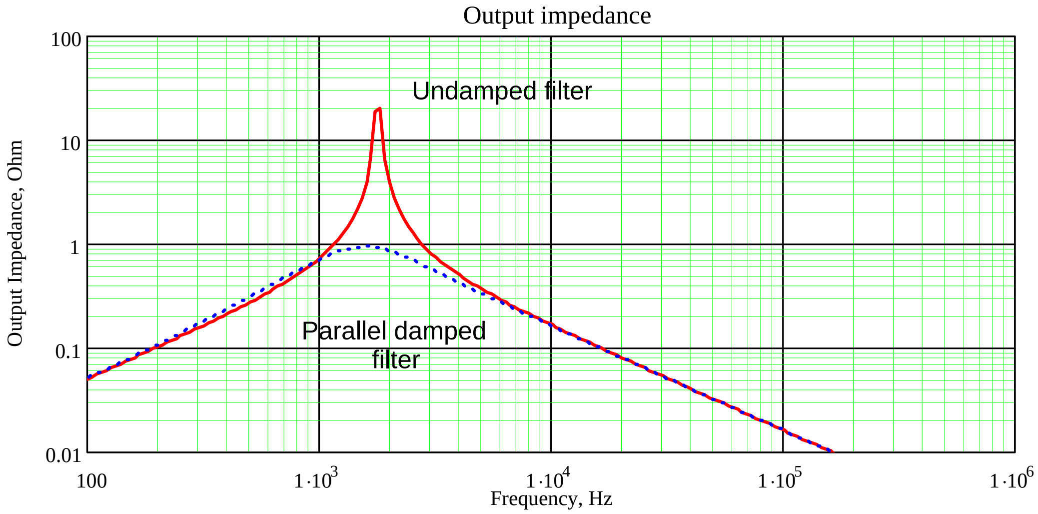


Figure 5 : Output impedance of the parallel damped filter.

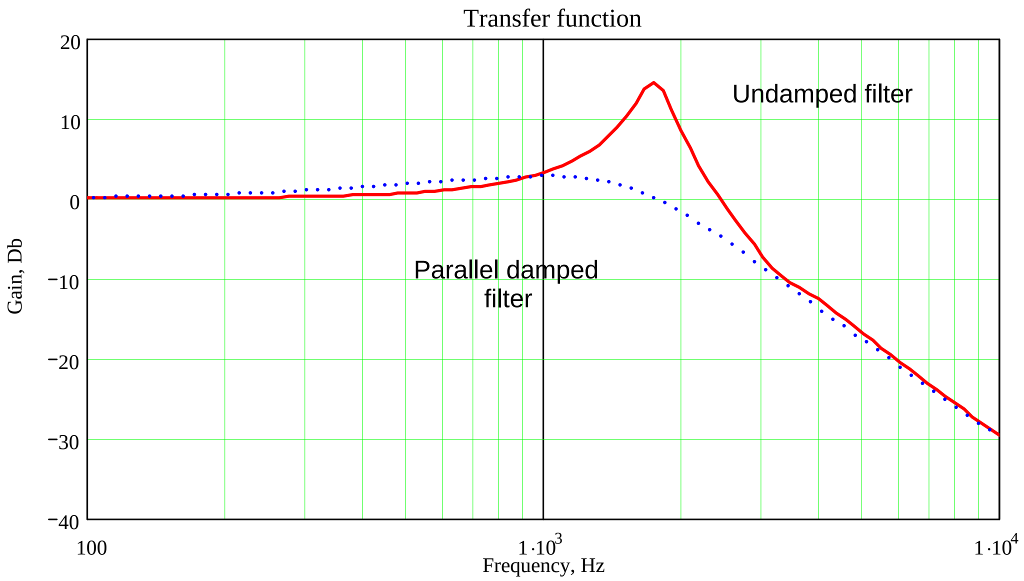


Figure 6 : Transfer function of the parallel damped filter.

SERIES DAMPED FILTER

Another way to obtain a damped filter is with a resistance Rd in series with an inductor Ld, all connected in parallel with the filter inductor L. (figure 7)

At the cutoff frequency, the resistance Rd has to be a higher value of the Ld impedance.

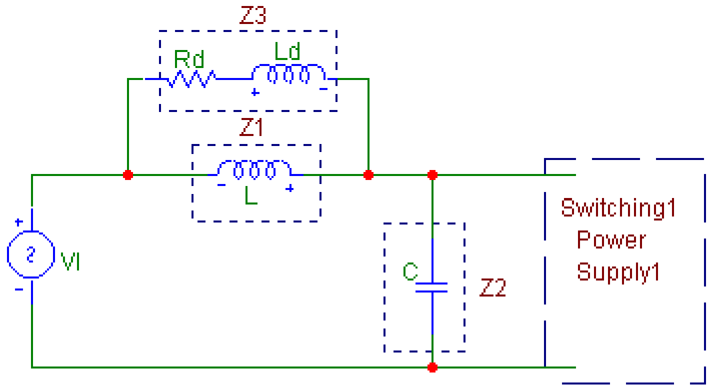


Figure 7 : Series damped filter

The output impedance and the transfer function of the filter can be calculated the same way as the parallel damped filter:

$$Z_{filter3}(s) := \frac{1}{\frac{1}{Z_1(s)} + \frac{1}{Z_2(s)} + \frac{1}{Z_3(s)}} = \frac{s \cdot L \cdot (R_d + L_d \cdot s)}{R_d + (L + L_d) \cdot s + L \cdot C \cdot R_d \cdot s^2 + L \cdot L_d \cdot C \cdot s^3} =$$

$$= \frac{s \cdot L}{1 + \frac{R_d \cdot C}{(n+1)} \cdot s + s^2 \cdot L \cdot C \cdot \frac{n}{n+1}}$$

$$F_{filter3}(s) := \frac{Z_2}{Z_2 + Z_{eq1.3}} = \frac{R_d + s \cdot (L + L_d)}{R_d + (L + L_d) \cdot s + L \cdot C \cdot R_d \cdot s^2 + L \cdot L_d \cdot C \cdot s^3} =$$

$$= \frac{1}{1 + \frac{R_d \cdot C}{(n+1)} \cdot s + s^2 \cdot L \cdot C \cdot \frac{n}{n+1}} \qquad \text{where} \quad L_d := n \cdot L$$

From the approximated transfer function of the series damped filter, the damping factor can be calculated as:

$$\zeta_3 := \frac{1}{2} \cdot \frac{R_d}{(n+1)} \cdot \frac{\sqrt{C}}{\sqrt{L}}$$

The peaking is minimized with a damping factor:

$$\zeta_{3opt} := \sqrt{\frac{n \cdot (3 + 4 \cdot n) \cdot (1 + 2 \cdot n)}{2 \cdot (1 + 4n)}}$$

The optimal damped resistance is:

$$R_d := 2\cdot\zeta_{3opt}\cdot(n+1)\cdot\frac{\sqrt{L}}{\sqrt{C}} = \frac{\sqrt{L}}{\sqrt{C}} \qquad \text{with} \qquad n := \frac{2}{15}$$

The disadvantage of this damped filter is that the high frequency attenuation is degraded. (See figure 10)

MULTIPLE SECTION FILTERS

Most of the time, a multiple section filter allows higher attenuation at high frequencies with less volume and cost, because if the number of single components is increased, it allows the use of smaller inductance and capacitance values. (Figure 8)

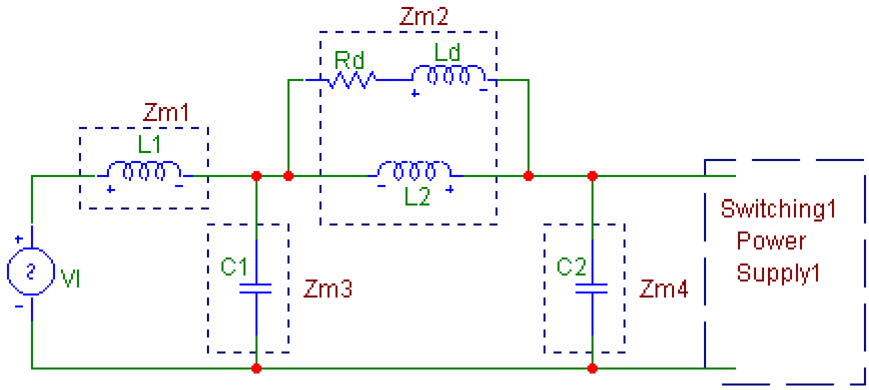


Figure 8 : Two section input filter

The output impedance and the transfer function can be calculated from the combination of each block impedance:

$$Z_{filter4}(s) := \frac{\left(\frac{Zm_1(s)\cdot Zm_2(s)}{Zm_1(s) + Zm_2(s)} + Zm_3(s)\right)\cdot Zm_4(s)}{\frac{Zm_1(s)\cdot Zm_2(s)}{Zm_1(s) + Zm_2(s)} + Zm_3(s) + Zm_4(s)} =$$

$$= \frac{s\left[(L_1+L_2)\cdot R_d + s\left[L_1\cdot(L_2+L_d)+L_2\cdot L_d\right] + s^2\cdot L_1\cdot L_2\cdot C_1\cdot R_d + s^3\cdot L_1\cdot L_2\cdot L_d\cdot C_1\right]}{R_d + s\cdot(L_2+L_d) + s^2\cdot R_d\cdot\left[(L_1+L_2)\cdot C_2 + L_1\cdot C_1\right] + s^3\cdot\left[C_2\cdot\left[L_1\cdot(L_2+L_d)+L_2\cdot L_d\right] + L_1\cdot C_1\cdot(L_2+L_d)\right] + s^4\cdot L_1\cdot L_2\cdot C_1\cdot C_2\cdot R_d + s^5\cdot L_1\cdot L_2\cdot L_d\cdot C_1\cdot C_2}$$

$$F_{filter4}(s) := \frac{Zm_4(s)}{\dfrac{Zm_1(s)\cdot Zm_2(s)}{Zm_1(s)+Zm_2(s)} + Zm_3(s) + Zm_4(s)}\cdot\frac{Zm_2(s)}{Zm_1(s)+Zm_2(s)} =$$

$$= \frac{\left(R_d + s(L_2+L_d)\right)}{R_d + s\cdot(L_2+L_d) + s^2\cdot R_d\cdot\left[(L_1+L_2)\cdot C_2 + L_1\cdot C_1\right] + s^3\cdot\left[C_2\cdot\left[L_1\cdot(L_2+L_d)+L_2\cdot L_d\right] + L_1\cdot C_1\cdot(L_2+L_d)\right] + s^4\cdot L_1\cdot L_2\cdot C_1\cdot C_2\cdot R_d + s^5\cdot L_1\cdot L_2\cdot L_d\cdot C_1\cdot C_2}$$

Figures 9 and 10 show the output impedance and the transfer function of the series damped filter compared with the undamped one.

The two-stage filter has been optimized with the following ratios:

$$L_1 := \frac{L}{2} \qquad L_2 := 7\cdot L_1 \qquad L_{d4} := \frac{L_2}{2} \qquad C_2 := 4\cdot C_1 \qquad R_{d4} := \sqrt{\frac{L_1}{4\cdot C_1}}$$

The filter provides an attenuation of 80dB with a peak filter output impedance lower than 2Ω.

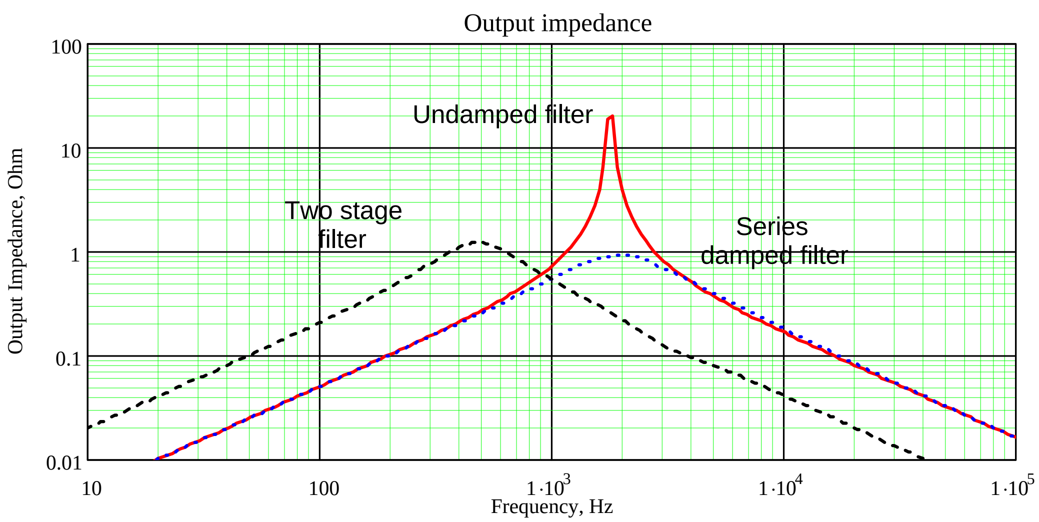


Figure 9 : Output impedance of the series damped filter, and two-stage damped filter.

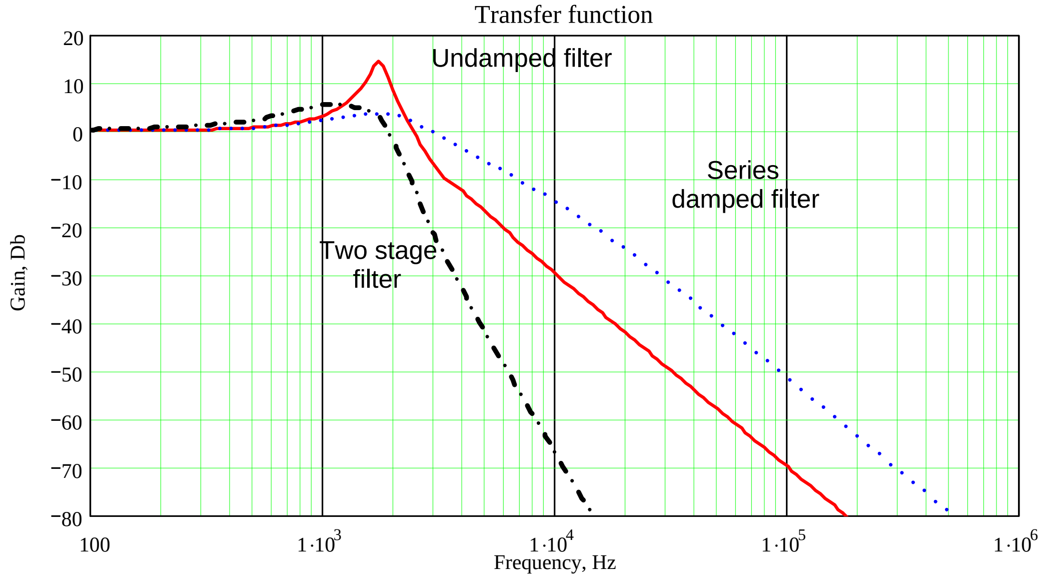


Figure 10 : Transfer function of the series damped filter, and two-stage damped filter.

The switching power supply rejects noise for frequencies below the crossover frequency of the feedback control loop and higher frequencies should be rejected from the input filter.

To be able to meet the forward filtering with a small solution, the input filter has to have the corner frequency around one decade below the bandwidth of the feedback loop.

CAPACITOR AND INDUCTOR SELECTION

Another important issue affecting the final performance of the filter is the right selection of capacitors and inductors. For high frequency attenuation, capacitors with low ESL and low ESR for ripple current capability must be selected. The most common capacitors used are the aluminum electrolytic type.

To achieve low ESR and ESL the output capacitor could be split into different smaller capacitors put in parallel to achieve the same total value.

Filter inductors should be designed to reduce parasitic capacitance as much as possible, the input and output leads should be kept as far apart as possible and single layer or banked windings are preferred.

At the National Semiconductor power web site, National.com/power, one can find all the information and tools needed to design a complete switching power supply solution. On the web site are datasheets, application notes, selection guides, and the WEBENCH® Power Designer supply design software.

REFERENCES

1. Rudolf P. Severns, Gordon E. Bloom "Modern DC to DC Switchmode Power Converter Circuits"
2. R.D. Middlebrook, "Design Techniques for Preventing Input Filter Oscillations in Switched-Mode Regulators"
3. Robert W. Erickson "Optimal Single Resistor Damping of Input Filters".
4. H. Dean Venable "Minimizing Input Filter"
5. Jim Riche "Feedback Loop Stabilization on Switching Power Supply"
6. Bruce W. Carsten "Design Techniques for the Inherent of Power Converter EMI"

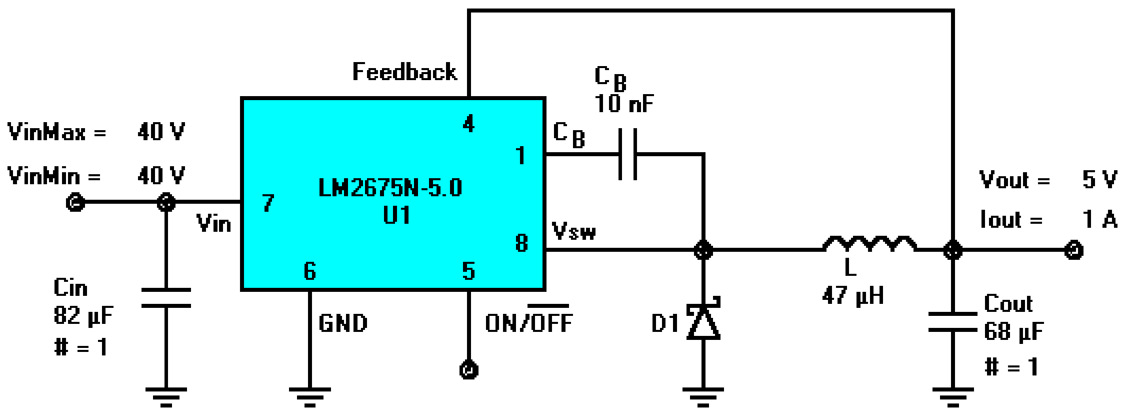
Appendix: Design Examples

Examples of filters using a basic step down simple switcher power supply

Downloads:

- Mathcad example EXE files (ZIP file)
- PTC® Mathcad website (links to PTC website)

Basic step-down simple switcher power supply:



Input parameters

Results

Maximum input voltage:

Vinput := 40V

Output current:

Iout := 1·A

Output voltage:

Vout := 5V

Output inductor:

Lo := 66µH

DC resistance:

$R_L := 0.088\Omega$

Output capacitor:

$Co := 68\mu F$

$ESR := 0.09\Omega$

Duty cycle:

$D := 0.458$

Output impedance:

$$Ro := \frac{Vout}{Iout}$$

$Ro = 5\Omega$

$i := 1 .. 2000$

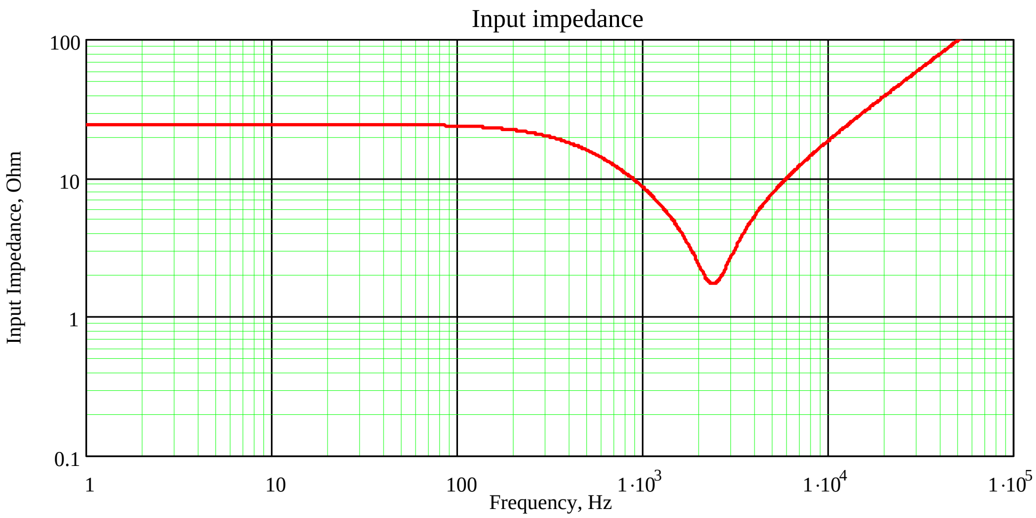
$$f_i := 100^{\frac{(i-200)}{500}}$$

$$w_i := f_i \cdot \frac{rad}{s}$$

$s_i := 2 \cdot \pi \cdot j \cdot w_i$

Input impedance of the power supply:

$$Zi_i := \left| \left(\frac{Ro + R_L}{D^2} \right) \cdot \left[\frac{1 + s_i \cdot \left[\frac{Lo}{Ro + R_L} + \left(ESR + \frac{Ro \cdot R_L}{Ro + R_L} \right) \cdot Co \right] + (s_i)^2 \cdot Lo \cdot Co \cdot \left(\frac{Ro + ESR}{Ro + R_L} \right)}{1 + s_i \cdot (Ro + ESR) \cdot Co} \right] \right|$$



Cross over frequency of the switching power supply:

$Fcross := 32kHz$

To meet the noise filtering requirements the input filter has to have the corner frequency around one decade below the bandwidth of the feedback loop of the power supply.

Cut off frequency of the input filter:

$fc := 5kHz$

Cut off frequency in radians:

$\omega c := fc \cdot 2\pi$

$\omega c = 3.142 \times 10^{4}\,\mathrm{Hz}$

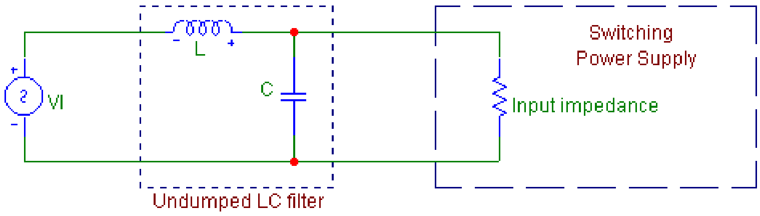
Maximum input impedance of the power supply:

$Rin := 25 \cdot \mathrm{ohm}$

Input Capacitance of the power supply:

$C := 15\mu F$

UNDAMPED LC FILTER



Inductance calculated:

$$L := \frac{1}{\omega c^{2} C}$$

$L = 0.068\,\mathrm{mH}$

Damping factor:

$$\zeta := \frac{L}{2 \cdot Rin \sqrt{L \cdot C}}$$

$\zeta = 0.042$

Inductor used:

$Lf := 33\mu H$

$Rf := 0.030 \cdot \Omega$

Capacitor used:

$Cf := 47\mu F$

$ESRci := 0.150\Omega$

Cut off frequency of the filter:

$$fc_{filter1} := \frac{1}{2 \cdot \pi \sqrt{Lf \cdot Cf}}$$

$fc_{filter1} = 4.041\,\mathrm{kHz}$

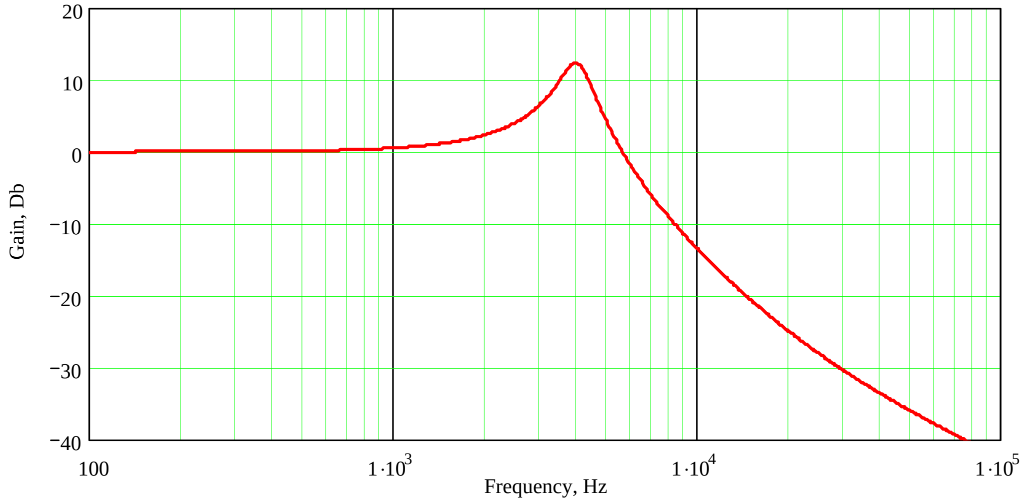
Transfer function:

$Z1_{i} := Rf + s_{i} \cdot Lf$

$$Z2_{i} := ESRci + \frac{1}{s_{i} \cdot Cf}$$

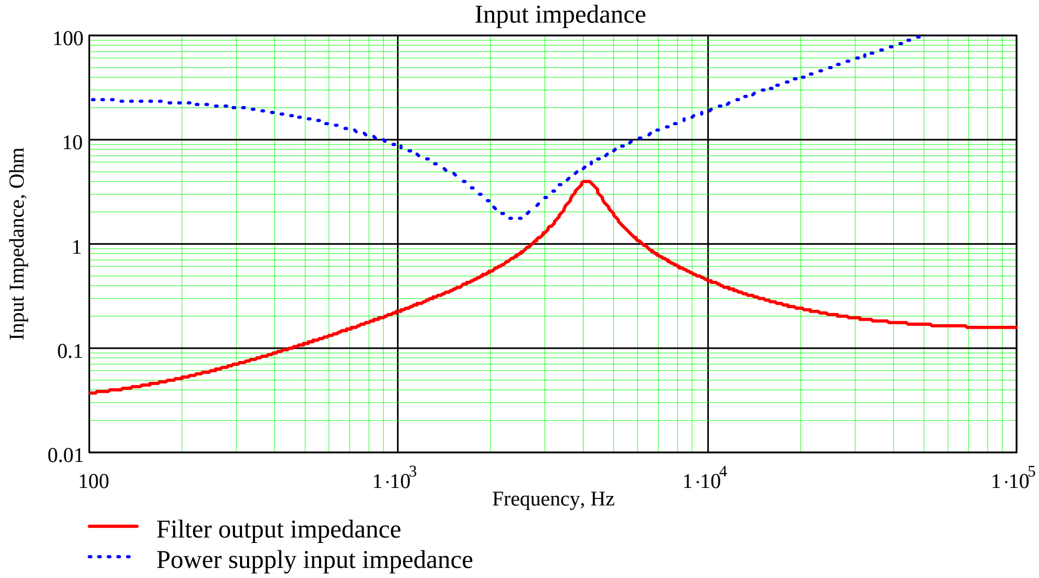
$$Z2eq_{i} := \frac{Z2_{i} \cdot Rin}{Z2_{i} + Rin}$$

$$\text{Filter}_i := 20 \cdot \log\left(\left|\frac{Z2eq_i}{Z2eq_i + Z1_i}\right|\right)$$



Filter output impedance:

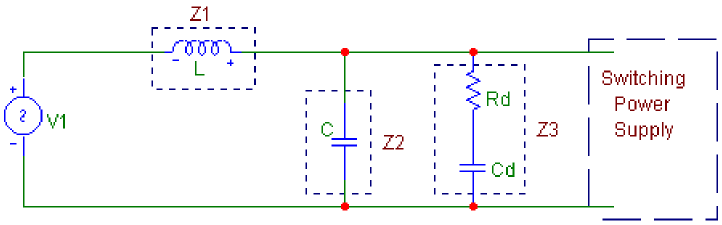
$$Zf_i := \left|\frac{Z1_i \cdot Z2_i}{Z1_i + Z2_i}\right|$$



In order to avoid oscillations it is important to keep the peak output impedance of the filter below the input impedance of the converter.
The two curves should not overlap.

PARALLEL DAMPED FILTER

In most of the cases a parallel damped filter easily meets the damping and impedance requirements.



The purpose of Rd is to reduce the output peak impedance of the filter at the cutoff frequency. The capacitor Cd blocks the DC component of the input voltage.

Damping resistance:

$$Rd := \sqrt{\frac{Lf}{Cf}}$$

$$Rd = 0.838\,\Omega$$

$$Cd := 4\cdot Cf$$

$$Cd = 188\,\mu F$$

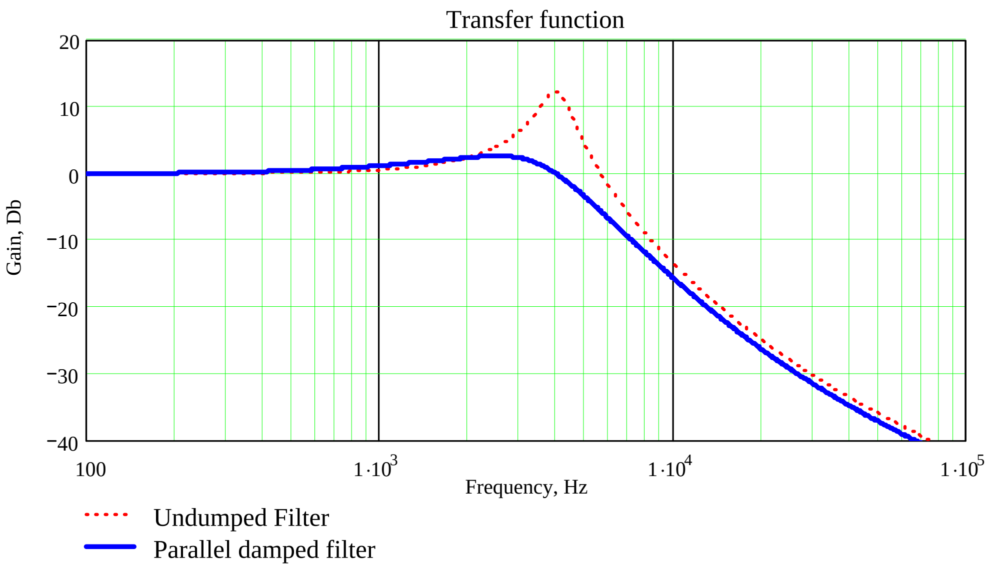
$$ESRcd := 0.200\Omega$$

$$Z3_i := \frac{1}{s_i\cdot Cd} + ESRcd + Rd$$

$$Z3eq2_i := \frac{Z2eq_i\cdot Z3_i}{Z2eq_i + Z3_i}$$

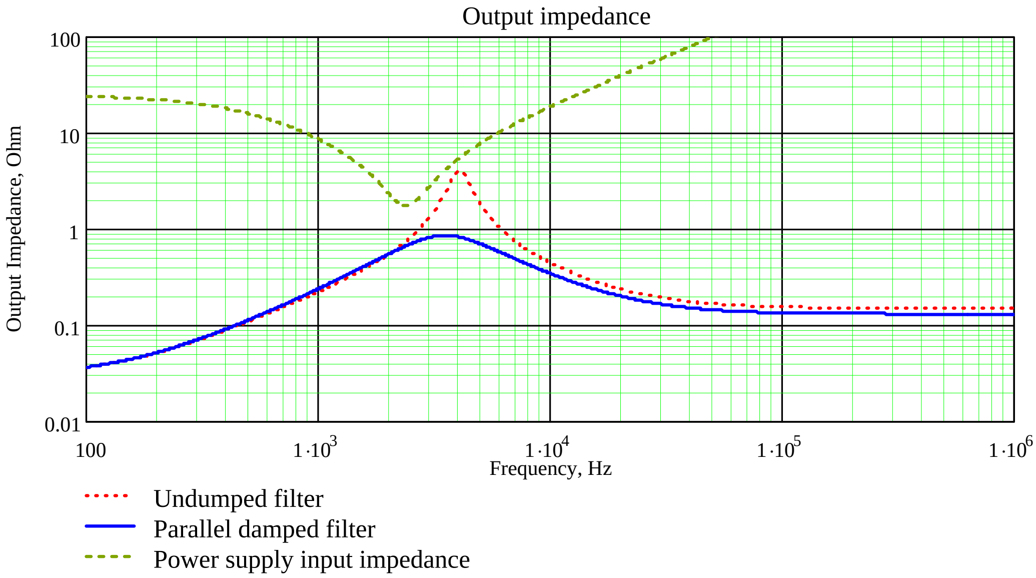
Transfer function:

$$Filter2_i := 20\cdot\log\left(\left|\frac{Z3eq2_i}{Z3eq2_i + Z1_i}\right|\right)$$

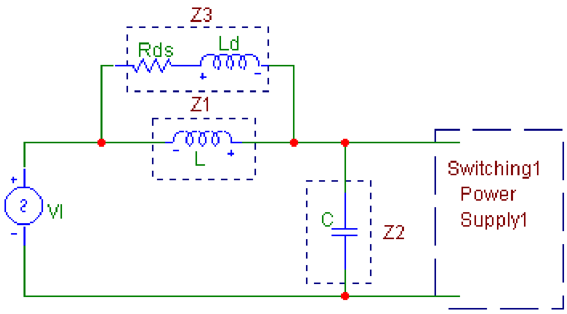


Filter output impedance:

$$Zf2_i := \left|\frac{Z1_i\cdot Z3eq2_i}{Z1_i + Z3eq2_i}\right|$$



SERIES DAMPED FILTER



$$s_r := \frac{1}{2\cdot\pi\cdot\sqrt{Lf\cdot Cf}}$$

Series inductor:

$$n_3 := \frac{2}{15}$$

$$Ld := Lf\cdot n_3$$

$$Ld = 4.4\,\mu H$$

Series damping resistance:

$$Rds := \frac{\sqrt{Lf}}{\sqrt{Cf}}$$

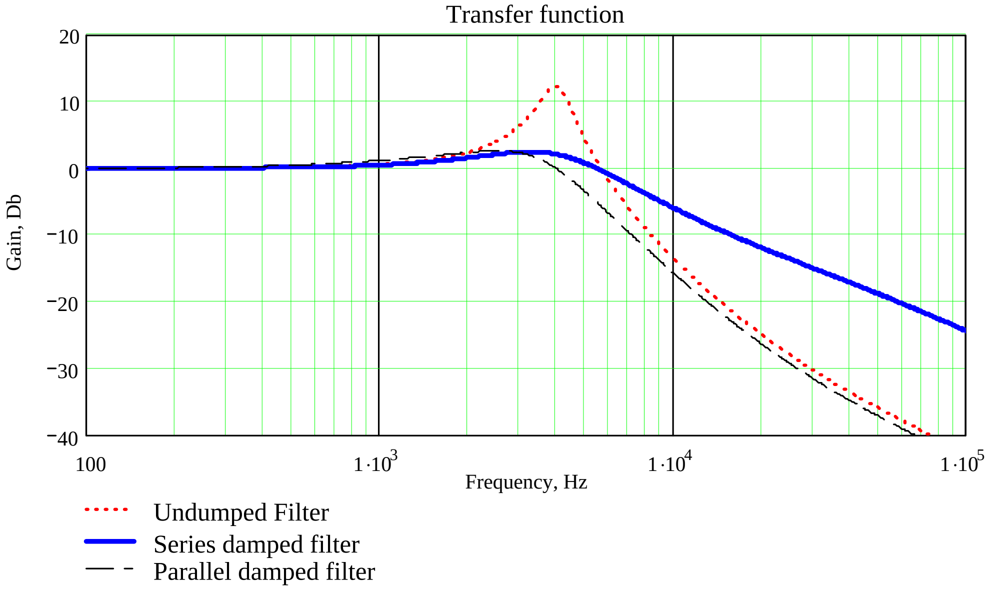
$$Rds = 0.838\,\Omega$$

$$Z3s_i := Rds + s_i\cdot Ld$$

$$Z13_i := \frac{Z1_i\cdot Z3s_i}{Z1_i + Z3s_i}$$

Transfer function:

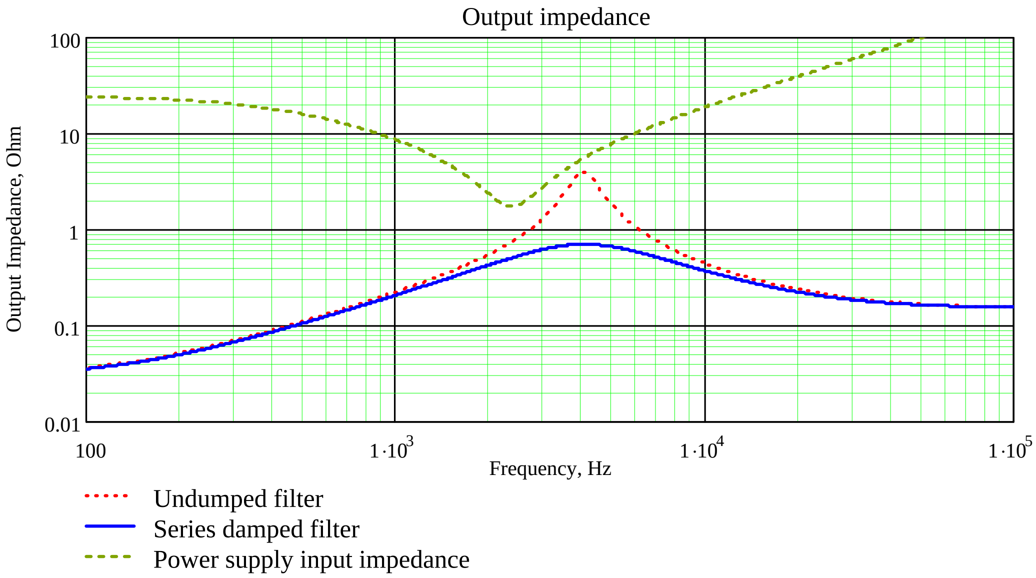
$$Filter3_i := 20 \cdot \log\left(\left|\frac{Z2_i}{Z2_i + Z13_i}\right|\right)$$



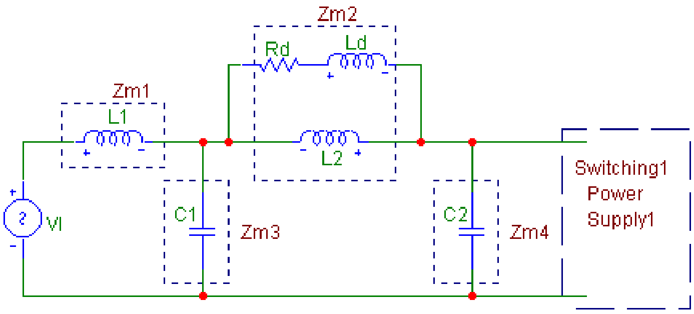
With the series damped filter the gain at high frequency is attenuated.

Filter output impedance:

$$Zf3_i := \left|\frac{Z2_i \cdot Z13_i}{Z2_i + Z13_i}\right|$$



MULTIPLE FILTER SECTIONS



First LC filter:

$$L1 := \frac{Lf}{4}$$

$$L1 = 8.25\,\mu H$$

$$RL1 := 0.1\Omega$$

$$C1 := \frac{Cf}{4}$$

$$C1 = 11.75\,\mu F$$

$$ESRc1 := 0.120\Omega$$

$$fm1 := \frac{1}{2\cdot\pi\cdot\sqrt{L1\cdot C1}}$$

$$fm1 = 16.165\,kHz$$

Second LC filter:

$$L2 := 7\cdot L1$$

$$L2 = 57.75\,\mu H$$

$$RL2 := 0.1\Omega$$

$$C2 := 4\cdot C1$$

$$C2 = 47\,\mu F$$

$$ESRc2 := 0.120\Omega$$

$$fm2 := \frac{1}{2\cdot\pi\cdot\sqrt{L2\cdot C2}}$$

$$fm2 = 3.055\,kHz$$

$$Rd4 := \sqrt{\frac{L1}{C2}}$$

$$Rd4 = 0.419\,\Omega$$

$$Ld4 := \frac{L1}{8}$$

$$Zm1_i := s_i\cdot L1 + RL1$$

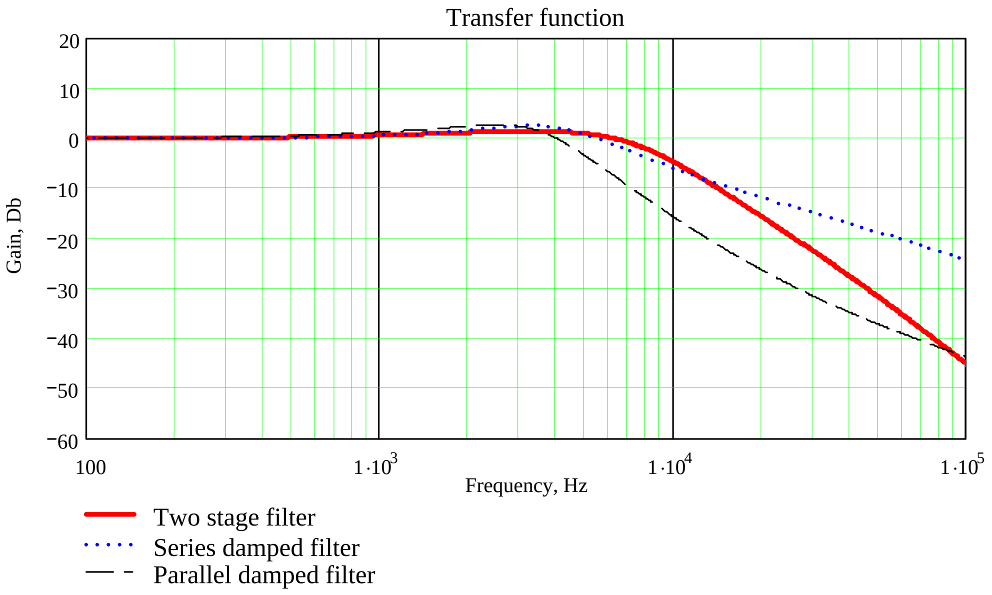
$$Zm2_i := \frac{1}{s_i\cdot C1} + ESRc1$$

$$Zm3_i := \frac{(Rd4 + s_i\cdot Ld4)\cdot(s_i\cdot L2 + RL2)}{(Rd4 + s_i\cdot Ld4) + s_i\cdot L2 + RL2}$$

$$Zm4_i := \frac{1}{s_i\cdot C2} + ESRc2$$

Transfer function:

$$Filter4_i := 20\cdot\log\left(\left|\frac{Zm4_i}{\frac{Zm1_i\,Zm2_i}{Zm1_i + Zm2_i} + Zm3_i + Zm4_i}\cdot\frac{Zm2_i}{Zm1_i + Zm2_i}\right|\right)$$



Filter output impedance:

$$Zf4_i := \left|\frac{Zm4_i\cdot\frac{Zm1_i\,Zm2_i}{Zm1_i + Zm2_i}\cdot Zm3_i}{\frac{Zm1_i\,Zm2_i}{Zm1_i + Zm2_i} + Zm3_i + Zm4_i}\right|$$