APPLICATION NOTE

Matching Differential Port Devices

Introduction

Differential signaling is the primary choice for a low power RF interconnection because it provides superior immunity to noise by offering twice the signal swing for a given supply voltage. The out of phase property of a balanced pair has several other benefits: it rejects any common mode interference signal, cancels the even order distortions such IM2, and reduces Electromagnetic Interference (EMI), EMI emission, and susceptibility.

However, this presents a challenge for RF designers to integrate differential devices together since the widely used S-parameter matching technique cannot simply be applied. Transmission line design is also different than a single-ended structure because of the differential mode of propagation.

This Application Note depicts a simple generic technique to match differential impedance devices and gives some matching circuit examples for the system shown in Figure 1 that uses the SKY65336-11 and SKY65337-11 Front-End Modules (FEMs). The results of differential matching are compared using elaborate tools such as a four-port network analyzer and ADS Electronics Design Automation (EDA) application software.

Refer to the respective Data Sheets for further information: SKY65336-11 (document #200939) and SKY65337-11 (document #200940).

Impedance Matching

Impedance matching is the practice of tuning a load impedance (Z) to the optimum impedance (Z_{opt}) of a connected device (see Figure 2). This requires three main steps:

1. Find the optimum impedance, Z_{opt}. This process is not detailed here but is usually achieved by tuning the load impedance of the circuit until the performance (i.e., the output power for a transmitter or the Noise Figure [NF] for a receiver) is met.
2. Measure the load impedance (Z) that needs to be matched (note that this is done in the same way that the optimum impedance is measured).
3. Determine the matching circuit that tunes the load (Z) to the optimum impedance, Z_{opt}.

Figure 1. Example of a Differential Interconnection Between a Zigbee®-Compliant Transceiver and an FEM
**Differential Impedance Measurement**

**Basic Two-Port Circuit**

A network analyzer is used to measure RF impedance. Each port of the two-port circuit shown in Figure 3 is alternately connected to a $Z_0$ impedance source and a $Z_0$ impedance load. The equipment measures and reports the matrix $S_{sm}$ as shown in Equation 1, which represents the single mode (sm) scattering parameters. The variables $a_1$ and $a_2$ represent the incident waves, and $b_1$ and $b_2$ represent the reflected waves.

\[
\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = S_{sm} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}
\]

Where:

\[
S_{sm} = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix}
\]

The relationship between the $S$-parameters and the impedance is given by Equation 2:

\[
Z_d = Z_0 \times \frac{(1 + s_{ii})}{(1 - s_{ii})} \quad (i \in \{1, 2\})
\]

Voltages and currents at the two nodes are calculated using Equation 3:

\[
\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \left( \begin{pmatrix} v_1^+ + v_1^- \\ v_2^+ + v_2^- \end{pmatrix} = \left( \begin{pmatrix} Z_1 \times i_1 \\ Z_2 \times i_2 \end{pmatrix} \right) \right) = \left( \begin{pmatrix} Z_1 \times (i_1^+ - i_1^-) \\ Z_2 \times (i_2^+ - i_2^-) \end{pmatrix} \right)
\]

Where $v_1^+$ and $i_1^+$ represent the forward voltage and current, respectively, and $v_1^-$ and $i_1^-$ represent the reverse voltage and current, respectively.

**Extension to Differential Circuits: Different Modes of Transmission**

For differential circuits, S-parameter theory has been extended to introduce the concept of mixed modes [Bockelman et. al. 1, 2]. Therefore, Figure 3 could represent either a two-port single ended circuit or a single-port mixed mode circuit.

The model has two modes of propagation: common mode and differential mode. For both modes, incident waves $a_c, a_d$, and reflected waves $b_c, b_d$ as well as voltages and currents $v_c, v_d, i_c, i_d$ are defined using Equations 4 through 10.

\[
a_c = \frac{1}{\sqrt{2}}(a_1 + a_2) \quad b_c = \frac{1}{\sqrt{2}}(b_1 + b_2)
\]

\[
a_d = \frac{1}{\sqrt{2}}(a_1 - a_2) \quad b_d = \frac{1}{\sqrt{2}}(b_1 - b_2)
\]

If:

\[
M = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}
\]

Then:

\[
\begin{pmatrix} b_d \\ b_c \end{pmatrix} = M \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \quad \begin{pmatrix} a_d \\ a_c \end{pmatrix} = M \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}
\]

\[
v_c = \frac{1}{2}(v_1 + v_2) \quad i_c = (i_1 + i_2)
\]

\[
v_d = \frac{1}{2}(v_1 - v_2) \quad i_d = \frac{1}{2}(i_1 - i_2)
\]

\[
\begin{pmatrix} v_d \\ v_c \end{pmatrix} = \begin{pmatrix} Z_d \times i_d \\ Z_c \times i_c \end{pmatrix}
\]
Mixed mode S-parameters are defined by:

\[
\begin{pmatrix}
 b_d \\
 b_c
\end{pmatrix} = S_{mm} \times \begin{pmatrix}
 a_d \\
 a_c
\end{pmatrix}
\]  

(11)

Where \( S_{mm} \) is defined by:

\[
S_{mm} = \begin{pmatrix}
 s_{dd} & s_{cd} \\
 s_{dc} & s_{cc}
\end{pmatrix}
\]  

(12)

And: \( s_{dd} = \) differential mode \( S \)-parameter, needed to determine the differential impedance.

\( s_{cc} = \) common mode \( S \)-parameter.

\( s_{dc}, s_{cd} = \) cross mode parameters.

Each of the cross mode parameters represent the amount of transfer from common to differential mode, and vice versa, that propagates through the circuit. For an ideal balanced circuit, note that the mixed terms \( s_{dc} \) and \( s_{cd} \) are zero.

The goal is to determine the differential mode circuit impedance \( Z_d \). For this purpose, only the differential mode propagation needs be evaluated (common mode propagation can be omitted). There is no RF common source (ac = 0) even if the DC supply can be present on each side of the balanced port.

Bockelman et. al. [2] demonstrate that mixed mode parameters can be derived from single mode parameters:

\[
(S_{mm}) = M \times S_{sm} \times M^{-1}
\]  

(13)

Therefore, \( S_{mm} \) can be redefined as:

\[
(S_{mm}) = \frac{1}{2} \times \begin{pmatrix}
 s_{11} - s_{12} - s_{21} + s_{22} & s_{11} + s_{12} - s_{21} - s_{22} \\
 s_{11} - s_{12} + s_{21} - s_{22} & s_{11} + s_{12} + s_{21} + s_{22}
\end{pmatrix}
\]  

(14)

Assuming the circuit shown in Figure 3 is perfectly balanced, \( S_{11} \) and \( S_{22} \) are equal as are \( S_{12} \) and \( S_{21} \). Therefore, from Equation 14, \( S_{dc} \) and \( S_{cd} \) are zero. Since \( S \)-parameters always normalize to a reference impedance, it is necessary to determine the value of the differential reference impedance \( Z_0 \). It is intuitively easy to determine. For a single mode operation, the reference (or characteristic) impedance is defined by:

\[
Z_0 = \frac{v_i^+}{i_i^+} = \frac{-v_i^-}{i_i^-}
\]  

(15)

Similarly, for a differential case:

\[
Z_{0d} = \frac{v_{i1}^+}{i_{i1}^+} = \frac{-v_{i2}^-}{i_{i2}^-}
\]  

(16)

Or:

\[
Z_{0d} = \frac{v_{i1}^+ - v_{i2}^+}{i_{i1}^+ - i_{i2}^+} = \frac{1}{2} (i_{i1}^+ - i_{i2}^+)
\]

If the circuit is matched, there is no return current. Therefore, according to Kirchoff's Law:

\[
i_{i1}^+ = -i_{i2}^-
\]

Therefore:

\[
Z_{0d} = \frac{v_{i1}^+ - v_{i2}^+}{i_{i1}^+ - i_{i2}^+} = \frac{v_{i1}^+}{i_{i1}^+} + \frac{v_{i2}^+}{i_{i2}^+} = \frac{v_{i1}^+}{i_{i1}^+} + \frac{v_{i2}^+}{i_{i2}^+}
\]

Or:

\[
Z_{0d} = 2 \times Z_0
\]  

(17)

Using Equation 2, differential impedance can be expressed as:

\[
Z_d = Z_{0d} \times \left(1 + s_{dd} \right)
\]

(18)

From Equations 14 and 17, \( Z_0 \) can be defined as:

\[
Z_d = 2Z_0 \times \left(\frac{2 + s_{11} - s_{12} - s_{21} + s_{22}}{2 - s_{11} + s_{12} + s_{21} - s_{22}}\right)
\]  

(19)

Using the SKY65336-11 FEM, the normalized (50 \( \Omega \) reference impedance) 2450 MHz single mode \( S \)-parameters of the differential transmit input were measured using a network analyzer and plugged into Equation 1:

\[
S_{sm} = \begin{pmatrix}
-0.083 + j \times 0.478 & -0.372 + j \times 0.199 \\
-0.361 + j \times 0.207 & -0.0714 + j \times 0.675
\end{pmatrix}
\]
Solving Equation 12 for $S_{dd}$ where the differential reference impedance, $Z_{0d}$, is 100 $\Omega$:

$$S_{dd} = -0.361 + j \times 0.374$$

And solving Equation 19 for $Z_d$:

$$Z_d = 36.6 + j \times 37.5 \, \Omega$$

The differential mode S-parameter, $S_{dd}$, was also simulated using the ADS EDA application software. The results were plotted on a Smith Chart as shown in Figure 4. The single-mode, S-parameters derived with software simulation agreed with measurements using a four-port network analyzer.

### Matching With Differential Impedance

The process of matching involves tuning the impedance (determined as described in the previous section) to a given impedance.

The use of discrete inductors and capacitors is an easy way to achieve impedance matching. If area is not a constraint, using transmission lines and stub tuner elements is a cost-competitive alternate solution. This method provides a lower loss but is not as flexible as using discrete elements since new matching means a new PCB design.

In both cases, the Smith Chart is a powerful and simple graphical tool that allows the navigation from one impedance to another, adding series and parallel matching elements.

Using a shunt element, a balanced circuit keeps its symmetry because the element is placed between the two ports. When a series component is introduced, the circuit is no longer symmetrical.

As shown in Figure 5, when the lumped elements $L$ and $C$ are added to the balanced load, $Z$ ($S_{11}$ and $S_{22}$ are equal), $S_{11}'$ and $S_{22}'$ of the matched load, $Z_{opt}$, are now different.

Based on Equations 12 and 14, mixed terms $S_{dc}$ and $S_{cd}$ of the matched circuit are no longer null.

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**Figure 4. Comparison of Simulated vs Measured Differential Input Impedance Using the SKY65336-11 FEM**

**Table:**

<table>
<thead>
<tr>
<th>S11calc freq = 2.450 GHz</th>
<th>S11sim freq = 2.450 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{11calc}$</td>
<td>$S_{11sim}$</td>
</tr>
<tr>
<td>$S_{dd} = -0.361 + j \times 0.374$</td>
<td>$S_{dd} = -0.351 + j \times 0.374$</td>
</tr>
<tr>
<td>Impedance = 36.68 + j37.5</td>
<td>Impedance = 38.15 + j38.9</td>
</tr>
</tbody>
</table>
The circuit symmetry can be easily realized by evaluating the impedance of the matched port, $Z_{\text{opt}}$, of circuit A which is given by:

$$Z_{\text{opt}} = X_C + (X_L \parallel Z) = \frac{1}{jC\omega} + (jL\omega \parallel Z)$$

(20)

Where: $X_C$ and $X_L$ represent the impedance (purely imaginary) of the ideal capacitor and inductor, respectively.

Equation 20 can be rewritten as:

$$Z_{\text{opt}} = \frac{1}{j2C\omega} + \frac{1}{j2C\omega} + (jL\omega \parallel Z)$$

This equation provides the impedance of circuit B and, therefore, demonstrates that circuits A and B are equivalent. However, only circuit B maintains the symmetry ($S_{1'1'} = S_{2'2'}$).

The impedance of the circuit shown in Figure 6 (C) can be calculated by:

$$Z_{\text{opt}} = X_L + (X_C \parallel Z) = jL\omega + (\frac{1}{jC\omega} \parallel Z)$$

Which can be rewritten as:

$$Z_{\text{opt}} = \frac{jL\omega}{2} + \frac{jL\omega}{2} + (\frac{1}{jC\omega} \parallel Z)$$

In this case, the circuit shown in Figure 6 (D) is equivalent to the circuit in Figure 6 (C).
Transmission Line Impedance Matching

When working with transmission line impedance matching, the simple transformation of the series element described above will not work. However, there is another simple technique available. As shown in Figure 7 (where \( Z_i \) and \( L_i \) denote the transmission line impedance and length, respectively), the balanced differential circuit is divided into two identical half single-ended structures. The dividing line is at the ground potential because of the circuit symmetry. The result is that the series elements of both single-ended and differential circuits are identical, although the shunt element is cut in half (including the loads, \( Z \) and \( Z_{opt} \)).

Rather than matching \( Z \) to \( Z_{opt} \), the new exercise becomes matching the half circuits, or matching \( Z/2 \) to \( Z_{opt}/2 \). Eventually, the fully differential matched circuit is derived by bringing the two half structures back together. Note that this technique can also be used with lumped elements as shown in Figure 7.

For example, a differential \( Z \) circuit and a single-ended \( Z/2 \) circuit are shown in Figure 8 (with \( Z = 38 + j \times 37 \ \Omega \) and \( Z/2 = 19 + j \times 18.5 \ \Omega \)). Note that the parameter \( E \) in the transmission line model refers to the electric length or phase shift expressed in degrees:

\[
E = 360 \frac{L}{\lambda}
\]
**Figure 9. Impedance Comparison of Differential and Half Single-Ended Matching Circuits**

The simulation results shown in Figure 9 demonstrate that the single-ended circuit is matched to $Z_{opt}/2$ and, using the transformation described above, the differential is actually matched to $Z_{opt}$. Note that the Smith Chart reference impedance of the single-ended circuit is 50 Ω and 100 Ω for the differential circuit.

**Table 1. Transmit and Receive Port Differential Impedances**

<table>
<thead>
<tr>
<th>Device Port</th>
<th>SKY65336-11</th>
<th>SKY65337-11</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_s + j(L_s \times 2 \times \pi \times f)$ (Ω)</td>
<td>$R_F (Ω) \parallel L_R (nH)$</td>
</tr>
<tr>
<td>Transmit</td>
<td>38 + j37</td>
<td>74</td>
</tr>
<tr>
<td>Receive</td>
<td>132 + j30</td>
<td>139</td>
</tr>
</tbody>
</table>

**Optimum Load Impedance ($Z_{opt}$)**

Various ZigBee-compliant transceivers are available with different RF port impedances. They also specify the optimum load impedance ($Z_{opt}$), which represents the impedance that the transceiver should see. An optimum reflection coefficient, $0.79 \angle 65°$, has been suggested [Ember, 4] (expressed in magnitude and phase) for maximum transmit power and best sensitivity. The reference impedance is 50 Ω. This corresponds to a load impedance ($Z_{opt}$) of $19.5 + j75$ Ω or $308 \Omega \parallel 5.2$ Ω.

**A Matching Example Using Lumped Elements**

In the following example, the SKY65336-11 transmit impedance of $Z = 38 + j37$ Ω is matched to the Ember transceiver $Z_{opt} = 19.5 + j75$ Ω. Both impedances are represented in the Smith Chart shown in Figure 10. The two traces (shown with arrows) show the course of impedance $Z$ by adding a shunt inductor (6.8 nH) and a series inductor (2.8 nH).
The matching network is transformed using the technique described in the previous section and the resulting differential structure is shown in Figure 11.

To verify this exercise, the two circuits are shown in Figure 12 and the simulation results are provided in Figure 13. Together, these Figures demonstrate the compliance of the transformation.

**A Matching Example Using a Combination of Lumped Elements and Transmission Lines**

Even the most compact, practical board design includes transmission lines to connect the different components together [Ember, 4]. Traces from the source and load devices to the lumped matching elements contribute to impedance matching and need to be taken into account especially for high frequencies.

For a differential circuit, the two traces have to be identical to maintain the symmetry, which means the same length, width, and distance to ground. The coupled microstrip lines structure commonly used is shown in Figure 14. The characteristic impedance of the differential mode in these transmission lines can be analyzed using the same mixed mode concept introduced earlier in this Application Note.

Assume the structure is symmetric, the differential and common modes propagate uncoupled, and the characteristic impedance of the differential mode is given by Chiariello et. al. [5]:

\[
Z_{\text{0d microstrip}} = 2Z_{\text{o microstrip}}
\]

Where \(Z_{\text{o microstrip}}\) is the odd mode impedance of the coupled microstrip line.
Figure 12. ADS-Lumped Element Matching Circuit Schematics

Figure 13. Comparison of the Matched Loads of the Two Circuits Shown in Figure 12
As a first order approximation, assume the ideal case where the distance between the two single-ended lines, \( S \), is one order of magnitude larger than their width, \( W \). As a result, unwanted coupling between them is negligible. In this case, the differential impedance is simply given by Bockelman et. al. [1]:

\[
Z_{0d\_microstrip} = 2Z_{0\_microstrip}
\]

Where \( Z_{0\_microstrip} \) is the characteristic impedance of the single microstrip line.

The transmission impedance needed to connect the different devices still needs to be determined. Usually, 50 \( \Omega \) trace impedance is used as a standard to interconnect single-ended devices. For differential devices, several standard impedances (e.g., 50, 75, 100 \( \Omega \)) are widely used. PCB stack-up constraints include minimum reliable trace width and PCB cost; both contribute to the final design.

Assume the following PCB stack-up:

- \( H = 8 \) mil
- \( \varepsilon_r = 4.3 \)
- \( \text{Cond} = 59.6\times10^6 \text{ Siemens/meter} \)
- \( t = 1.4 \) mil
- \( \tan\delta = 0.02 \)

A 75 \( \Omega \) reference impedance transmission line design has narrower traces compared to a 50 \( \Omega \) line. That allows such a design to be spaced out more to minimize the coupling, which is always difficult to estimate. The matching circuit shown in Figure 15 is composed of two identical 75 \( \Omega \) transmission lines, TL1 and TL2, and one shunt inductor, L1, that tune the load impedance \( Z/2 \) to \( Z_{opt}/2 \).

Since a 75 \( \Omega \) trace impedance is used for matching in this example, the Smith Chart reference impedance should also be 75 \( \Omega \) so that when a transmission line is added to the load, \( Z \), the impedance navigates on a constant VSWR circle shown in Figure 16.
To create a microstrip line with an impedance of 75 Ω and an electrical length of 10°, Equations 21 and 22 are used to compute the width (W = 6 mils) and the length of the microstrip line (L = 80 mils).

$$W = \frac{(5.98 \times He^{-87(\sqrt{\varepsilon_r+1.41)} - t)}}{0.8}$$  \hspace{1cm} (21)

Where $H$ is the dielectric thickness (8 mils), $\varepsilon_r$ is the dielectric relative permittivity, and $t$ is the conductor thickness.

$$L = \frac{E}{T \times f_0 \times 360}$$  \hspace{1cm} (22)

Where $f_0$ is the frequency (2.45 GHz), $E$ is the electrical length (10°), and $T$ is the propagation delay:

$$T = 85 \times e^{-15 \sqrt{(0.475 \times \varepsilon_r + 0.67 \frac{s}{mil})}}$$

The actual single-ended structure is shown in Figure 17.

Assuming there is no coupling between the two single-ended transmission lines, the differential structure as described in the previous section is derived by combining the two single-ended structures as shown in Figure 18.

Eventually, the results are compared on a Smith Chart (see Figure 19) and it can be confirmed that the differential matched load – $S(5,5)$ on the plot – is $Z_{opt} (19.5 + j75 \Omega)$. Note that this point on the chart is slightly different because of the approximation of the transmission line (negligible coupling between the two microstrip lines). However, because the matched load is very close, the approximation has no effect on performance.

**Conclusions**

Common opinion is that matching differential port devices is a difficult task compared to matching single-ended devices, and can only be achieved using advanced and expensive tools such as four-port differential network analyzers. This Application Note demonstrates that matching differential port devices does not require any additional tools and that once the different tasks have been clearly identified, it becomes a fairly simple exercise.

Determining the differential load can be achieved using two-port network analyzer measurements together with a simple calculation as expressed in Equation 19.

Matching techniques can also be performed using a Smith Chart and applying simple circuit transformations as depicted in Figures 6 and 7.
**Figure 17.** Actual Single-Ended Matching Circuit Schematic Using Distributed and Lumped Elements

**Figure 18.** Actual Differential Matching Circuit Schematic Using Distributed and Lumped Elements

**Figure 19.** Impedance of Single-Ended Ideal/Actual and Differential Matching Circuits Using a Combination of Distributed and Lumped Elements
References


