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The Sepic Converter

The most basic converter that we looked at last month is the buck converter. It is so named because it always steps down, or bucks, the input voltage. The output of the converter is given by:

\[ V_o = \frac{1}{D} V_i \]

Interchange the input and the output of the buck converter, and you get the second basic converter – the boost. The boost always steps up, hence its name. The output voltage is always higher than the input voltage, and is given by:

\[ V_o = \frac{1}{D} V_i \]

What if you have an application where you need to both step up and step down, depending on the input and output voltage? You could use two cascaded converters – a buck and a boost. Unfortunately, this requires two separate controllers and switches. It is, however, a good solution in many cases.

The buck-boost converter has the desired step up and step down functions:

\[ V_o = -\frac{D}{1-D} V_i \]

The output is inverted. A flyback converter (isolated buck-boost) requires a transformer instead of just an inductor, adding to the complexity of the development.

One converter that provides the needed input-to-output gain is the Sepic (single-ended primary inductor converter) converter. A Sepic converter is shown in Fig. 1. It has become popular in recent years in battery-powered systems that must step up or down depending upon the charge level of the battery.

Analyzing the Sepic Converter

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\[ V_o = -\frac{D}{1-D} V_i \]

When the power switch is turned off, both inductors provide current to the load capacitor. The input current is non-pulsating, a distinct advantage in running from a battery supply.
The PWM Switch Model in the Sepic Converter

The best way to analyze both the AC and DC characteristics of the Sepic converter is by using the PWM switch model, developed by Dr. Vatché Vorprérian in 1986. Some minor circuit manipulations are first needed to reveal the location of the switch model, and this is shown in Fig. 4.

First, capacitor C1 is moved to the bottom branch of the converter. Then, inductor L2 is pulled over to the left, keeping its ends connected to the same nodes of the circuit. This reveals the PWM switch model of the converter, with its active, passive, and common ports, allowing us to use well-established analysis results for this converter.

For more background on the PWM switch model, the text book “Fast Analytical Techniques for Electrical and Electronic Circuits” [1] is highly recommended.

DC Analysis of the Sepic Converter

Fig. 5 shows the equivalent circuit of the Sepic converter with the DC portion of the PWM switch model in place. The DC model is just a 1:1 transformer. We replace the inductors with short circuits, and the capacitors with open circuits for the DC analysis. You can, if you like, include any parasitic resistances in the model [2], but that’s beyond the scope of this article.

After the circuit is manipulated as shown in the figure, we can write the KVL equation around the outer loop of the converter:

\[ V_s + V_i - \frac{1}{D} \frac{dV_o}{dt} = 0 \]

Rearranging gives:

\[ V_s = \frac{1}{D - 1} \frac{V_i}{D} \frac{dV_o}{dt} \]

And the DC gain is given by:

\[ G_d = \frac{V_o}{V_i} = \frac{1}{1 - D} \frac{1}{D} \frac{1}{C_1} + \frac{1}{L_2} \]

Here we see the ability of the converter to step up or down, with a gain of 1 when D=0.5. Unlike the buck-boost and Cuk converters, the output is not inverted.

AC Analysis of the Sepic Converter

You won’t find a complete analysis of the Sepic converter anywhere in printed literature. What you will find are application notes with comments like, “the Sepic is not well-understood.” Despite the lack of documentation for the converter, engineers continue to use it when applicable.

Proper small-signal analysis of the Sepic converter is a difficult analytical task, only made practical by advanced circuit analysis techniques originally developed by Dr. David Middlebrook and continued by Vorprérian [1].

If you’re going to build a Sepic, as a minimum, you need to understand the control characteristics. Fortunately, Vorprérian’s work is now available for this converter, and you can download the complete analysis notes [2].

The simplified analysis of the Sepic converter, derived in detail in [1], ignores parasitic resistances of the inductors and capacitors, and yields the following result for the control-to-output transfer function:

\[ \frac{V_o}{V_i} = \frac{1}{1 - D} \frac{1}{D} \frac{1}{C_1} + \frac{1}{L_2} (v_i + C_i) \]

Where

\[ \omega_2 = \sqrt{\frac{1}{L_2} \frac{D}{D} \left( \frac{1}{D} + \frac{1}{C_1} \right)} \]

The small-signal AC sources are included in the switch model, and we can either solve the analysis by hand, or use PSpice to plot desired transfer functions. The hand analysis is crucial for symbolic expressions and design equations.
There are several possibilities. First, the dynamic and step load requirements on the system may be very benign, with no reason to design a loop with high bandwidth. This allows the loop gain to be reduced below 0 dB before the extreme phase delay of the second resonance.

Secondly, in many practical cases, the parasitic resistances of the circuit move the RHP zeros to the left half plane, greatly reducing the phase delay. This can also be done with the addition of damping networks to the power stage, a topic beyond the scope of this article.

Thirdly, some engineers do not build a proper Sepic. In some application notes, the two inductors are wound on a single toroidal core, which provides almost unity coupling between the two. In this case, the circuit no longer works as a proper Sepic. Don’t fall into this design trap - the circuit will be far from optimum.

Additional Reading

As you can see from these expressions, the “simplified” analysis is anything but simple. Including the parasitic resistances greatly complicates the analysis, but may be necessary for worst-case analysis of the Sepic converter. The analysis of this converter involves the use of the powerful extra element theorem, and Vorpérian’s book on circuit analysis techniques.

In addition to the inevitable fourth-order denominator of the Sepic, the most important features to note in the control transfer function are the terms in the numerator. The first term is a single right-half-plane (RHP) zero. Right-half-plane zeros are a result of converters where the response to an increased duty cycle is to initially decrease the output voltage.

When the power switch is turned on, the first inductor is disconnected from the load, and this directly gives rise to the first-order RHP zero. Notice that the expression only depends on the input inductor, L1, the load resistor, R, and the duty cycle.

The complex RHP zeros arise from the fact that turning on the switch disconnects the second inductor from the load. These zeros will actually move with the values of parasitic resistors in the circuit, so careful analysis of your converter is needed to ensure stability under all conditions.

PSpice Modeling of the Sepic Converter
The analytical solution above does not include all of the parasitic circuit elements. As you will see from [1], there is a prodigious amount of work to be done even without the resistances.

We can also use PSpice to help understand the Sepic better. Fig. 7 shows the circuit model for a specific numerical application of the Sepic, and it includes resistances which will affect the stability of the converter, sometimes in dramatic ways.

The PSpice file listing can be downloaded from [2] so you can reproduce these results to analyze your own Sepic converter.

Fig. 8 shows the result of the PSpice analysis. The two resonant frequencies predicted by the hand analysis can clearly be seen in the transfer function plot. What is remarkable is the extreme amount of phase shift after the second resonance. This is caused by the delay of the second pair of poles, and the additional delay of the complex RHP zeros. The total phase delay through the converter is an astonishing 630 degrees.

What is remarkable is the extreme amount of phase shift after the second resonance. This is caused by the delay of the second pair of poles, and the additional delay of the complex RHP zeros. The total phase delay through the converter is an astonishing 630 degrees. Controlling this converter at a frequency beyond the second resonance is impossible.

Summary
The Sepic converter definitely has some select applications where it is the topology of choice. How do designers get away with building such a convert-

Figure 7. Analysis can also be done with PSpice. This figure shows a specific design example for a 15 W converter. Parasitic resistances are included in the PSpice model.

Figure 8. This shows the control-to-output transfer function for the Sepic converter. With low values of damping resistors, the converter has four poles, and three right-half-plane zeros. This results in an extreme phase delay of 630 degrees!
Since 2000, Ridley Engineering has provided hands-on laboratory workshops for power supply design engineers. Now, Ridley Engineering Europe will continue this trend with a focus on Europe.

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When designs become insurmountable in the lab, contact us for consulting services to assist in reaching production more efficiently.
Analysis of the Sepic Converter

by

Dr. Vatché Vorpérian
Notes on the small-signal analysis of the

The isolated sepic converter with synchronous rectifiers

Fig. 1

Fig. 2

To get from Fig 1 to Fig 2 replace the
transformer with its equivalent circuit model:

\[ L_1 \rightarrow \frac{L_s}{n^2} \]

reflect \( V_{in} \), \( L_1 \), \( C_1 \), & \( Q_1 \) as follows to the
secondary side:

\[ L_1 = L_1 n^2 \]
\[ Q_1 \rightarrow Q_{1-ref} = \left\{ \begin{array}{l l} V_{DS-ref} = n V_{DS} \\ I_{D-ref} = \frac{I_D}{n} \end{array} \right. \]

\[ C_f = \frac{C_1}{n^2} \]
\[ V_g = V_{in} n \]
- now redraw L₁ around C₁ and slide Q₂ next to Q₁-ref in the return line like this

This is the PWM switch
Replace the PWM switch with its equivalent circuit model

\[ r_e = r_c_1 + r_c_2 \parallel R \]

Set the small-signal sources to zero to obtain the dc model
I. Determination of the conversion ratio and the dc operating point of
the PWM switch

\[ I_0 = \frac{I_g}{D} - \frac{I_g}{D'} \Rightarrow \frac{I_g}{I_0} = \frac{D}{D'} \quad \text{also} \quad \frac{I_c}{I_0} = \frac{1}{D'} \]

As in the buck-boost converter, the current conversion is not affected by the parasitic elements and is given by KCL at point 'p':

\[ \frac{I_g}{I_0} = \frac{I_g}{D} \]

We use now the efficiency to determine the conversion ratio

\[ M = \frac{V_o}{V_g} = \frac{I_g}{I_0} \cdot \eta \]

\[ \eta = \frac{P_{out}}{P_{out} + P_{Lost}} \]

\[ P_{out} = I_0^2R \]

\[ P_{Lost} = I_0^2r_{L2} + I_g^2r_L + I_c^2r_c \]

\[ \Rightarrow \eta = \frac{1}{1 + \frac{r_{L2}}{R} + \frac{r_L}{R} \left( \frac{D}{D'} \right)^2 + \frac{r_c}{R} \left( \frac{D}{D'} \right)^2} \]

\[ M = \frac{D}{D'} \frac{1}{1 + \frac{r_{L2}}{R} + \frac{r_L}{R} \left( \frac{D}{D'} \right)^2 + \frac{r_c}{R} \left( \frac{D}{D'} \right)^2} \]

Operating point:

\[ I_c = \frac{I_g}{D'} \]

\[ V_{op} = V_g + V_o + I_0r_{L2} - I_gr_L \]

\[ = V_g + V_o \left( 1 + \frac{r_{L2}}{R} - \frac{D}{D'} \frac{r_L}{R} \right) \]
2. Determination of $D(s)$

With $\hat{v}_g$ and $\hat{d}$ set to zero, we obtain the circuit above from we will determine $D(s)$ for the small signal transfer functions:

$$\frac{\hat{v}_g}{\bar{v}_g} = M \frac{N_I(s)}{D(s)}$$

$$\frac{\hat{v}_o}{\bar{d}} = K_D \frac{N_d(s)}{D(s)}$$

$$Y_{in}(s) = \frac{\hat{v}_g}{\bar{v}_g} = G_{in} \frac{N_v(s)}{D(s)}$$

$$Z_o(s) = \frac{\hat{v}_T}{\bar{i}_T} = R_o \frac{N_o(s)}{D(s)}$$; $\hat{i}_T$ is test current source connected at the output
Fast analytical techniques for ELECTRICAL and ELECTRONIC CIRCUITS
by Vatché Vorpérian
Jet Propulsion Laboratory
California Institute of Technology
Cambridge University Press
www.cambridge.org

Today, the only method of circuit analysis known to most engineers and students is nodal or loop analysis. Although this works well for obtaining numerical solutions, it is almost useless for obtaining analytical solutions in all but the simplest cases.

In this unique book, Vorpérian describes remarkable alternative techniques to solve, almost by inspection, complicated linear circuits in symbolic form and obtains meaningful analytical answers for any transfer function or impedance.

Although not intended to replace traditional computer based methods, these techniques provide engineers with a powerful set of tools for tackling circuit design problems. They also have great value in enhancing students understanding of circuit operation. The numerous problems and worked examples in this book make it an ideal textbook for senior/graduate courses, or a reference book.

This book will show you how to:
- Use less algebra and do most of it directly on the circuit diagram.
- Obtain meaningful analytical solutions to complex circuits with reactive elements and dependent sources by reducing them to a set of simple and purely resistive circuits which can be analyzed by inspection.
- Analyze feedback amplifiers easily using the simplest and most natural formulation.
- Analyze PWM converters easily using the model of the PWM switch.

Originally developed and taught at institutions and companies around the world by Professor David Middlebrook at Caltech, the extended and new techniques described in this book are an indispensable set of tools for linear electronic circuit analysis and design.

Publisher's note: Dr. Vatché Vorpérian is one of the rare few researchers who delight in the process of analysis of analog circuits, and in finding simple and elegant solutions to seemingly insurmountable problems. I have observed, on numerous occasions, his ability to derive models and equations overnight. In this latest book for the electrical engineer, he reveals many of his techniques. Much of it is applied to power conversion circuits—which is one of the few remaining disciplines where hand analysis is crucial to the development of circuit topologies and new technologies. It’s a ‘must have’ text for anyone serious about the field of power electronics.

Vatché’s major contributions to our field include the PWM switch model, the ZCS and ZVS quasi-resonant switch model, and the analysis of the series and parallel resonant converter.

Corrections to Previous Articles in SPM

Winter 2002 issue, Designer Series’ Part VII. We published a formula for temperature dependence of the R material. A minus sign was omitted in front of the linear coefficient of the 5th order polynomial. The correct equation should be as follows:

\[ P_{core} = (-3.626 \ln f + 28.32) f^{1.729} AB^{2.99575 + 2.83332} g(T) \]

where

\[ g(T) = 1.83877T^5 - 4.194T^4 + 2.0955T^3 + 2.7933T^2 - 4.1658T + 2.6345 \]

was +4.1658T in the original article.

\[ T' = T / 100 \] where T is in degrees C.

Thanks to Phil Cooke of Analog Devices for finding this error. We apologize for any inconvenience this may have caused.

Designer Series Part III, January 2001. We published an article on loop gain crossover frequency. The flyback converter of Figure 1a had a typographical error - the output voltage of the converter should have been 12 V, not 24 V. If you run the calculation for the RHP zero with 24 V output, you would have a frequency of 13.8 kHz instead of 20 kHz, and the maximum duty cycle would be 0.62.

Thanks to Fred Waechter of Phihong USA for finding this error.
The New AP300 Frequency Response Analyzer
from AP Instruments, Inc.

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The AP300 is manufactured by AP Instruments, Inc. and distributed worldwide by Ridley Engineering, Inc. All accessories are manufactured by Ridley Engineering, Inc. For more information, visit our website.
Since the converter is of fourth order, we have:

\[ D(s) = 1 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 \]  

(1)

For a properly designed fourth-order converter, the denominator consists of two quadratic factors whose resonances are well separated and almost entirely damped by the load. The parasitic resistances have almost no effect on the two resonant frequencies of \( D(s) \) and contribute very little to the damping of the resonances under normal loading conditions. Therefore we can write:

\[ D(s) = \left( 1 + \frac{s}{\omega_{o1} Q_1} + \frac{s^2}{\omega_{o1}^2} \right) \left( 1 + \frac{s}{\omega_{o2} Q_2} + \frac{s^2}{\omega_{o2}^2} \right) \]  

(2)

Expanding Eq. (2) and comparing to Eq. (1) under the assumption of moderate to high \( Q \) and well-separated resonances, we get:

\[ a_1 \sim \frac{1}{\omega_{o1} Q_1} \]  

(3a)

\[ a_2 \sim \frac{1}{\omega_{o1}^2} \]  

(3b)

\[ a_3 \sim \frac{1}{\omega_{o1} Q_1 \omega_{o2}} + \frac{1}{\omega_{o2} Q_2 \omega_{o1}} \]  

(3c)

\[ a_4 \sim \frac{1}{\omega_{o1}^2 \omega_{o2}^2} \]  

(3d)

We will determine these coefficients according to Eqs. (4-7) given below and perform all necessary approximations as we go along. The reference circuit is shown on the next page.

\[ a_1 = \frac{L_{f1}}{R^{(1)}} + \frac{L_{f2}}{R^{(2)}} + C_{f2} R^{(3)} + C_{f1} R^{(4)} \]  

(4)
reference circuit indicating normal port conditions

\[ a_2 = \frac{L_{f_1}}{R^{(1)}} \frac{L_{f_2}}{R^{(2)}} + \frac{L_{f_1}}{R^{(1)}} C_{f_2} R^{(3)} + \frac{L_{f_1}}{R^{(1)}} C_{f_1} R^{(4)} \]

\[ + \frac{L_{f_2}}{R^{(2)}} C_{f_2} R^{(2)} + \frac{L_{f_2}}{R^{(2)}} C_{f_1} R^{(2)} + C_{f_2} R^{(3)} C_{f_1} R^{(3)} \]  

\[ a_3 = \frac{L_{f_1}}{R^{(1)}} \frac{L_{f_2}}{R^{(2)}} C_{f_2} R^{(21)} + \frac{L_{f_1}}{R^{(1)}} \frac{L_{f_2}}{R^{(2)}} C_{f_1} R^{(21)} \]

\[ + \frac{L_{f_1}}{R^{(1)}} C_{f_2} R^{(31)} C_{f_1} R^{(31)} + \frac{L_{f_2}}{R^{(2)}} C_{f_2} R^{(32)} C_{f_1} R^{(32)} \]  

\[ a_4 = \frac{L_{f_1}}{R^{(1)}} \frac{L_{f_2}}{R^{(2)}} C_{f_2} R^{(21)} C_{f_1} R^{(21)} \]  

The order in which the elements, or the ports, are taken in any one term is immaterial.

For example:

\[ \frac{L_{f_1}}{R^{(1)}} \frac{L_{f_2}}{R^{(2)}} C_{f_1} R^{(21)} = \frac{L_{f_1}}{R^{(1)}} C_{f_1} R^{(21)} \frac{L_{f_2}}{R^{(21)}} \]  

\[ \frac{L_{f_1}}{R^{(1)}} \frac{L_{f_2}}{R^{(1)}} C_{f_1} R^{(41)} = \frac{L_{f_1}}{R^{(1)}} C_{f_1} R^{(41)} \frac{L_{f_2}}{R^{(41)}} \]
i) Determination of $\alpha$,

$$R^{(\alpha)} = r_L + \frac{r_c}{D^2} + \left(\frac{D'}{D}\right)^2 (R + r_L) \approx \left(\frac{D'}{D}\right)^2 R$$
\[ R^{(2)} = r_{L_2} + R + \left( \frac{D}{D'} \right)^2 \left( \frac{r_c}{D_2} + r_{L_1} \right) \approx R \]

\[ R^{(3)} = \frac{r_{C_2} + R}{\left( r_{L_2} + \left( \frac{D}{D'} \right) \left( \frac{r_c}{D_2} + r_{L_1} \right) \right)} \]
\[ \approx r_{C_2} + r_{L_2} + \frac{r_c}{D'} + r_{L_1} \left( \frac{D}{D'} \right)^2 \]
$R^{(4)} = r_{C_1} + R^{(4')}$

According to the EET

$$R^{(4')} = R_0 \left( 1 + \frac{R_n}{r_c/D^2 + R(D'/D)^2} \right)$$

$$R_0 = r_{L_1} + r_{L_2}, \quad R_d = r_{L_1} + \left( \frac{D'}{D} \right)^2 r_{L_2}$$

$$R_n = \frac{r_{L_1} r_{L_2}}{D^2}$$

$\therefore R^{(4)} \approx r_{C_1} + r_{L_1} + r_{L_2}$
**Determination of \( a_2 \)**

In order to determine \( a_2 \), we need to determine the following:

\[
R_{(1)}^{(2)}, R_{(1)}^{(4)}, R_{(1)}^{(3)}, R_{(2)}^{(4)}, R_{(2)}^{(3)}, R_{(1)}^{(4)}
\]

According to the reference circuit below, all of the above can be deduced from \( R_{(1)}^{(1)} \), \( R_{(2)}^{(2)} \), \( R_{(3)}^{(3)} \) and \( R_{(4)}^{(4)} \) as follows:

\[
R_{(1)}^{(n)} = \lim_{L_1 \to \infty} R_{(1)}^{(n)} \quad n = 2, 3, 4
\]

\[
R_{(2)}^{(n)} = \lim_{L_2 \to \infty} R_{(2)}^{(n)} \quad n = 3, 4
\]

\[
R_{(3)}^{(4)} = \frac{R_{(4)}}{R_{2} || R}
\]

**Reference circuit indicating normal port conditions**
Now in its 18th year of development, POWER 4-5-6 Plus is the most powerful design, analysis, and simulation software available for power supply development. The latest version includes topology selection, detailed magnetics design, capacitor selection, and feedback design in addition to voltage-mode or current-mode control.

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The magnetics designer in POWER 4-5-6 provides a library of cores and wire types. Our advanced proximity loss analysis feature shows the effect of ac winding losses, to ensure that you choose the best combination of wire sizes and winding layers for minimal dissipation. Entering new cores into the library is simple and fast.

And now, a special version of the software is available for AP300 Frequency Response Analyzer users that predicts the response of your power supply and compares it with data collected from the AP300. They communicate with each other to show measurements overlaid on theoretical curves.
we now have

\[
R_{(0)}^{(2)} \rightarrow \infty
\]

\[
R_{(1)}^{(3)} = R_{c_2} + R \approx R
\]

\[
R_{(2)}^{(3)} = R_{c_2} + 1/R \approx R
\]

\[
R_{(1)}^{(4)} = R_{c_1} + \frac{R}{D^2} + R \left( \frac{D'}{D} \right)^2 + \frac{R}{D^2} \approx R \left( \frac{D'}{D} \right)^2
\]

\[
R_{(2)}^{(4)} = R_{c_1} + \frac{R}{D^2} + \frac{R}{D^2} + R \approx R
\]

\[
R_{(3)}^{(4)} = R_{c_1} + R_0 \frac{1 + \frac{R}{c_1 D^2} \frac{R}{c_2 1/R (D')^2}}{1 + \frac{c_1 D^2 + c_2 1/R (D')^2}{R_0}}
\]

where \( R_0, R_0, \) and \( R_0 \) are the same as before given for \( R^{(4)} \).

Hence, ignoring parasitic elements, we have

\[
\alpha_2 = \frac{L_{f_1}}{R \left( \frac{D'}{D} \right)^2} \frac{L_{f_2}}{\infty} + \frac{L_{f_1}}{R \left( \frac{D'}{D} \right)^2} \cdot \frac{R_{c_2}}{R} + \frac{L_{f_1}}{R \left( \frac{D'}{D} \right)^2} \cdot \frac{R_{c_1} \left( \frac{D'}{D} \right)^2}{R}
\]

\[
+ \frac{L_{f_2}}{R} \cdot \frac{R_{c_2}}{R} + \frac{L_{f_2}}{R} \cdot \frac{R_{c_1}}{R} + \frac{L_{f_2}}{R} \cdot \frac{R_{c_1} \left( \frac{D'}{D} \right)^2}{R}
\]

\[
\alpha_1 = L_{f_1} \left( \frac{D'}{D} \right)^2 + C_{f_1} \right) + L_{f_2} \left( C_{f_1} + C_{f_2} \right)
\]

\[
\omega_{01} = \frac{1}{\sqrt{\alpha_2}} = \frac{1}{\sqrt{L_{f_1} \left( \frac{D'}{D} \right)^2 + C_{f_1} + L_{f_2} \left( C_{f_1} + C_{f_2} \right)}}
\]
**Determination of \( a_3 \)**

In order to determine \( a_3 \), we need to determine the following:

\[
R^{(3)}_{(2)} = \lim_{r_2 \to \infty} R^{(3)}_{(2)} = \frac{r_2 + R}{\bar{r}} = R
\]

\[
R^{(4)}_{(2)} = \lim_{r_2 \to \infty} R^{(4)}_{(2)} = \infty
\]

\[
R^{(4)}_{(3)} = \lim_{r_2 \to \infty} R^{(4)}_{(3)} = \frac{r_{C_1} + \frac{r_{L_2} + r_{C_1}||r + r_{C_2}||r}{D^2}}{D^2}
\]

\[
R^{(4)}_{(32)} = \lim_{r_2 \to \infty} R^{(4)}_{(32)} = \frac{r_{C_1} + r_{C_2}||r}{D^2}
\]

\[
\therefore \quad a_3 = \frac{L_{f_1} \cdot L_{f_2}}{(D')^2} \cdot \frac{C_{f_1}}{R} \cdot \frac{R_{(3)}}{R} \cdot \frac{R_{(4)}}{R} \cdot \frac{C_{f_2}}{R_{(3)}} + \frac{L_{f_1} \cdot L_{f_2}}{(D')^2} \cdot \frac{C_{f_1}}{R} \cdot \frac{R_{(3)}}{R} \cdot \frac{R_{(4)}}{R} \cdot \frac{C_{f_2}}{R_{(3)}} \xrightarrow{\text{indeterminate}} 0
\]

The indeterminancy can be removed by changing the order in which the ports are taken:

\[
\frac{L_{f_1}}{R_{(1)}} \cdot \frac{L_{f_2}}{R_{(4)}} \cdot \frac{C_{f_1}}{R_{(3)}} \cdot \frac{R_{(2)}}{R_{(1)}} = \frac{L_{f_1}}{R_{(1)}} \cdot \frac{C_{f_1}}{R_{(4)}} \cdot \frac{L_{f_2}}{R_{(4)}} \cdot \frac{R_{(3)}}{R_{(4)}}
\]

\[
1 \quad 2 \quad 4 \quad 1 \quad 4 \quad 2
\]
$R^{(2)}_{(41)}$ is determined from Fig. (a) above which can be transformed to Fig. (b) by reflecting $R$ through $(0\prime D)$. It follows from Fig. (b) that

$$R^{(2)}_{(41)} = L_2 + D^2\left[\frac{c_1 + c}{D^2} + R\left(\frac{D'}{D}\right)^2\right]$$

$$= L_2 + c + D^2 c_1 + R D' = R 0'^{\prime 2}$$

$$\therefore \quad q_3 \approx \frac{Lf_1}{(0')^2 R} \cdot \frac{C_f}{Lf_1} \cdot \frac{R(D')^2}{R 0'^{\prime 2}} \cdot \frac{Lf_2}{R D'^2}$$

$$q_3 \approx \frac{L f_1 L f_2 C_f}{D'^2 R}$$
Determination of $\alpha_4$

Since from $\alpha_3$ we already have the expression of the term in which the ports are taken in the order 1, 4, 2, we will write $\alpha_4$ as

$$\alpha_4 = \frac{L_1}{R^{(1)}} \cdot \frac{C_1}{r_2} \cdot \frac{R^{(4)}}{R^{(2)}} \cdot \frac{L_2}{R^{(2)}} \cdot \frac{C_2}{r_3} \cdot \frac{R^{(3)}}{R^{(124)}}$$

$$= \frac{1}{4} \cdot \frac{C_2}{2} \cdot \frac{R^{(3)}}{3}$$

$$\alpha_4 = \alpha_3 C_2 R^{(3)}_{(124)}$$

and $R^{(3)}_{(124)} = r_c + R \approx R$

$$\therefore \alpha_4 = \frac{L_1 L_2 C_1 C_2}{D'^2}$$

Since $\alpha_4 = 1/\omega_0^2$, we have

$$\omega_0^2 = \frac{D'^2}{L_1 L_2 C_1 C_2} \left[ \frac{L_1}{D} \left( \frac{C_1}{D} \right)^2 + C_1 \right] + \frac{L_2}{D} \left( \frac{C_1 + C_2}{D} \right)$$

$$\omega_0 = \sqrt{\frac{1}{L_2 \cdot C_1 / D'^2} \parallel \frac{C_2}{D'^2} + \frac{1}{L_1 \cdot C_1 \parallel C_2}}$$
Determination of $Q_1$

\[ a_1 \approx \frac{1}{\omega_0 Q_1} \Rightarrow Q_1 = \frac{1}{a_1 \omega_0} \]

\[ a_1 = \frac{L_f f_1}{R} \left( \frac{D}{D'} \right)^2 + \frac{L_f}{R} \]

\[ \Rightarrow Q_1 = \frac{R}{\omega_0 \left( \frac{L_f f_1}{R} \left( \frac{D}{D'} \right)^2 + \frac{L_f}{R} \right)} \]

Determination of $Q_2$

\[ a_3 \approx \frac{1}{\omega_0 Q_1 \omega_0^2} + \frac{1}{\omega_0 Q_2 \omega_0^2} \]

\[ = \frac{1}{\omega_0^2 \omega_0^2} \left[ \frac{\omega_0}{Q_1} + \frac{\omega_0}{Q_2} \right] \]

\[ = a_4 \left( \frac{\omega_0}{Q_1} + \frac{\omega_0}{Q_2} \right) \]

\[ \Rightarrow \frac{\omega_0}{Q_2} = \frac{a_3}{a_4} - \frac{\omega_0}{Q_1} = \frac{1}{C_{f_2} R} - \omega_0 \left( L_{f_2} + L_{f_1} \left( \frac{D}{D'} \right) \right)^2 \]

\[ = \frac{\omega_0^2}{R} \left( \frac{1}{C_{f_2} \omega_0} - L_{f_2} - L_{f_1} \left( \frac{D}{D'} \right)^2 \right) \]

\[ = \frac{\omega_0^2}{R} \left( L_{f_1} \left( \frac{D}{D'} \right)^2 + L_{f_2} + \frac{L_{f_1} C_{f_2}}{C_{f_2}} + \frac{L_{f_2} C_{f_1}}{C_{f_2}} - L_{f_2} \left( \frac{D}{D'} \right)^2 \right) \]
\[
\frac{\omega_{02}}{Q_2} = \frac{\omega_{01}^2}{R} \frac{C_{f_1}}{C_{f_2}} \left( L_{f_1} + L_{f_2} \right)
\]

\[
Q_2 = \frac{R}{\omega_{02} \left( L_{f_1} + L_{f_2} \right) \frac{C_{f_1}}{C_{f_2}} \left( \frac{\omega_{01}}{\omega_{02}} \right)^2}
\]

Putting all the results together, we have:

\[
D(s) = 1 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4
\]

\[
a_1 = \left( L_{f_1} \left( \frac{D}{D^2} \right)^2 + L_{f_2} \right) \frac{1}{R}
\]

\[
a_2 = L_{f_1} \left( C_{f_1} + \left( \frac{D}{D^2} \right) C_{f_2} \right) + L_{f_2} \left( C_{f_1} + C_{f_2} \right)
\]

\[
a_3 = \frac{L_{f_1} L_{f_2} C_{f_1}}{RD^2}
\]

\[
a_4 = L_{f_1} L_{f_2} C_{f_1} C_{f_2} \cdot \frac{1}{D^2}
\]

\[
D(s) = \left( 1 + \frac{s}{\omega_{01} Q_1} + \left( \frac{s}{\omega_{01}} \right)^2 \right) \left( 1 + \frac{s}{\omega_{02} Q_2} + \left( \frac{s}{\omega_{02}} \right)^2 \right)
\]

\[
\omega_{01} = \frac{1}{\sqrt{L_{f_1} \left( C_{f_2} \left( \frac{D}{D^2} \right)^2 + C_{f_1} \right) + L_{f_2} \left( C_{f_1} + C_{f_2} \right)}}
\]

\[
Q_1 = \frac{R}{\omega_{01} \left( L_{f_1} \left( \frac{D}{D^2} \right)^2 + L_{f_2} \right)}
\]

\[
\omega_{02} = \frac{1}{\sqrt{L_{f_2} \frac{C_{f_1}}{D^2} \left\| \frac{C_{f_2}}{D^2} \right\|} + \frac{1}{L_{f_1} \frac{C_{f_1}}{\left\| C_{f_2} \right\|}}}
\]
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3. Determination of the line-to-output transfer function

The line-to-output transfer function is of the form

\[ \frac{\hat{V}_o(s)}{\hat{V}_g(s)} = M \frac{N_g(s)}{D(s)} \]

in which \( M \) and \( D(s) \) have already been determined and \( N_g(s) \) corresponds to the null conditions in the response \( \hat{V}_o(s) \) to the excitation \( \hat{V}_g(s) \).

The first null is given by the zero of the impedance branch, \( \frac{1}{r_c + j\omega C_f} \), connected across the response \( \hat{V}_o(s) \) so that the first factor of \( N_g(s) \) is

\[ N_g(s) = (1 + j\omega C_f) N_2(s) \]

The second null condition, \( N_2(s) \), is given by \( i_f(s) = 0 \) which we will investigate next.
With \( \hat{i}_f = 0 \), KCL at node "p" yields \( \hat{i}_a = \hat{i}_c = 0 \). With \( \hat{v}_o(s) = 0 \), we see that the voltage drop across the impedance branch \( r_c + 1/sC_f \) is \( \hat{v}_{ap} \)

\[
\hat{v}_{ap} = \hat{i}_i \left( r_c + \frac{1}{sC_f} \right)
\]

Since \( \hat{i}_f = 0 \), \( \hat{i}_i \) also flows through the impedance branch \( L_2 + sL_f_2 \). With \( \hat{v}_o(s) = 0 \) and the voltage drop across \( r_c \) being zero, the voltage drop across the branch \( L_2 + sL_f_2 \) is \( \hat{v}_{cp} \)

\[
\hat{v}_{cp} = -\hat{i}_i \left( L_2 + sL_f_2 \right)
\]

Since \( \hat{v}_{cp} = D \hat{v}_{ap} \), we have

\[
\hat{i}_i \left( r_c + \frac{1}{sC_f} \right) D = -\left( L_2 + sL_f_2 \right) \hat{i}_i \Rightarrow 1 + sC_f \left( D r_c + L_2 \right) + s^2 L_f_2 C_f = 0
\]

\[
D_{2} (s) = 1 + sC_f \left( D r_c + L_2 \right) + s^2 L_f_2 C_f
\]
4. Determination of the control-to-output transfer function

The control-to-output transfer function is given by

\[
\hat{V}_o(s) = K_D \frac{N_d(s)}{D(s)}
\]

Although \(K_D\) can be determined from the circuit above by letting \(s \to 0\) (inductors short and capacitors open), we can derive it from the expression of the conversion ratio

\[
V_o = N(0) V_g \Rightarrow K_D = \frac{dV_o}{dD} = V_g \frac{dN}{dD} = V_g \frac{d}{dD} \left( \frac{D}{1-D} \right)
\]

\(K_D = \frac{1}{D^2}\)

\(N_d(s)\) corresponds to the null conditions in the response, \(\hat{V}_i(s)\), to the excitation sources \((V_o/D)d\) and \(I_c d\). The first
The second null condition in the control-to-output transfer function

null is given by the zero of the impedance connected across \(v_o(s)\). This impedance consists of \(C_f + 1/5C_{f2}\), so that the first factor in \(N_d(s)\) is \((1 + s/C_2 C_{f2})\).

\[
N_d(s) = (1 + s/C_2 C_{f2}) N_2(s)
\]

In which \(N_2(s)\) corresponds to the second null condition shown in the circuit above given by the condition \(i_f(s) = 0\).

Referring to the circuit above, we have at ground node \(c\), according to KCL,

\[
\hat{i}_c(s) = \hat{i}_1 + \hat{i}_2
\]

With \(\hat{i}_f(s) = 0\) and KCL at node \(p\), we have

\[
I_c \Delta = D \hat{i}_c - \hat{i}_c = -DI_c
\]
Equations (1) and (2) given

\[ i_1^\wedge + i_2^\wedge + \frac{V_c}{D} = 0 \]  \hspace{1cm} (I)

Next, we will assume \( C_1 = C_2 = C_r = \frac{C}{2} = 0 \). The voltage across \( L_f \) is the same as the voltage drop across \( C_f \) and \( L_f \), which, with \( f > 0 \), can be written as:

\[ i_2^\wedge S L_f = i_1^\wedge \left( S L_f + \frac{1}{S C_f} \right) \]  \hspace{1cm} (3)

\[ i_2^\wedge S^2 L_f C_f - i_1^\wedge \left( 1 + S^2 L_f C_f \right) = 0 \]  \hspace{1cm} (II)

Finally, with \( V_0(s) = 0 \), the voltage across port "a-p" of the PWM switch is

\[ \frac{V_0}{D} = i_1^\wedge \frac{1}{S C_f} = V_{ap} \]  \hspace{1cm} (4)

and the voltage across port "c-p" is

\[ V_{cp} = -i_1^\wedge S L_f \]  \hspace{1cm} (5)

Since \( V_{cp} = D V_{ap} \), we have

\[ \left[ \frac{V_0}{D} = i_1^\wedge \frac{1}{S C_f} \right] D = -i_1^\wedge S L_f \]  \hspace{1cm} (6)

\[ \therefore \quad S C_f V_{ap} + i_1^\wedge \left( D + S^2 L_f C_f \right) = 0 \]  \hspace{1cm} (7)
Putting equations (I)-(III) together ($V_D = V_{OP}$)

\[\hat{i}_1 + \hat{i}_2 + \frac{I_c}{D} \hat{d} = 0 \]  
\[-\hat{i}_1 (1 + s^2 L_2 C_f) + \hat{i}_2 s^2 L_1 C_f = 0 \]  
\[\hat{i}_1 (D + s^2 L_2 C_f) + s C_f V_{OP} \hat{d} = 0 \]

With $\hat{i}_1, \hat{i}_2$ and $\hat{d}$ different from zero, the only way the above can be satisfied is to have their determinant vanish:

\[\begin{vmatrix}
1 & 1 & I_c/D' \\
-(1 + s^2 L_2 C_f) & s^2 L_1 C_f & 0 \\
D + s^2 L_2 C_f & 0 & s C_f V_{OP}
\end{vmatrix} = 0 \]  

or

\[s^3 L_2 L_1 C_f - \frac{V_{OP}}{I_c} D' \left[ L_1 C_f + L_2 C_f \right] s^2 + s L_2 C_f D - \frac{V_{OP}}{I_c} D' = 0 \]

\[
\frac{V_{OP}}{I_c} = \frac{V_g + V_o}{I_o/D'} = D' \frac{V_o}{I_o} \left( 1 + \frac{V_g}{V_o} \right) = D' R \left( 1 + \frac{1}{M} \right)
\]

\[\therefore \frac{V_{OP}}{I_c} = \frac{D'^2 \cdot R}{D} \]
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Substitution of Eq. (9) in (8) yields the desired result, \( N_2(s) \):

\[
N_2(s) = 1 - s L_1 \left( \frac{D}{D'} \right)^2 \frac{1}{R} + s^2 C_1 \left( L_1 + L_2 \right) - s^3 \frac{L_2 L_1 C_1}{R} \left( \frac{D}{D'} \right)^2
\]  

(10)

\( N_2(s) \) factors as follows:

i) Normal load conditions

\[
N_2(s) \approx \left( 1 - s L_1 \left( \frac{D^2}{D'^2} \right) \frac{1}{R} \right) \left( 1 - s \frac{C_1 \left( L_1 + L_2 \right) R \left( \frac{D'}{D} \right)^2 + s^2 L_2 C_1}{L_1} \right)
\]  

(11)

The first factor corresponds to the expected RHP zero due to the pulsating output filter current during \( D' T_s \).

ii) No load (Synchronous rectification)

\[
N_2(s) = 1 + s^2 C_1 \left( L_1 + L_2 \right)
\]  

(12)

In this case the RHP zero behavior disappears as expected.

iii) Very light load (Synchronous rectification)

\[
N_2(s) \approx \left[ 1 - s L_1 \left( \frac{D}{D'} \right)^2 \frac{1}{R} + s^2 C_1 \left( L_1 + L_2 \right) \right] \left( 1 - s \frac{L_2 L_1 C_1}{R} \left( \frac{D}{D'^2} \right) \right)
\]  

(13)
5. Determination of the open-loop output impedance

\[ Z_0(s) = R_0 \frac{N_0(s)}{D(s)} \]

where \( D(s) \) is the same as before and \( R_0 \) is determined from the adjacent circuits.

\[ R_U = R \parallel \left( \frac{r_L}{D} \right) \left( \frac{r_L + r_c}{D^2} \right) \]
Circuit for the determination of the denominator of the output admittance or the numerator of the output impedance

Since the output impedance is the reciprocal of the output admittance we have

\[ Z_0(s) = R_0 \frac{N_0(s)}{D(s)} \quad \frac{Y_0(s)}{Z_0(s)} = \frac{1}{C_0} \frac{D(s)}{N_0(s)} \]

where we see that the denominator of \( Y_0(s) \) is \( N_0(s) \) which can be determined by setting the excitation of \( Y_0(s) \) to zero. The excitation of \( Y_0(s) \) is a test voltage source connected at the output which upon setting equal to zero results in the circuit shown above. It follows immediately that

\[ N_0(s) = \lim_{R \to 0} D(s) \]
Before taking the limit, we realize that one of the factors of \( N_0(s) \) must be \( 1 + sC_2C_f \), because this is the zero of an impedance branch connected directly across the output:

\[
N_0(s) = (1 + sC_2C_f) N_0'(s)
\]

\[
N_0'(s) = \lim_{R \to 0} \frac{D(s)}{C_2 \to 0 \quad C_f \to 0}
\]

In taking this new limit, we will first ignore all the parasitic elements and obtain

\[
N_0'(s) = b_5 + b_3 s^3 = b_5 (1 + \frac{b_3 s^2}{b_1})
\]

which implies that \( N_0'(s) \) has a zero at the origin and an undamped resonance at \( \omega_0 = \sqrt{b_1/b_3} \).

The zero at the origin is consistent with the fact that the low-frequency asymptote of \( Z_0(s) \), \( R_0 \), goes to zero when all the parasitic elements go to zero.

Next we determine \( \omega_0 \):

\[
\omega_0^2 = \frac{b_1}{b_3} = \lim_{C_f \to 0} \left( \frac{a_1}{a_3} \right) = \frac{D'^2}{L_f L_c_f} \left( \frac{1}{L_f + \left( \frac{D'}{D} \right)^2 L_f} \right)
\]
It follows that

\[ \omega_{0z} = \sqrt{\frac{1}{C_1 \frac{L_2}{D^2} || \frac{L_1}{D^1}}} \]

This is very consistent with the physical picture, without parasitic elements shown below.

\[
\omega_{0z} = \sqrt{\frac{1}{C_1 \frac{L_2}{D^2} || \frac{L_1}{D^1}}} \]

\[
\left\{ \begin{array}{c}
\frac{L_2}{D^2} \\
\frac{L_1}{D^1} \\
\end{array} \right. 
\]
Finally, we include the parasitic elements to determine the zero at very low frequencies and the damping of the resonance at \( \omega_{02} \). Now we have

\[
N_0'(s) = 1 + b_1 s + b_2 s^2 + b_3 s^3 \\
\approx (1 + b_1 s) \left( 1 + \frac{b_2}{b_1} s + \frac{b_3}{b_1} s^2 \right) \\
\approx (1 + \frac{s}{s_{21}}) \left( 1 + \frac{s}{\omega_{02} Q_2} + \frac{s^2}{\omega_{02}^2} \right)
\]

\[
b_1 \approx \lim_{R \to 0} \frac{2f_1}{\frac{L_1}{R_1 + \frac{R_c}{D^2} + \frac{L_2 (D')^2}{D}}} + \frac{L_2}{R_2 + \frac{L_1 (D')^2}{D} + \frac{R_c}{D^2}} + \frac{C_f \left[ \frac{R_c}{L_{11} + L_{12}} + \frac{D^2 L_{11} + D' L_{12}}{R_c + L_{11} L_{12}} \right]}{R_c + L_{11} L_{12}} = 0
\]

\[
b_1 \approx \frac{L_1 + (D')^2}{R_1 + \frac{R_c}{D^2} + (D')^2} \frac{L_2}{R_2 + \frac{R_c}{D^2} + (D')^2} \\
S_{21} = \frac{1}{b_1}
\]
\[ b_2 = \lim_{C_f \to 0} a_2 = C_f (L_{f_1} + L_{f_2}) \]

\[ \frac{1}{\omega_0 z Q_z} = \frac{b_2}{b_1} \Rightarrow Q_z = \frac{b_1}{b_2 \omega_0 z} \]

making the necessary substitutions, we get

\[ Q_z = \frac{\omega_0 z L_{f_1} L_{f_2}}{r_C + r_{l_1} D^2 + r_{l_2} D'^2} \]

Putting all the results together, we have:

\[ N_0(s) = \left(1 + \frac{s}{s_z_1}\right) \left(1 + \frac{s}{s_z_2}\right) \left(1 + \frac{s}{\omega_0 z Q_z} + \frac{s^2}{\omega_0^2 z^2}\right) \]

\[ s_z_1 = \frac{r_{l_1} + r_C D^2 + r_{l_2} (D')^2}{L_{f_1} + \left(\frac{D'}{D}\right)^2 L_{f_2}} \]

\[ s_z_2 = \frac{1}{r_{l_2} C_{f_2}} \]

\[ \omega_0 z = \sqrt{\frac{1}{C_f \frac{L_{f_2}}{D^2} \frac{L_{f_1}}{D'^2}}} \]
6. Determination of the open-loop input admittance

The input admittance is of the form

\[ Y_{in}(s) = G_{in} \frac{N_i(s)}{D(s)} \]

\( D(s) \) is the same as before while \( G_{in} \) is determined from the adjacent circuit

\[ G_{in} = \frac{1}{L_1 \left( \frac{c}{D} \right)^2 + \left( \frac{D'}{D} \right)^2 (R+R_L)} \]
Circuit for the determination of the denominator of the input impedance or the numerator of the input admittance.

Since the input admittance is the reciprocal of the input impedance, we have:

\[ Y_{in}(s) = C_{in} \frac{N_i(s)}{D(s)} \quad Z_{in}(s) = R_{in} \frac{D(s)}{N_i(s)} \]

whence we see that \( N_i(s) \) is the denominator of \( Z_{in}(s) \) which can be determined from the circuit by reducing the excitation of \( Z_{in}(s) \) to zero. The excitation of \( Z_{in}(s) \) is a test current source connected at the input which upon reducing to zero results in the circuit shown above. Whence it follows that

\[ N_i(s) = \lim_{n_i \to 0} D(s) \]
Next we determine the coefficients \( a_i \) in the limit \( L_1 \to \infty \):

\[
\lim_{L_1 \to \infty} a_1 \rightarrow \infty
\]

\[
\lim_{L_1 \to \infty} R^{(1)} \rightarrow \infty
\]

\[
\lim_{L_1 \to \infty} R^{(2)} \rightarrow \infty
\]

\[
\lim_{L_1 \to \infty} R^{(3)} \rightarrow C_2 + R
\]

\[
\lim_{L_1 \to \infty} R^{(4)} \rightarrow C_1 + \lim_{L_1 \to \infty} \left( C_1 + \frac{r_{L_2}}{L_1} \right) \frac{1 + \frac{r_{L_2}}{L_1}}{1 + \frac{r_{L_2} + (D')^2 L_2}{L_1}}
\]

\[
= C_1 + \frac{r_{L_1}}{L_1} \frac{1 + \frac{r_{L_2}}{L_1}}{L_1}
\]

\[
= C_1 + \frac{r_{L_2}}{L_1}
\]

\[
\lim_{L_1 \to \infty} R^{(4)} = C_1 + \frac{r_{C} + r_{L_2} + (D')^2 R}{D^2}
\]

\[
\lim_{L_1 \to \infty} \alpha_1 = C_2 (C_2 + R) + C_1 \left( C_1 + \frac{r_{C} + r_{L_2} + (D')^2 R}{D^2} \right)
\]

\[
\approx \left[ C_2 + C_1 \left( \frac{D'}{D} \right)^2 \right] R
\]
\[
\lim_{\ell_2 \to \infty} a_2
\]

In this limit only the following two terms survive

\[
\lim_{\ell_2 \to \infty} a_2 = \lim_{\ell_2 \to \infty} \left[ \frac{L_{f_2}}{R(2)} C_{f_1} R^{(4)}_{(2)} \right] + \lim_{\ell_2 \to \infty} \left[ C_{f_1} C_{f_2} R^{(3)} R^{(4)}_{(3)} \right] = \frac{L_{f_2} C_{f_1}}{D^2} + \frac{C_{f_1} C_{f_2} (r_{c_2} + R)(r_{c_1} + \frac{r_{c_1} + r_{c_2} + D^2 R_{11} R_{22}}{D^2})}{D^2}
\]

\[
\lim_{\ell_2 \to \infty} a_3
\]

In this limit only one term survives

\[
\lim_{\ell_2 \to \infty} a_3 = \lim_{\ell_2 \to \infty} \left[ \frac{L_{f_2}}{R(2)} C_{f_2} R^{(3)} C_{f_1} R^{(4)}_{(32)} \right] = \frac{L_{f_2} C_{f_2} C_{f_1} (r_{c_2} + R) \lim_{\ell_2 \to \infty} \left[ \frac{R_{B(2)}}{R(2)} \right]}{D^2} = \frac{L_{f_2} C_{f_2} C_{f_1} R}{D^2} \frac{r_{c_1} + D^2}{D^2} = \frac{L_{f_2} C_{f_2} C_{f_1} R}{D^2}
\]
\[ \lim_{L_z \to \infty} q_z = 0 \]

Putting all the results together we get

\[ N_i(s) = 1 + sR \left( C_f + C_f \left( \frac{D'}{D} \right)^2 \right) + s^2 \frac{L_z C_f}{D^2} + s^3 \frac{L_z C_f C_f R}{D^2} \]

In order to determine the approximate factors of \( N_i(s) \) analytically, we study the circuit at low and high frequencies.

At low frequencies we have:

\[ L_2 \text{ at low frequencies is a short.} \]

\[ \tau = \frac{R(D')^2}{R(D')^2} \left[ C_f + C_f \left( \frac{D'}{D} \right)^2 \right] = R \left( C_f + C_f \left( \frac{D'}{D} \right)^2 \right) \]
Hence, having identified the low-frequency factor, we investigate the possibility of factoring \( N_i(s) \) as follows:

\[
N_i(s) \approx \left( 1 + \frac{s}{S_{2i}} \right) \left( 1 + \frac{s}{\omega_i Q_i} + \frac{s^2}{\omega_i^2} \right)
\]

where

\[
S_{2i} = \frac{1}{R \left( C_{f2} + C_{f1} \left( \frac{D'}{D} \right)^2 \right)}
\]

\[
\omega_i^2 = \frac{C_{f2} + C_{f1} \left( \frac{D'}{D} \right)^2}{L_{f2} C_{f1} C_{f2} / D^2} = \frac{1}{L_{f2} C_{f1} \parallel \frac{C_{f2}}{D^2}}
\]

\[
Q_i = \frac{R \left( C_{f2} + C_{f1} \left( \frac{D'}{D} \right)^2 \right)}{\omega_i C_{f1} L_{f2}}
\]

\[
D^2 = \omega_i R C_{f2}
\]

This resonance can be verified by examining the circuit without any damping as shown below.
*SEPIC CONVERTER WITH PWM SWITCH MODEL

*SEPIC CONVERTER FROM POWER SYSTEMS DESIGN EUROPE NOVEMBER 2006 ISSUE

*Dr. Ray Ridley’s Design Tips

\[
\begin{align*}
\text{Vin} & \quad 1 \quad 100 \quad AC \quad 0 \\
RL1 & \quad 1 \quad 2 \quad 0.001 \\
L1 & \quad 2 \quad 4 \quad 10E-05 \\
RL2 & \quad 4 \quad 3 \quad 0.001 \\
L2 & \quad 3 \quad 0 \quad 10E-05 \\
RC1 & \quad 100 \quad 7 \quad 0.003000 \\
C1 & \quad 7 \quad 0 \quad 6.80E-04 \\
RC2 & \quad 5 \quad 6 \quad 0.001000 \\
C2 & \quad 6 \quad 0 \quad 2.20E-03 \\
R & \quad 5 \quad 0 \quad 1 \\
Vc & \quad 11 \quad 0 \quad AC \quad 1 \\
Rvc & \quad 11 \quad 0 \quad 10MEG \\
X1 & \quad 100 \quad 5 \quad 4 \quad 11 \\
\end{align*}
\]

.AC DEC 100 10Hz 10000Hz
.PRINT AC VDB(5) VP(5)
.PROBE

*PWM SWITCH MODEL

*SWITCH MODEL PARAMETER VALUES: Vap=-Vo/D  Ic = -Io/D'

.SUBCKT PWMCCM 1 2 3 4

E2 7 1 4 0 -41.667
G1 1 2 4 0 -37.5
Fxf 7 2 Vxf 0.6
Exf 9 2 7 2 0.6
Vxf 9 3 0
Rvc 4 0 10MEG

.ENDS
.END

-PWMCCM Subcircuit-
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