Noise In Photodiode Applications

By Art Kay and Bryan Zhao (赵 伟)

June 1, 2011

Contents

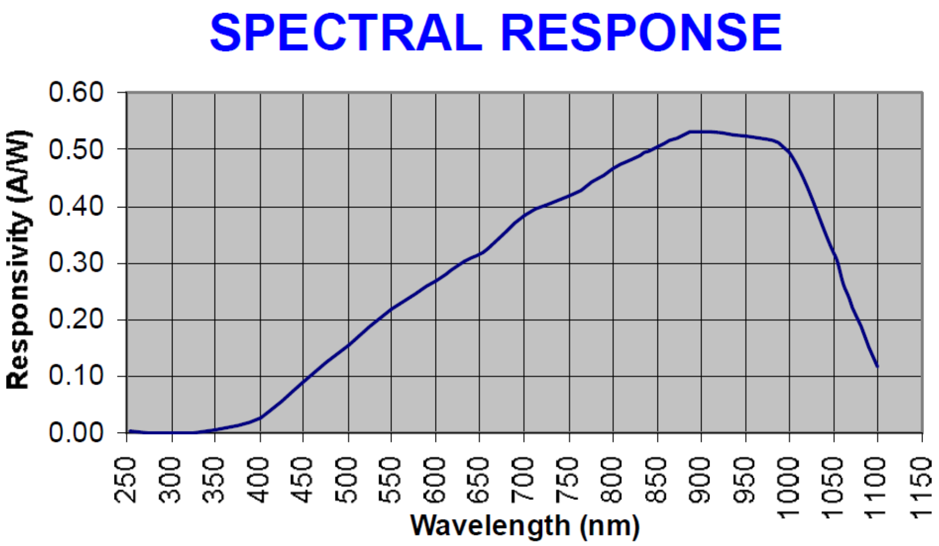
Photodiode Basics

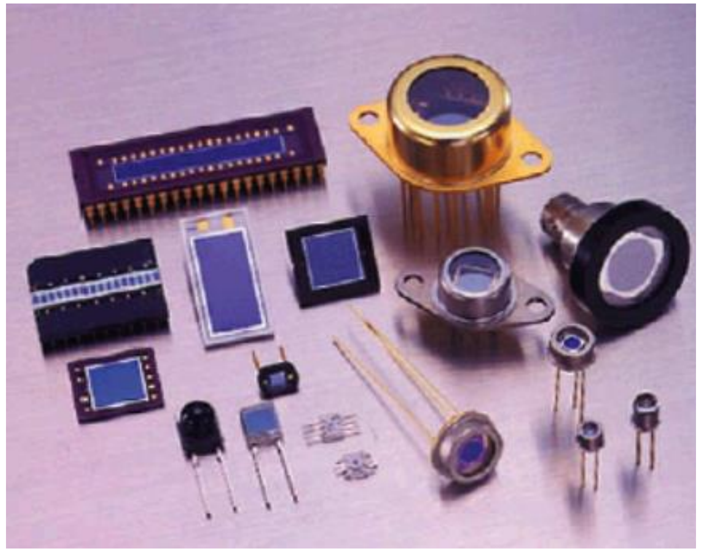
Why Photodiode Noise?

- Noise is a key parameter in photodiode design
  - Wide bandwidth (integrate more noise)
  - Low signal levels (noise more critical)
- Photodiode amplifier noise is more complex
  - Parasitic capacitance and sensor capacitance
  - Poles and zeros
  - Gain peaking

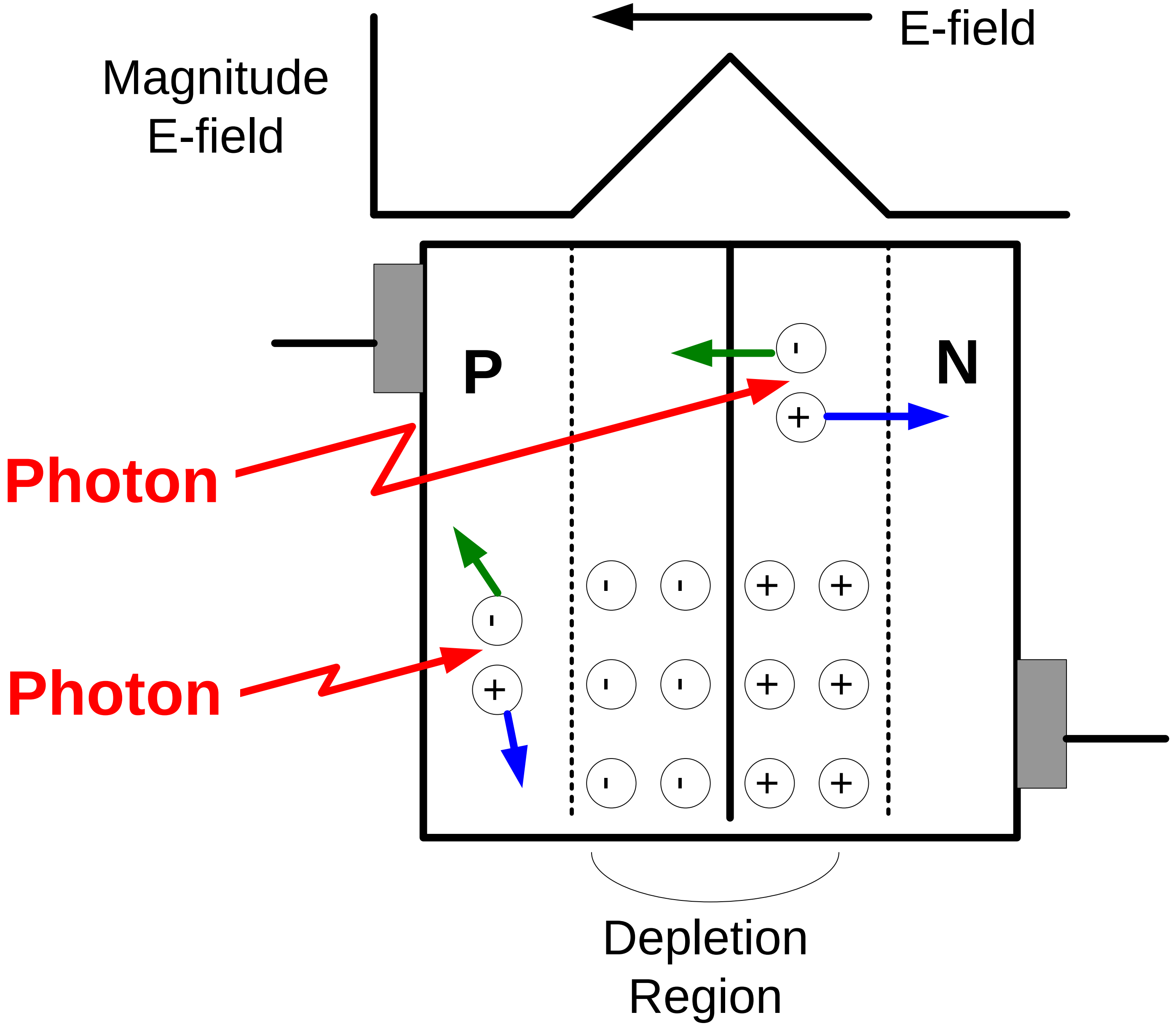
Photodiode Basics

- **Introduction**
- Photodiodes convert light into current or voltage.
- **Photodiode type**
- PIN photodiode – wide spectral range (less selective), PIN = p-doping, intrinsic material, n-doping (most popular)
- PN photodiode – more wavelength selective
- APD (Avalanche photodiode) – sensitive to low light, fast





Basic Photodiode Physics



Photodiode model

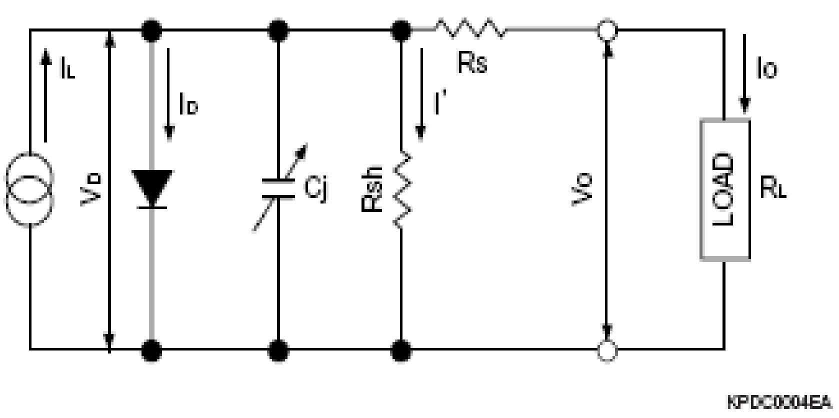


Figure1.4 Photodiode Equivalent Circuit

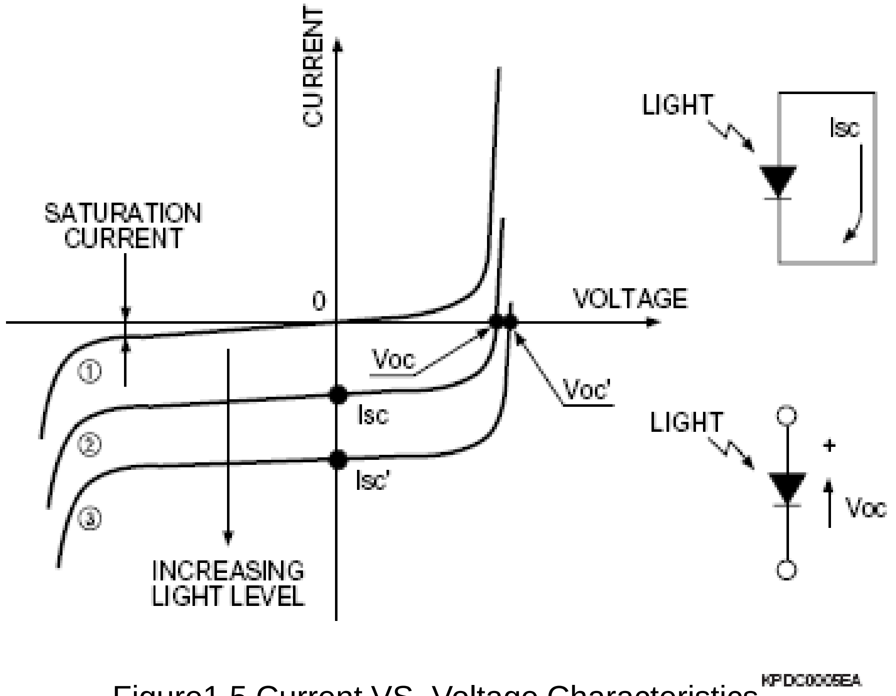


Figure1.5 Current VS. Voltage Characteristics

Output current is given as :

$$I_o := I_L - I_D - I' = I_L - I_S\left(e^{\frac{eV_D}{kT}} - 1\right) - I'$$

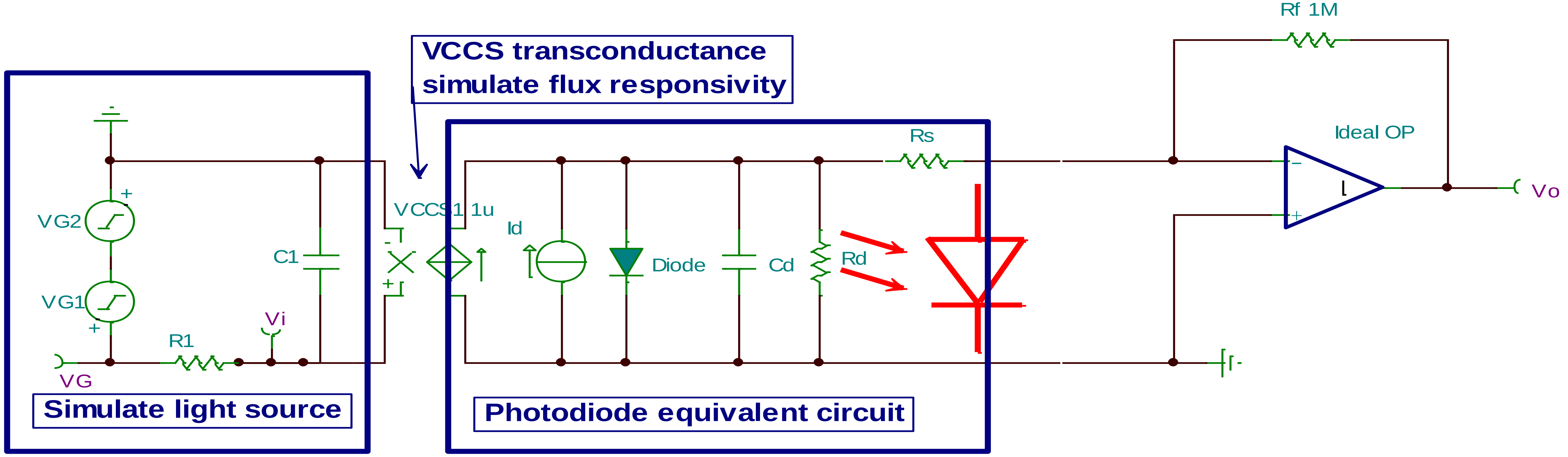
I_S : Photodiode reverse saturation current
e: electron charge
k: Boltzmann's constant
T: Absolute temperature of the photodiode

I_L = light current
I_D = dark current
I' = leakage current

The open circuit voltage Voc is the output voltage when Io equals 0. Thus Voc becomes:

$$V_{oc} := \frac{kT}{c} \ln\left(\frac{I_L - I'}{I_S} + 1\right)$$

Photodiode and Control Source TINA model


VCCS transconductance simulate flux responsivity

Simulate light source

Photodiode equivalent circuit

Light exciting source:

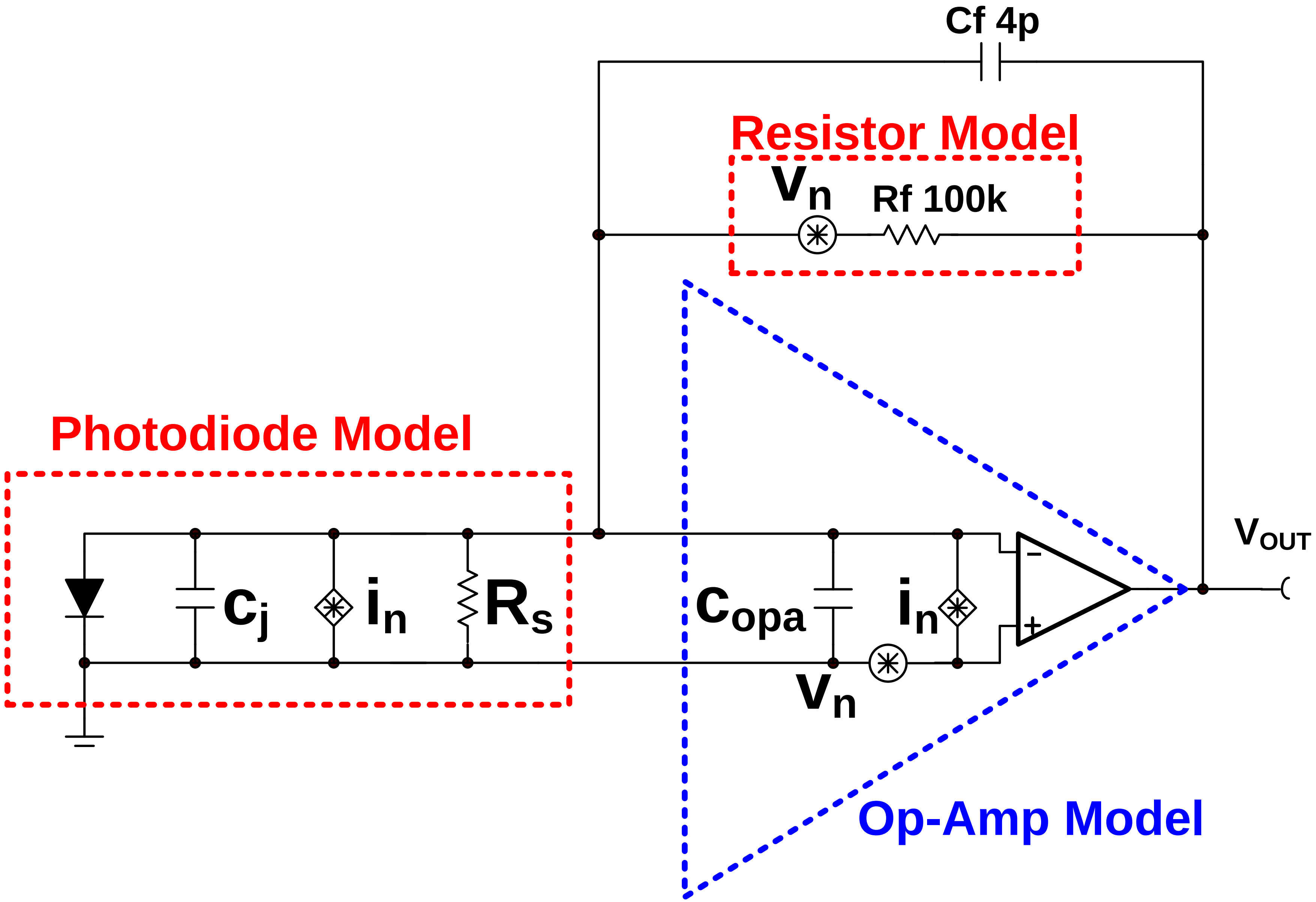
1) Use VG1 and VG2 voltage sources to simulate light power wave.
2) Use R1 and C1 shape light signal
3) The Voltage Control Current Source (VCCS1) simulates photodiode sensitivity.

Photodiode Equivalent Circuit:

1), Current Source Id simulates Dark current
2), Diode is a ideal diode
3), Cd and Rd simulate photodiode's junction capacitor and dark Resistance.
4), Rs is series resistor, which is far smaller than Rd.

Photodiode Noise Theory

Photo-Diode Amp Noise Model



Photodiode noise

Thermal (Johnson Noise)

$$i_j = \sqrt{\frac{4k_b \cdot T_n}{R_{sh}}}$$

Shot noise (dark)

$$i_{sD} = \sqrt{2q \cdot I_D}$$

Shot noise (w . Light)

$$i_{sL} = \sqrt{2q \cdot I_L}$$

Total Diode Current Noise

$$i_n = \sqrt{i_j^{\,2} + i_{sD}^{\,2} + i_{sL}^{\,2}}$$

k_b Boltzmann constant $1.38*10^{-23}$ J/K

q Electron Charge $1.6*10^{-19}$ C

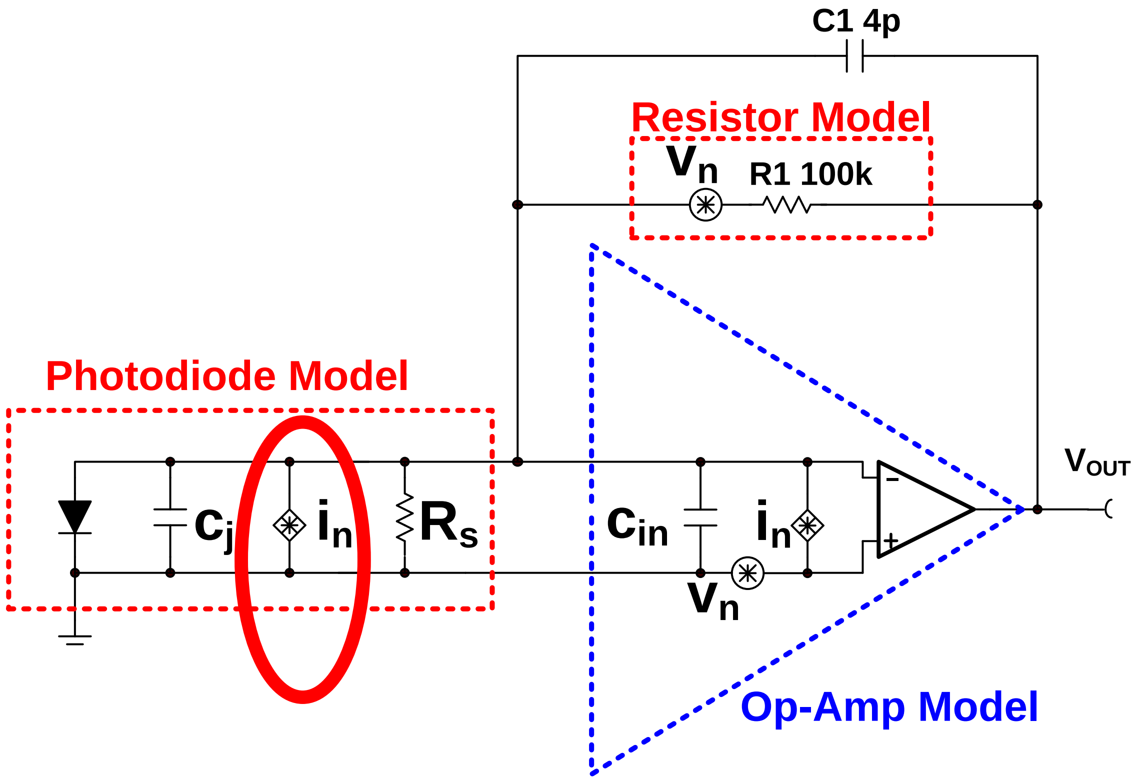
T_n Temperature in Kelvin (25C)

f_p Transconductance bandwidth

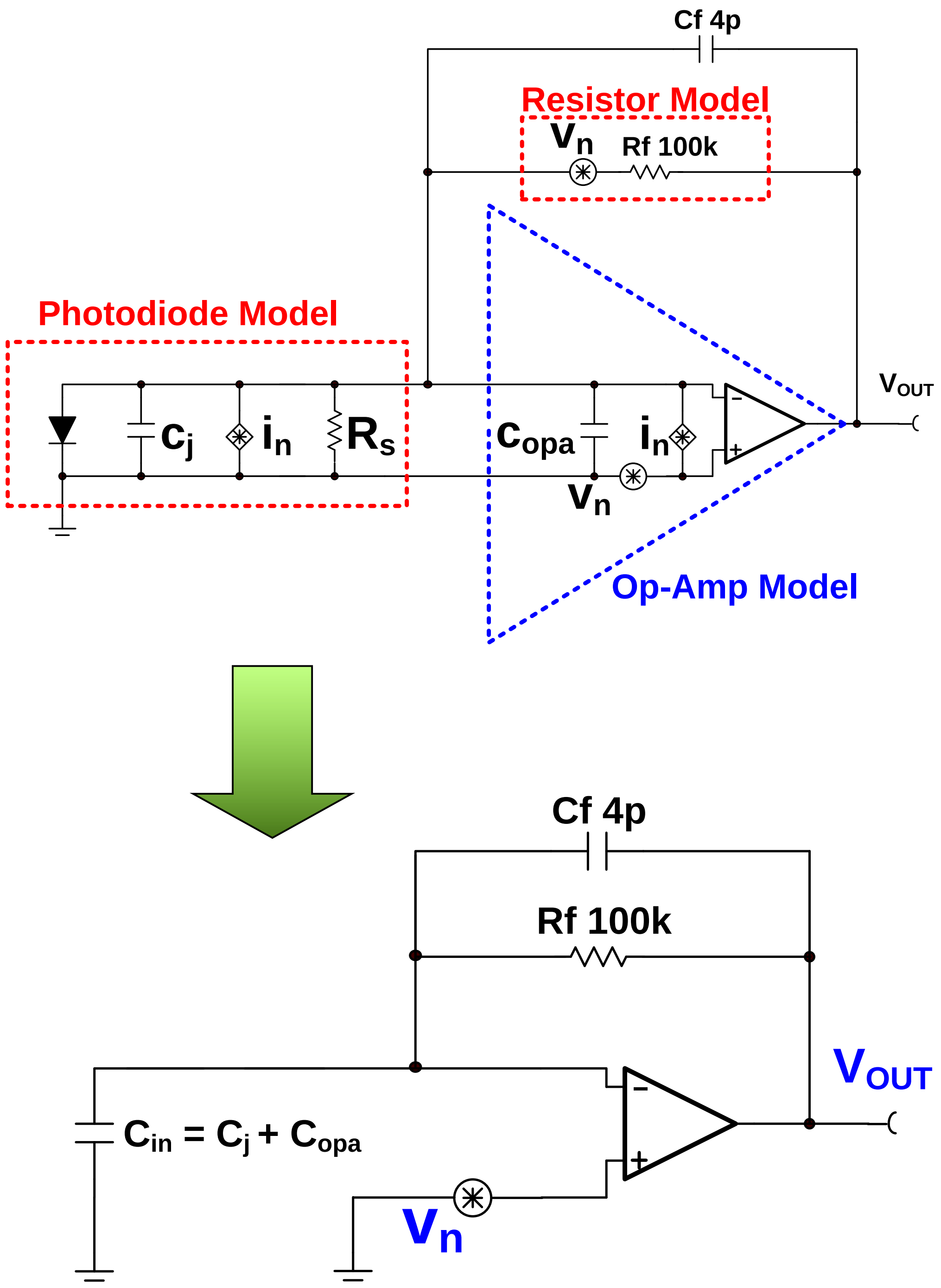
R_{sh} Shunt Resistance in photodiode

I_D Dark Current in photodiode

I_L Photo current in photodiode



Noise Gain



Simplify the model to compute Noise Gain

$$\text{Noise\_Gain} = \frac{V_{out}}{V_n}$$

Gain seen by the noise voltage source.

Noise Gain

Nodal Analysis on transimpedance amp

$$\frac{V_n}{\frac{1}{s\cdot C_{in}}} + \frac{(V_n - V_{out})}{R_f} + \frac{V_n - V_{out}}{\frac{1}{s\cdot C_f}} = 0$$

Solve for noise gain Vout / Vn

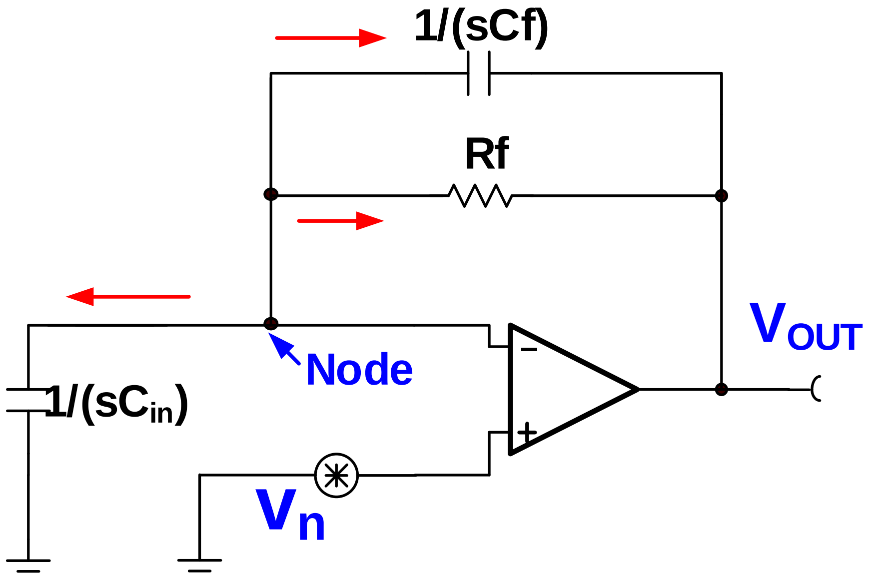
$$\frac{V_{out}}{V_n} = \frac{R_f\cdot(C_f + C_{in})\cdot s + 1}{C_f\cdot R_f\cdot s + 1}$$

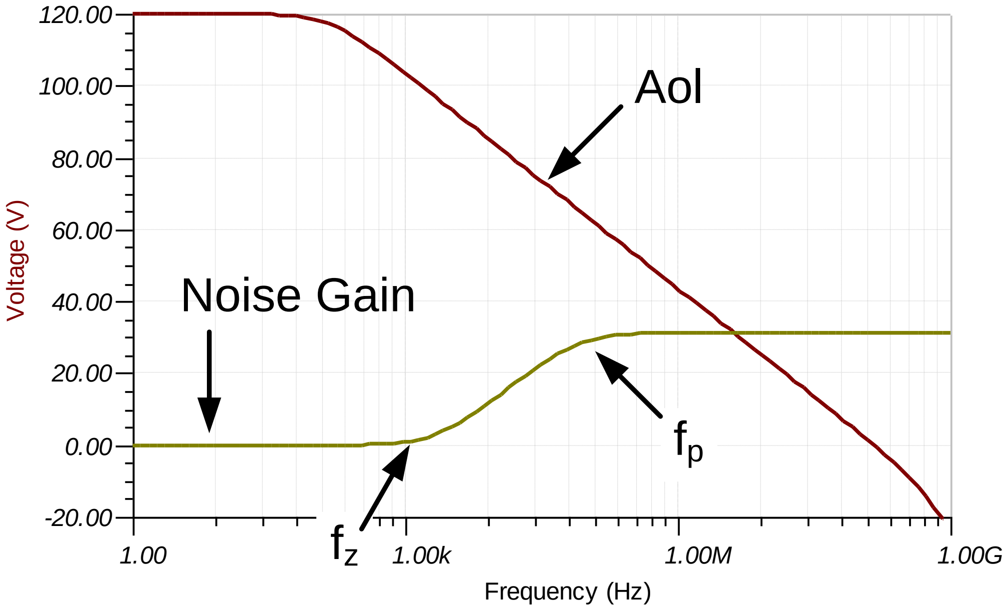
The numerator contains a **Zero**

$$f_z = \frac{1}{2\pi\, R_f\cdot(C_f + C_{in})}$$

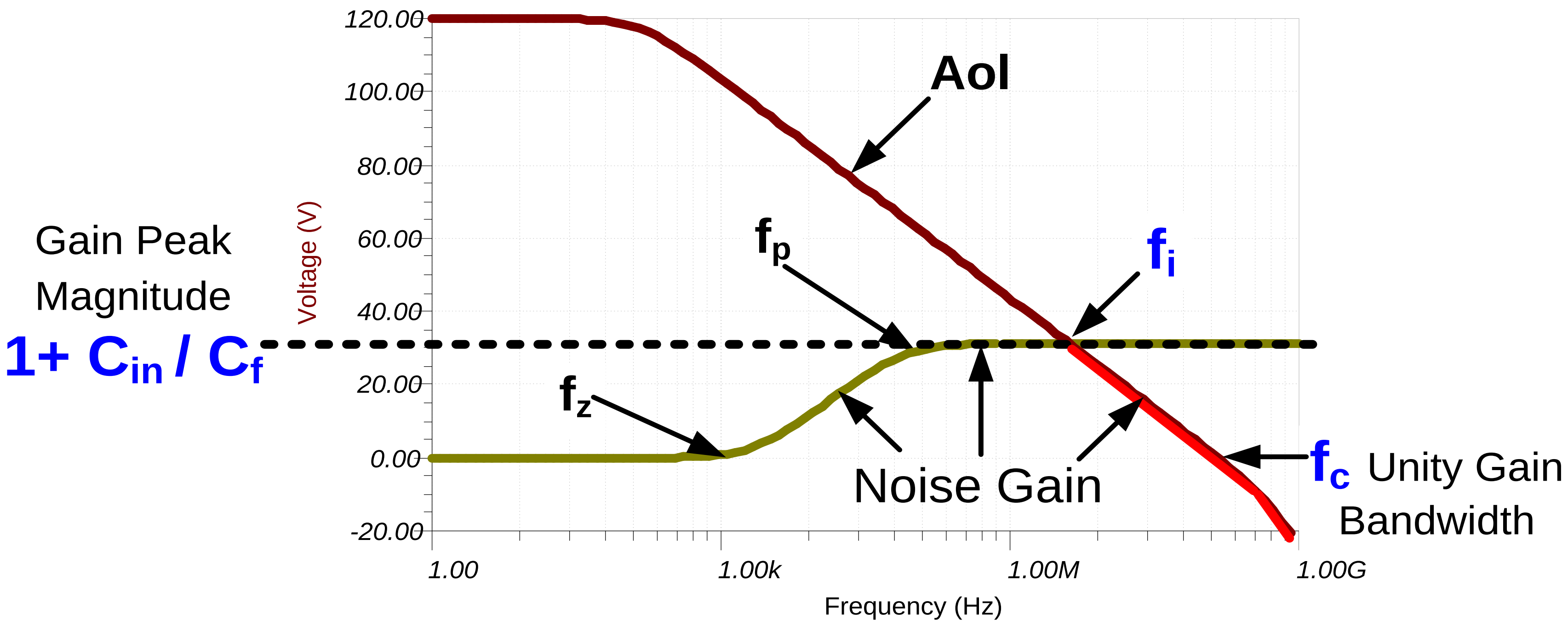
The denominator contains a **Pole**

$$f_p = \frac{1}{2\pi\, R_f\cdot C_f}$$



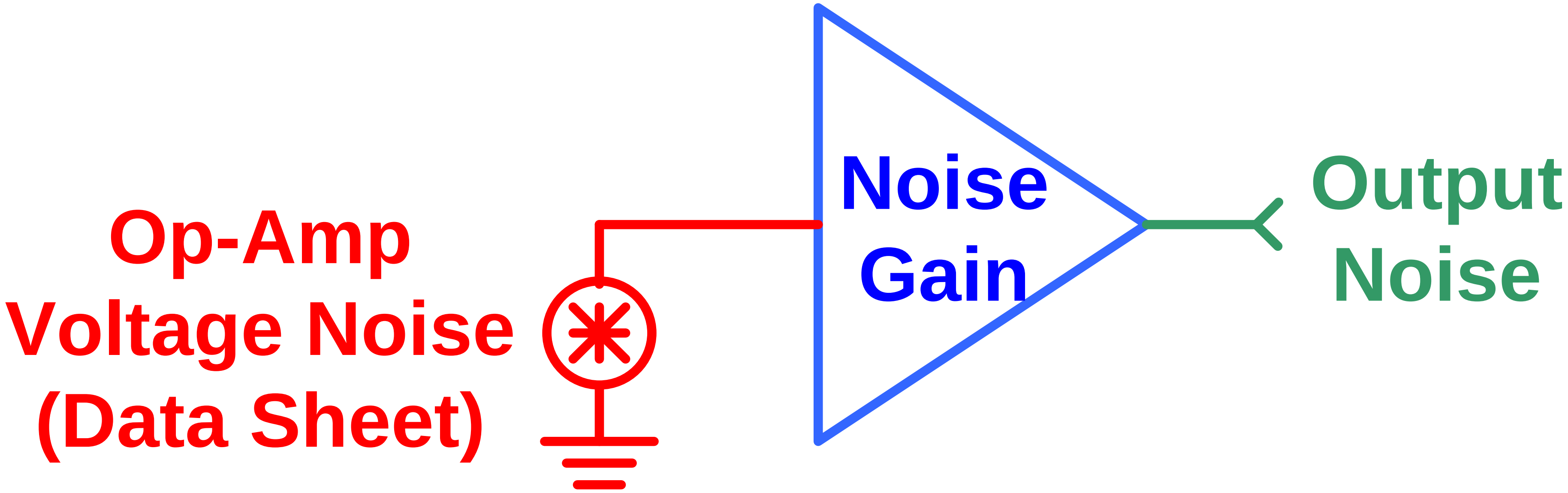


Noise Gain



$f_i = \frac{C_f}{C_i + C_f} \cdot f_c$	Intersection of the noise gain curve with the Aol Curve
f_c	Unity Gain Bandwidth from Op-Amp Data Sheet
$GPM = 1 + \frac{C_{in}}{C_f}$	Gain Peak Magnitude

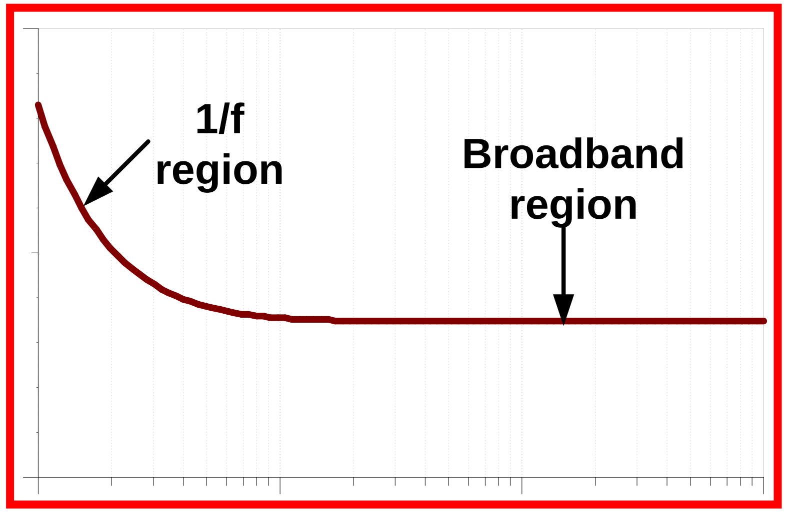
Noise Gain

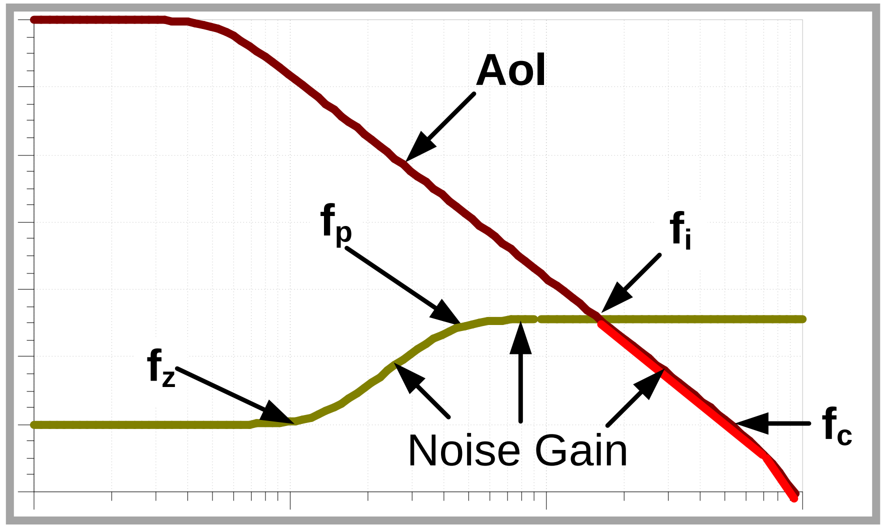


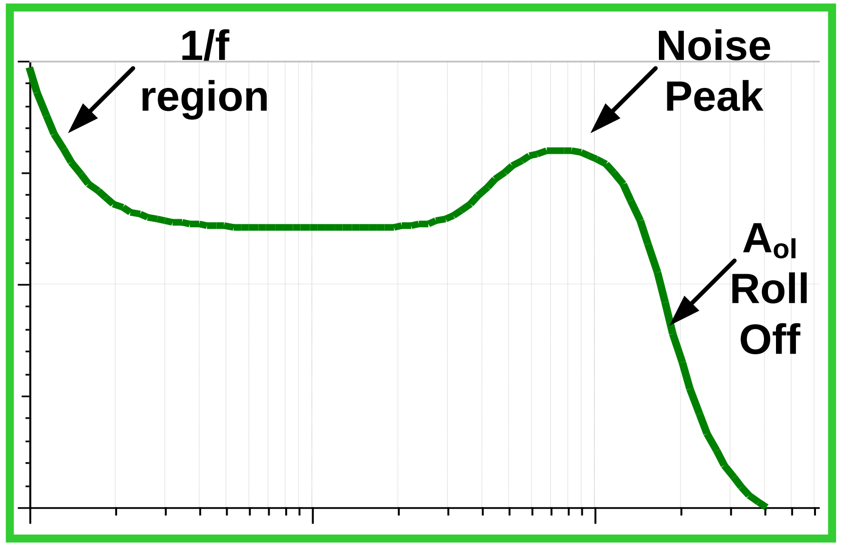
Op-Amp Voltage Noise (Data Sheet)

Noise Gain

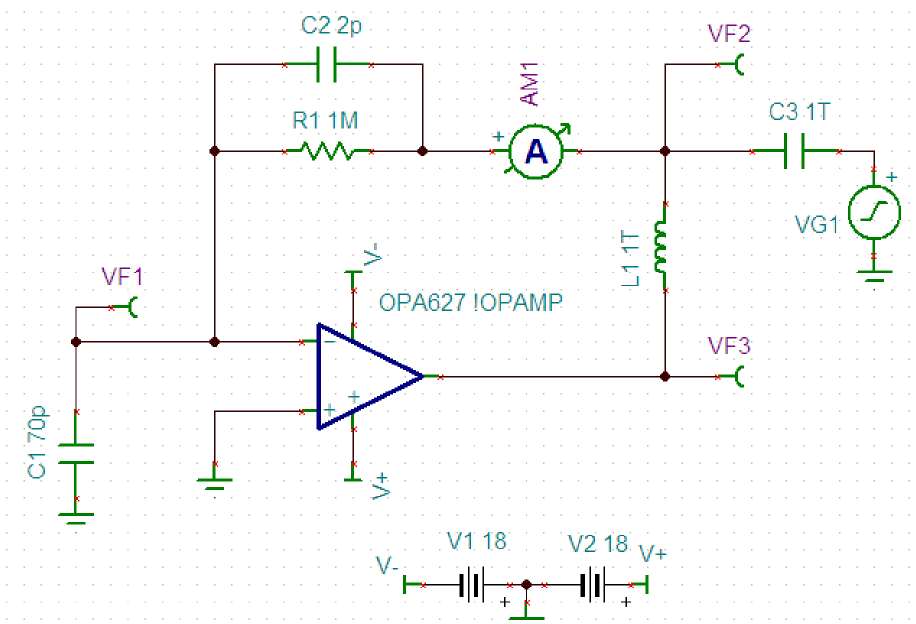
Output Noise







Simulating Noise gain and noise bandwidth

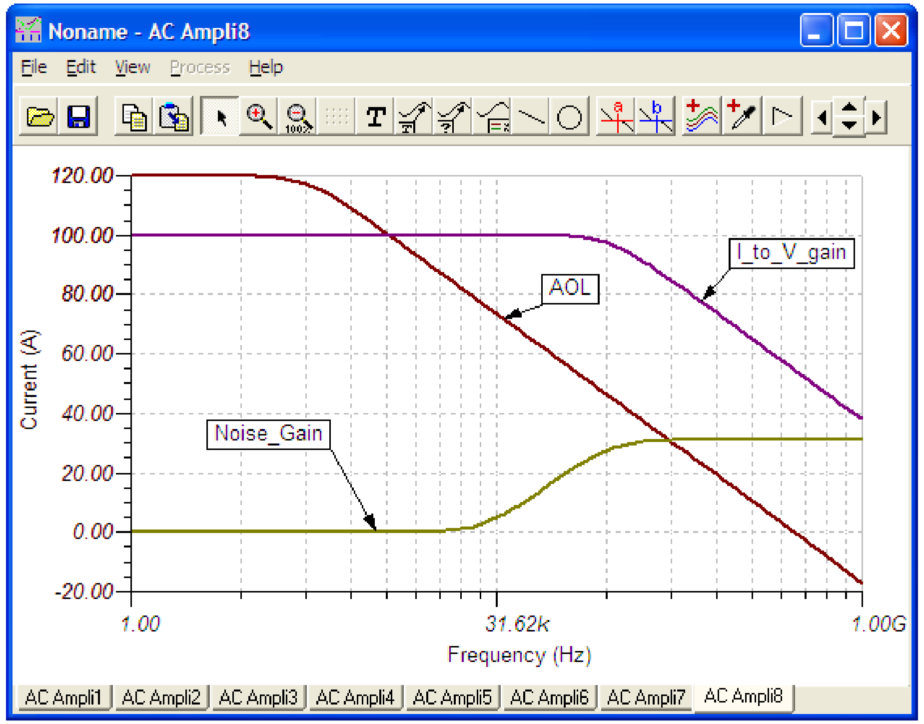


$$A_{ol} = \frac{VF3}{VF1}$$

$$Noise\_Gain = \frac{1}{\beta} = \frac{VF2}{VF1}$$

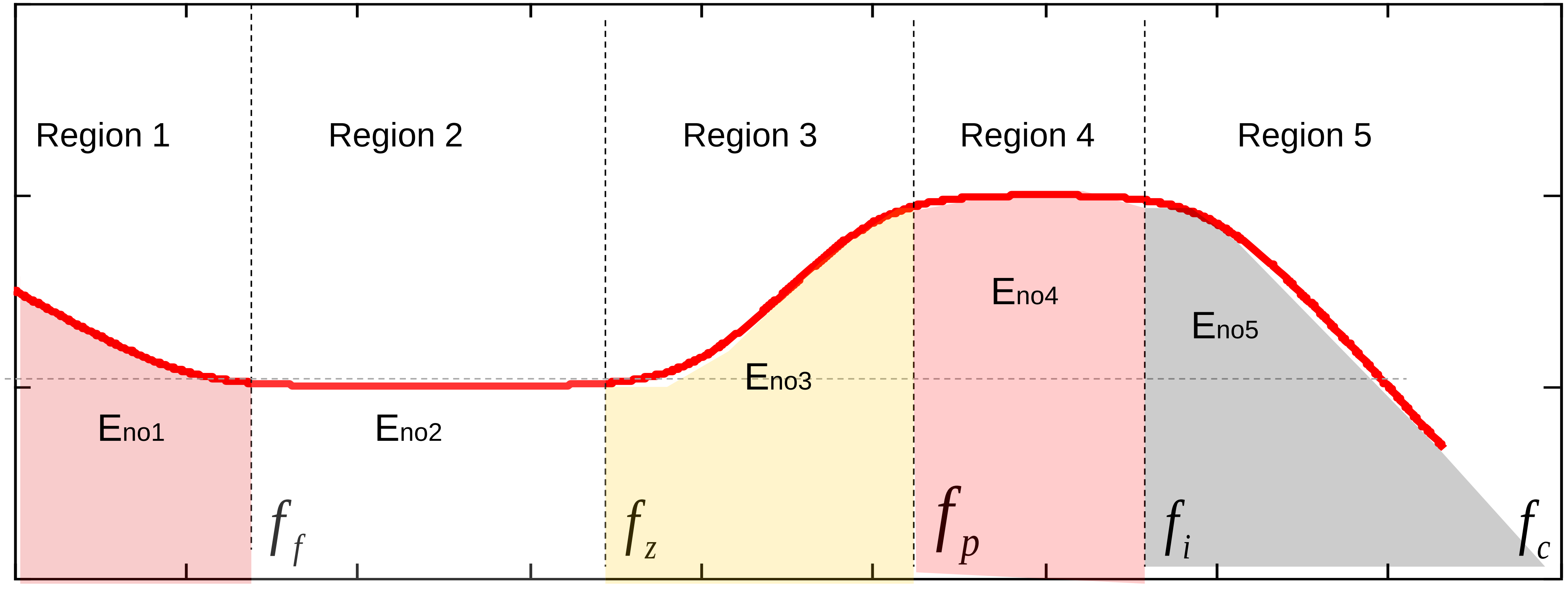
$$I\_to\_V\_Gain = \frac{VF3}{AM1}$$

- Break the loop to measure Aol, 1/B, and I to V Gain



Voltage Noise eni , eno and En

$$e_{no} = A_n \cdot e_{ni} = \frac{\left(1 + \frac{s}{\omega_z}\right) \cdot \sqrt{1 + \frac{\omega_f}{s}}}{\left(1 + \frac{s}{\omega_p}\right) \cdot \left(1 + \frac{s}{\omega_i}\right)}$$



Voltage Noise e_{ni} , e_{no} and E_{no}

Region 1 noise: $$E_{noe1}^{2} = \int_{f_L}^{f_f} \frac{e_{nif}^{2} \cdot f_f}{f} d_f = e_{nif}^{2} f_f \ln \frac{f_f}{f_L}$$

Region 2 noise: $$E_{noe2}^{2} = \int_{f_f}^{f_z} e_{nif}^{2} d_f = e_{nif}^{2} \left(f_z - f_f \right)$$

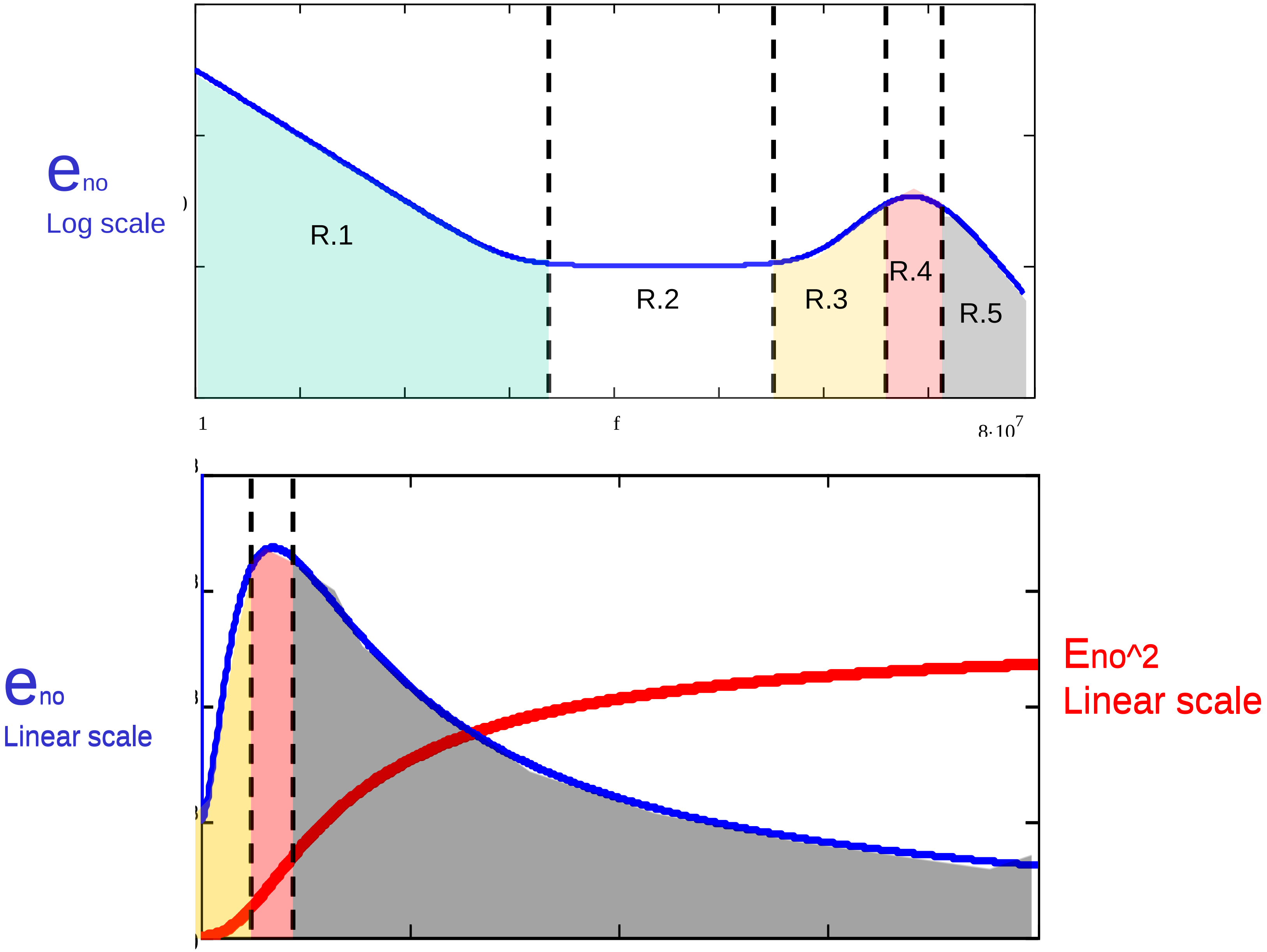
Region 3 noise: $$E_{noe3}^{2} = \int_{f_z}^{f_p} \frac{e_{nif}^{2} \cdot f^{2}}{f_z^{2}} d_f = \left(\frac{e_{nif}}{f_z} \right)^{2} \frac{f_p^{3} - f_z^{3}}{3}$$

Region 4 noise: $$E_{noe4}^{2} = \int_{f_p}^{f_i} \left(\frac{e_{nif}}{\beta} \right)^{2} d_f = \left(e_{nif} \cdot \frac{C_i + C_f}{C_f} \right)^{2} \left(f_i - f_p \right)$$

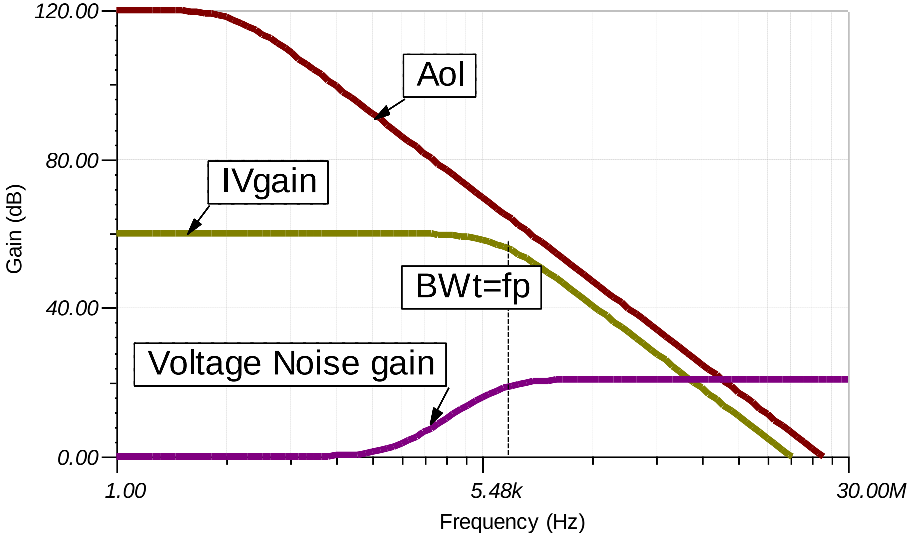
Region 5 noise: $$E_{noe5}^{2} = \int_{f_i}^{\infty} \left(\frac{e_{nif} f_c}{f} \right)^{2} d_f = \frac{\left(e_{nif} f_c \right)^{2}}{f_i}$$

Total voltage noise: $$E_{noe}^{2} = E_{noe1}^{2} + E_{noe2}^{2} + E_{noe3}^{2} + E_{noe4}^{2} + E_{noe5}^{2}$$

Voltage Noise e_{ni} , e_{no} and E_{no}



Resistor Noise and Current Noise



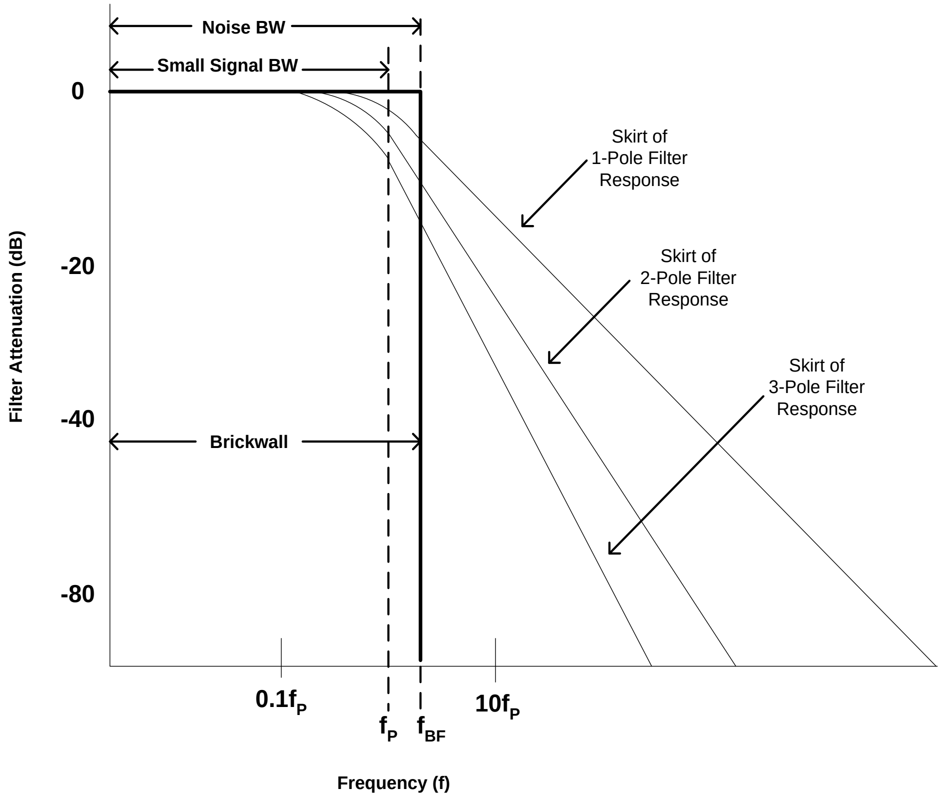
$$BW_n = K_n \cdot f_p$$

$$E_{noR} = \sqrt{4K \cdot T \cdot R_f \cdot BW_n}$$

$$E_{no1} = i_{ni} \cdot R_f \cdot BW_n$$

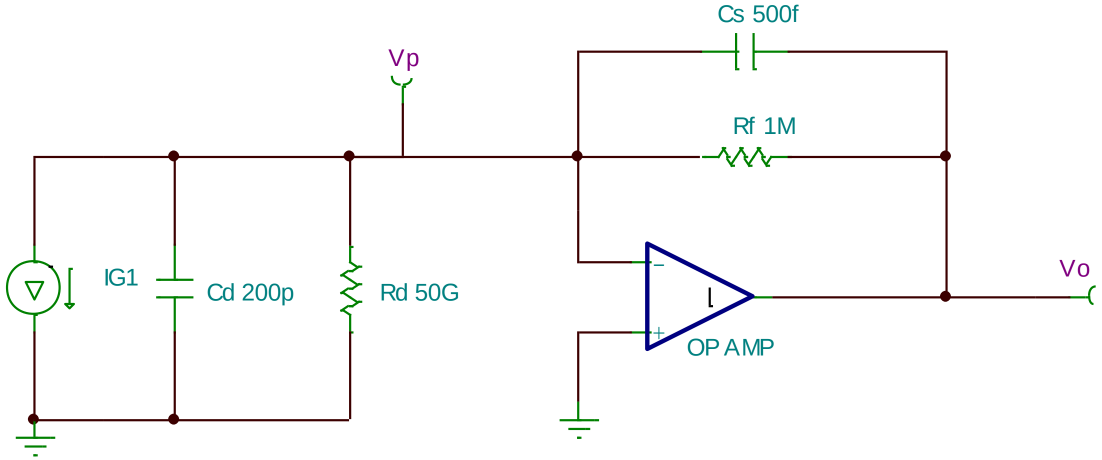
Current noise and resistor noise are limited by the transimpedance (I-V gain) bandwidth

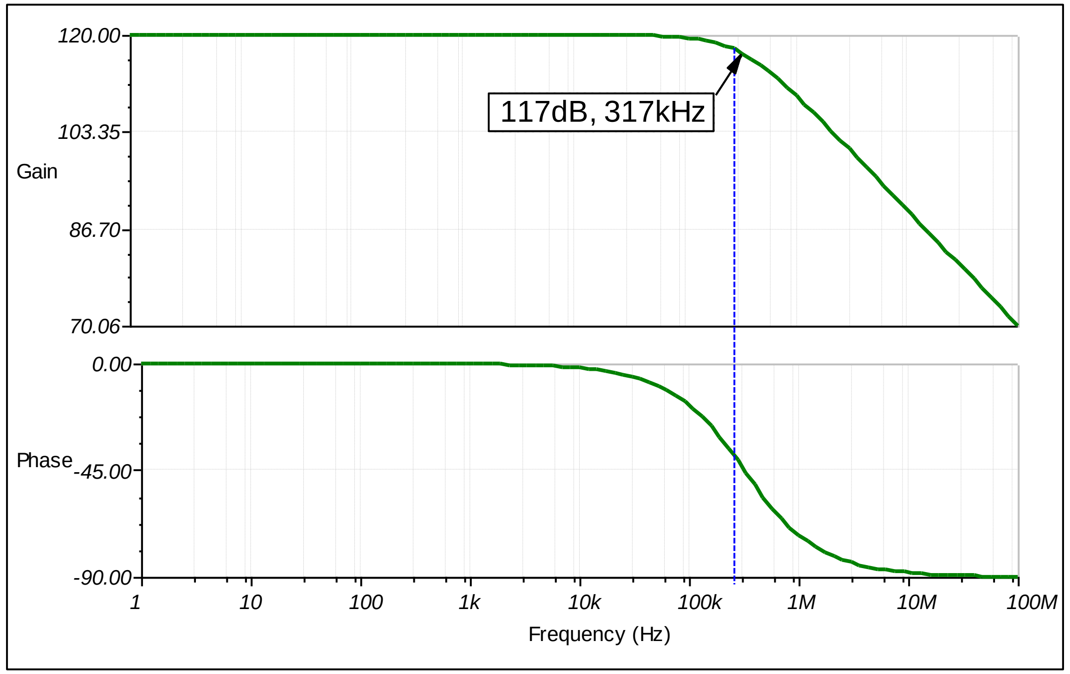
Poles	Kn
1	1.57
2	1.22
3	1.16



Bandwidth and Stability

Parasitic Capacitance Limits the Bandwidth

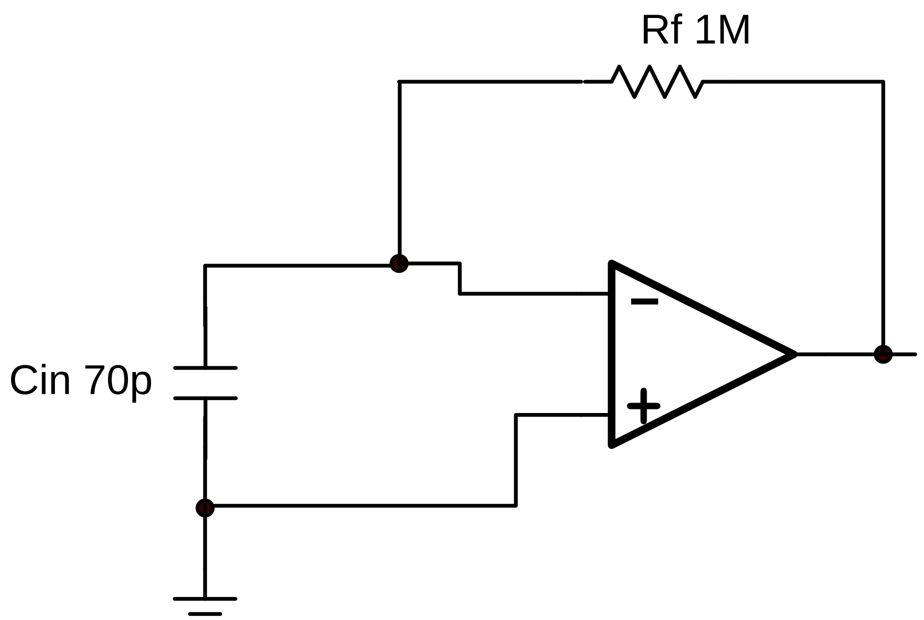


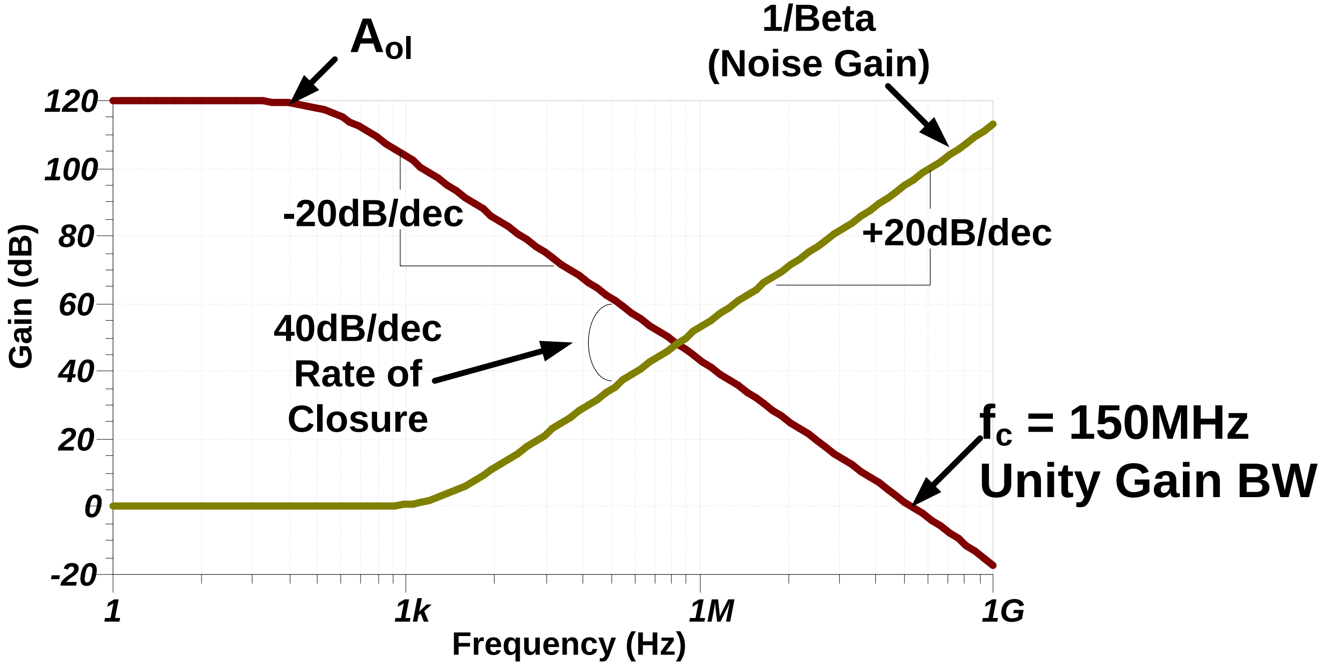


$$f_p = \frac{1}{2\pi\ R_f \cdot C_f} = 318\text{kHz}$$

- Max bandwidth with Min Cf
- Low Cf may be unstable
- Wide BW increases noise
- As shown Cf=Cs (stray cap)

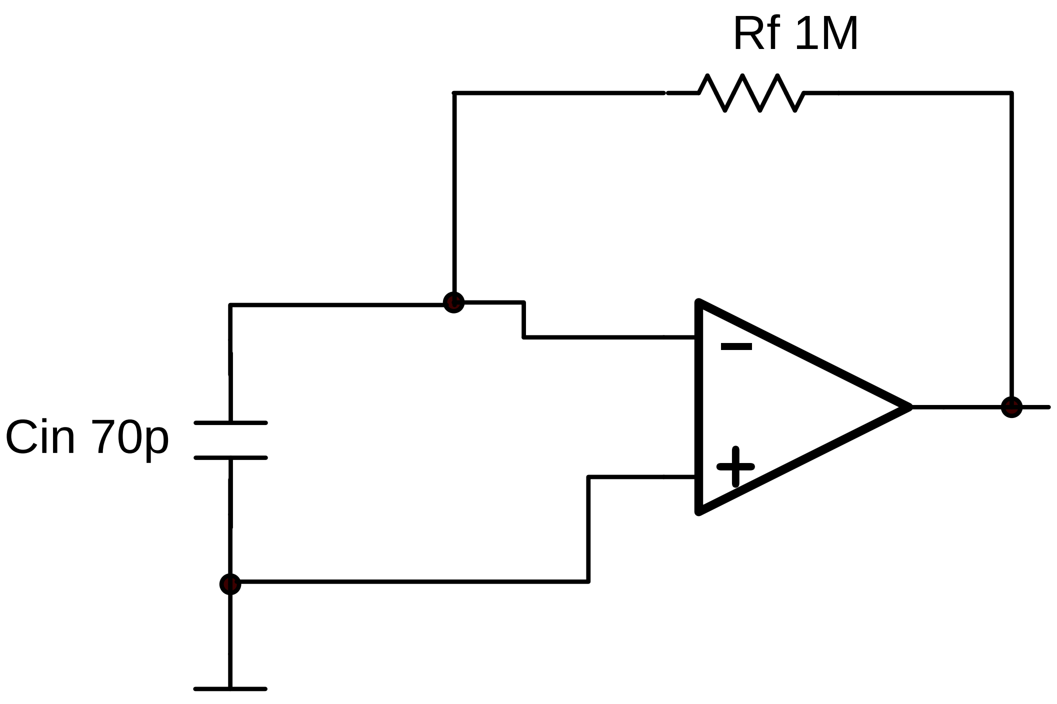
Feedback Capacitance Required for Stability

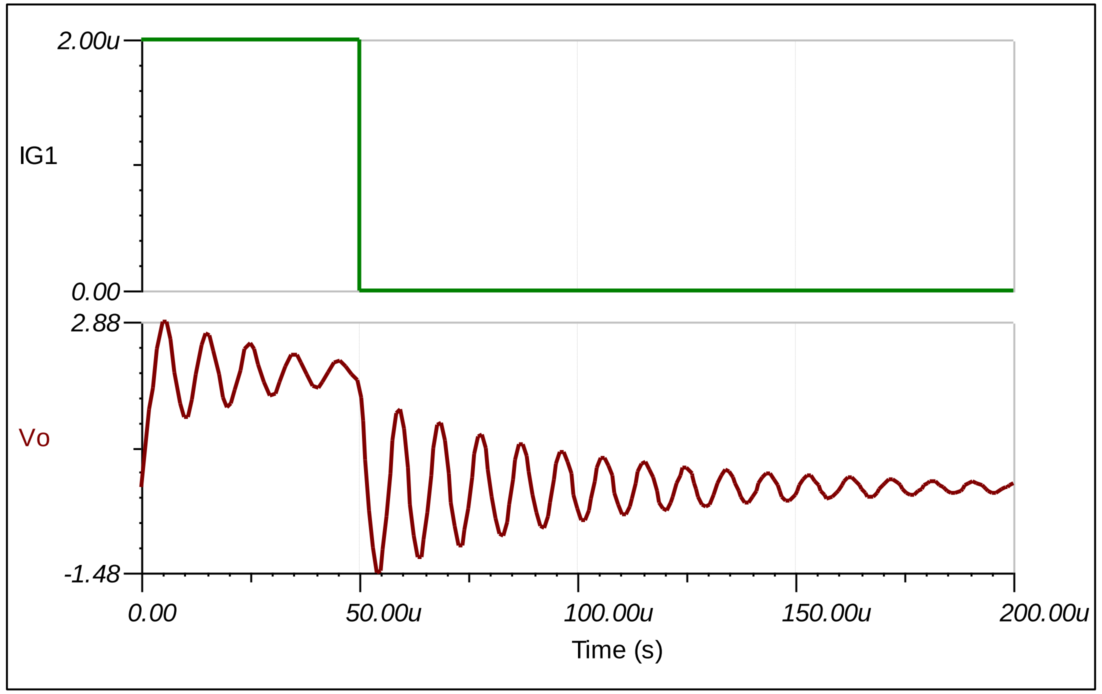




- Noise Gain is key to stability
- Also called 1/Beta (in stability analysis)
- ROC = Rate of Closure
- ROC = (Aol slope) – (1/Beta slope)
- Unstable when ROC > 20dB/decade

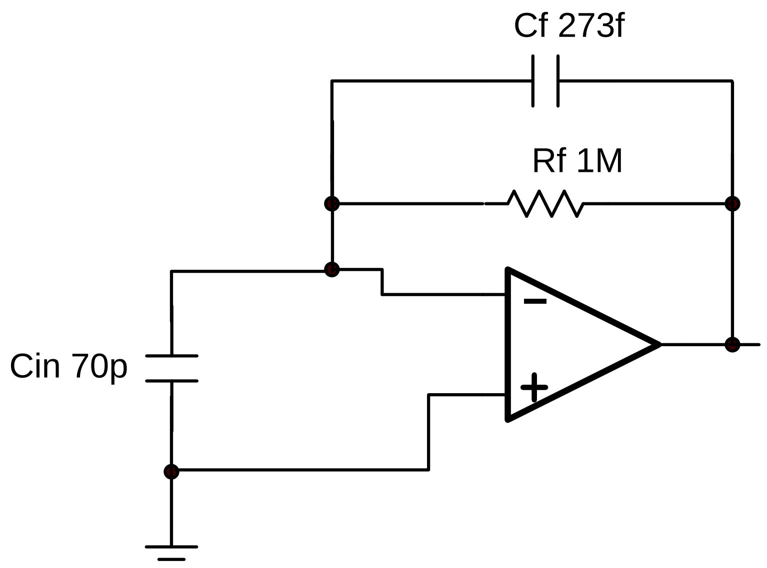
Feedback Capacitance Required for Stability

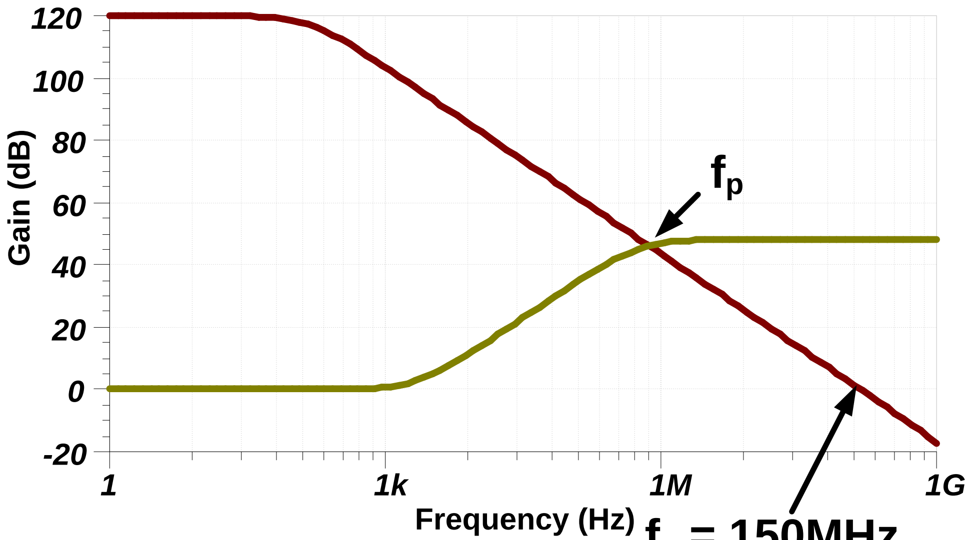




Applying a Step Input shows instability at output

Choosing a Minimum Cf for Stability





f_c = 150MHz
Unity Gain BW

$f_c = 150\text{MHz}$ Op-amp Unity Gain Bandwidth

$C_{in} = 70\text{pF}$ Total input capacitance

$R_f = 1\text{M}\Omega$ Feedback resistance

$$C_f = \sqrt{\frac{C_{in}}{2\pi \cdot R_f \cdot f_c}} = 272.5\text{fF}$$

Simplified equation for minimum feedback cap
Assumes Cin >> Cf

$$C_c = \frac{1}{2\pi \cdot R_f \cdot f_c}$$

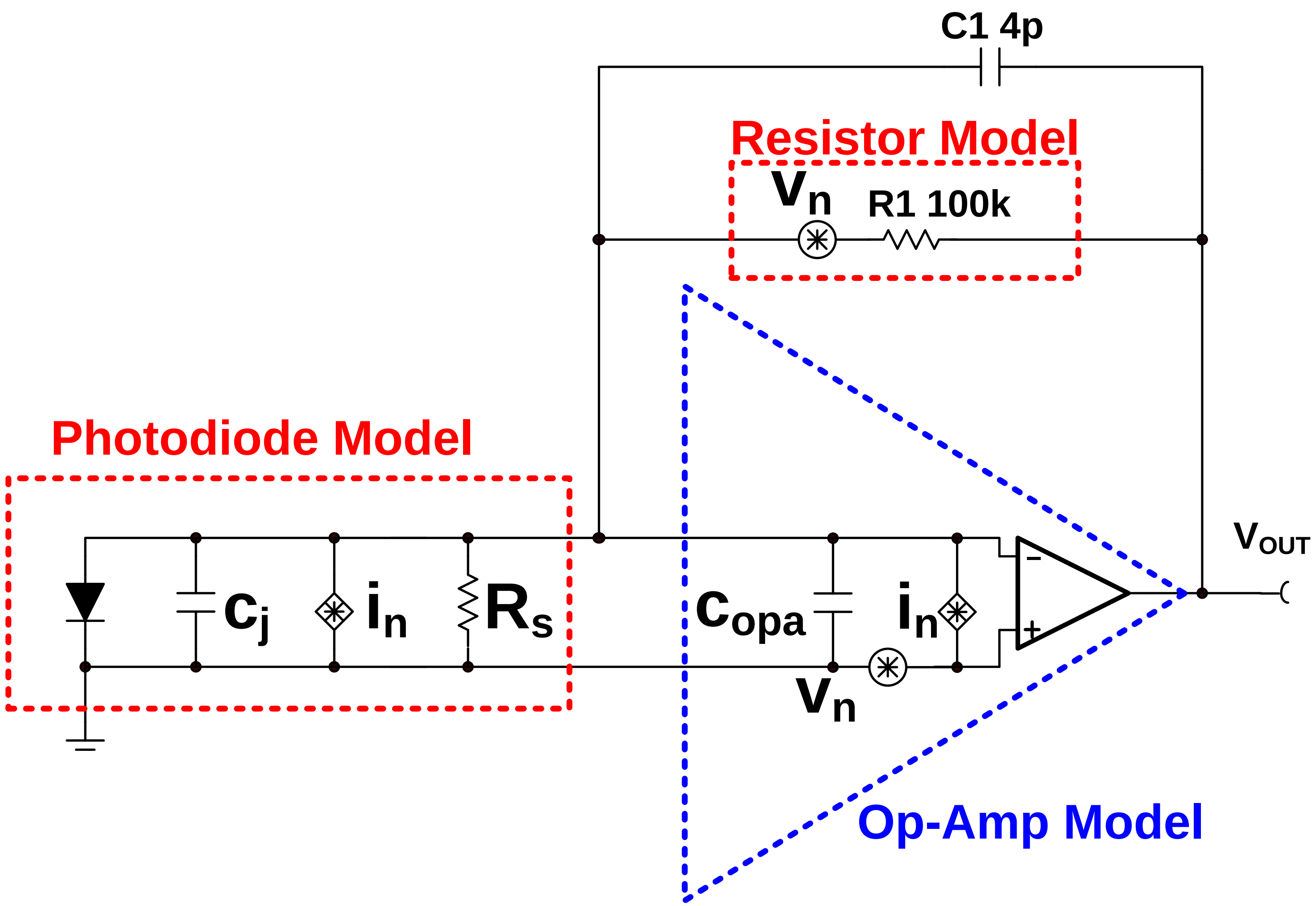
Intermediate calculation used in more exact formula

$$C_{fe} = \frac{C_c}{2} \cdot \left(1 + \sqrt{1 + \frac{4C_{in}}{C_c}}\right) = 273.1\text{fF}$$

More exact formula for feedback capacitance

OPA827 Hand Calculation

Noise Model for Simple Transimpedance Amp



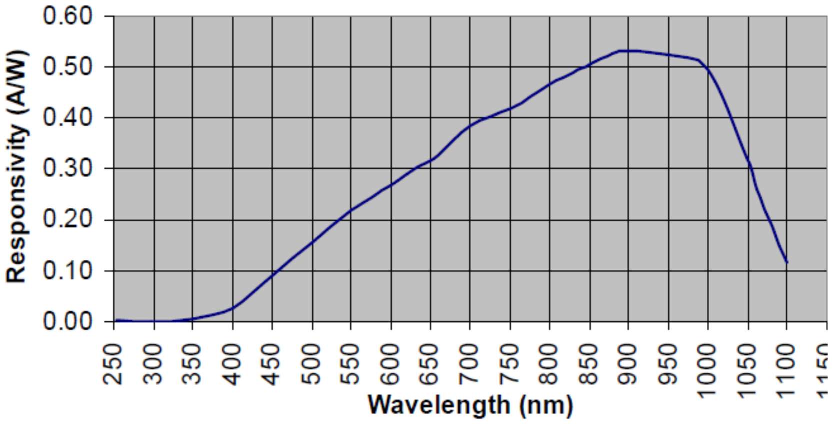
Example Photodiode: PDB-C158

ABSOLUTE MAXIMUM RATING (TA)= 23°C UNLESS OTHERWISE NOTED

SYMBOL	PARAMETER	MIN	MAX	UNITS
V_{BR}	Reverse Voltage		50	V
T_{STG}	Storage Temperature	-40	+100	°C
T_O	Operating Temperature	-40	+80	°C
T_S	Soldering Temperature*		+260	°C

* 1/16 inch from case for 3 seconds max.

SPECTRAL RESPONSE



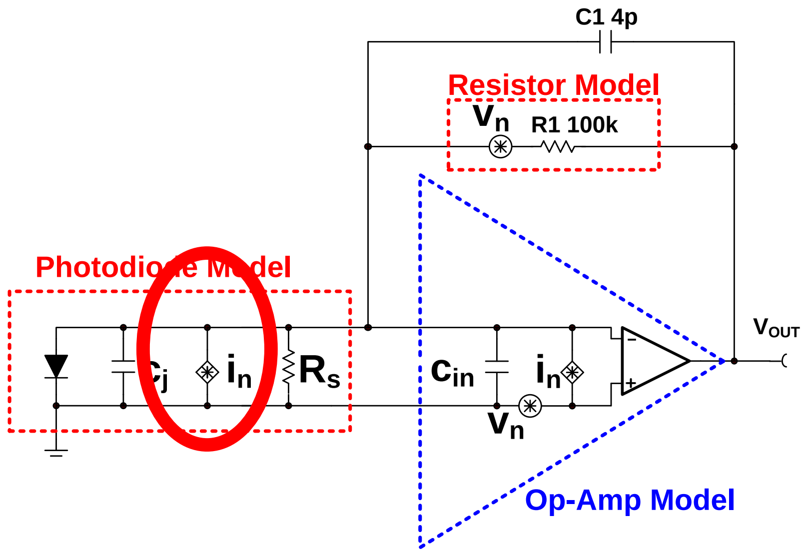
ELECTRO-OPTICAL CHARACTERISTICS RATING (TA)= 23°C UNLESS OTHERWISE NOTED

SYMBOL	CHARACTERISTIC	TEST CONDITIONS	MIN	TYP	MAX	UNITS
I_{SC}	Short Circuit Current	H = 100 fc, 2850 K	100	145		μA
I_D	Dark Current	V_R = 10 V		2	30	nA
R_{SH}	Shunt Resistance	V_R = 10 mV	100	150		MΩ
C_J	Junction Capacitance	V_R = 10 V, f = 1 MHz		10	25	pF
λrange	Spectral Application Range	Spot Scan	400		1100	nm
V_{BR}	Breakdown Voltage	I = 10 μA	30	75		V
NEP	Noise Equivalent Power	V_R = 10V @ λ = Peak		4.4×10^{-14}		W/ √ Hz
t_r	Response Time	RL = 1KΩ, V_R = 10 V		50		nS

Unfortunately Cj is not specified at Vr=0V.

We called the manufacturer for this info Cj=70pF for Vr=0V

Calculate Diode Current Noise



Thermal (Johnson Noise)

$$\sqrt{\frac{4k_b \cdot T_n}{R_{sh}}} = 10.472 \times 10^{-15} \frac{A}{\sqrt{Hz}}$$

Shot noise (dark)

$$\sqrt{2q \cdot I_D} = 25.314 \times 10^{-15} \cdot \frac{A}{\sqrt{Hz}}$$

Shot noise (w. Light)

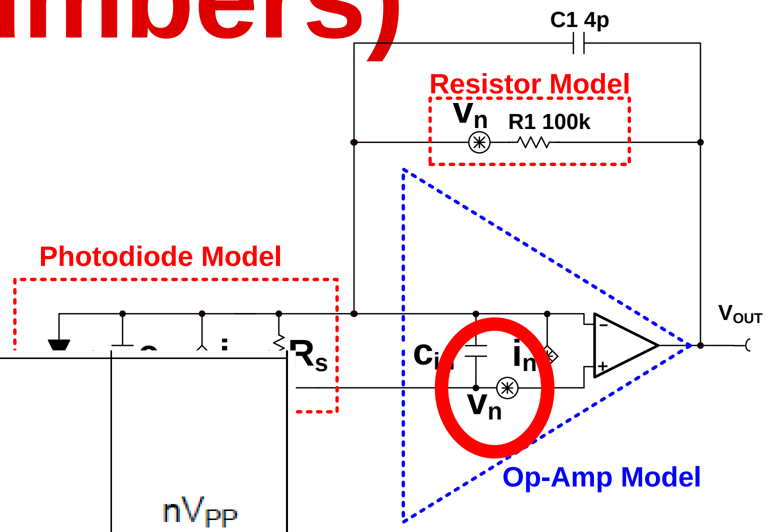
$$i_{sL} := \sqrt{2q \cdot I_L} = 0$$

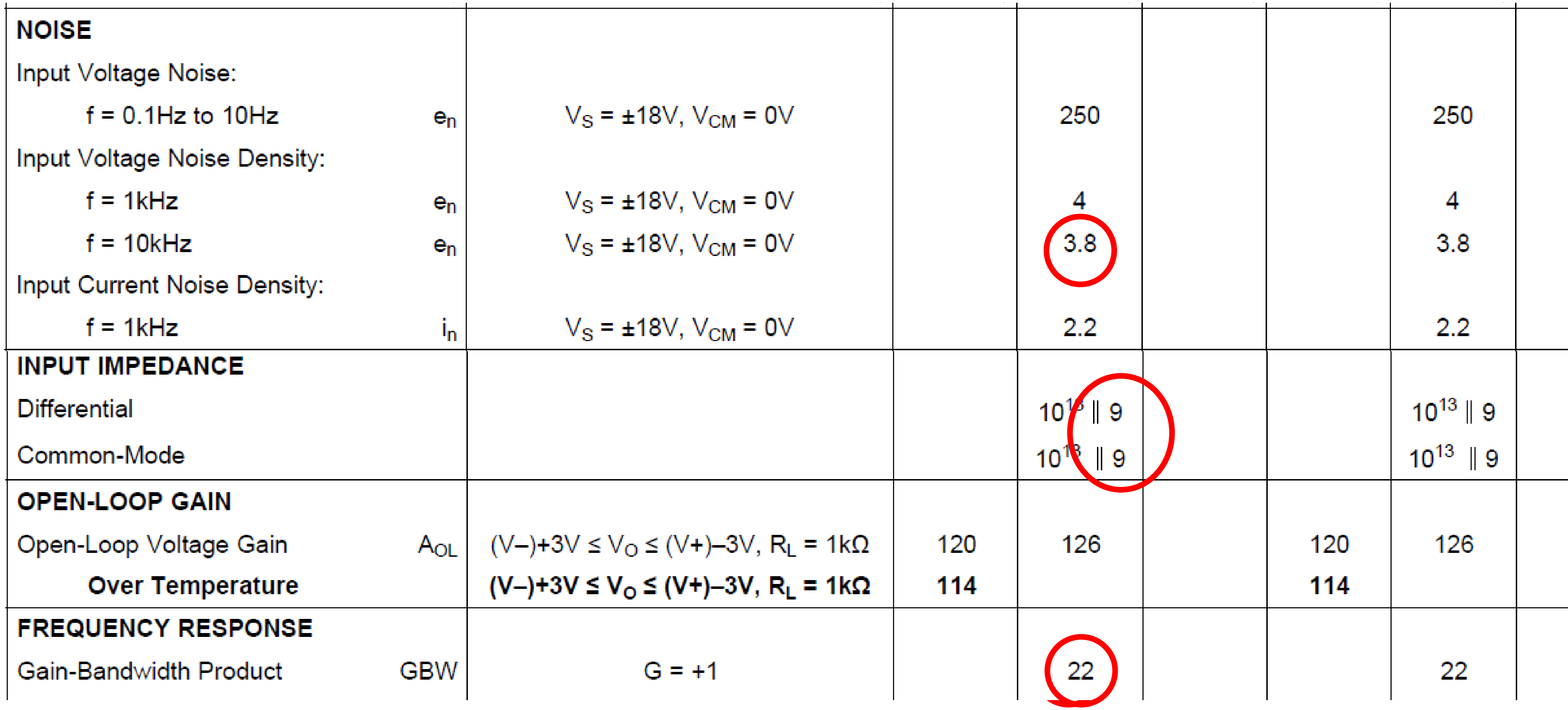
Total Diode Current Noise

$$\sqrt{i_j^{2} + i_{sD}^{2} + i_{sL}^{2}} = 27.395 \times 10^{-15} \cdot \frac{A}{\sqrt{Hz}}$$

$k_b := 1.38\,10^{-23} \frac{J}{K}$	Boltzmann constant
$q := 1.602\,10^{-19} C$	One electron Charge
$T_n := 298K$	Temperature in Kelvin (25C)
$f_p := 397.887 \times 10^{3}\ Hz$	Transconductance bandwidth
$R_{sh} := 150\,10^{6}\Omega$	Shunt Resistance in photodiode
$I_D := 2 \cdot 10^{-9} A$	Dark Current in photodiode
$I_L := 0A$	Photo current in photodiode (our measurements are dark)

OPA827 Noise Hand Calculation (key numbers)

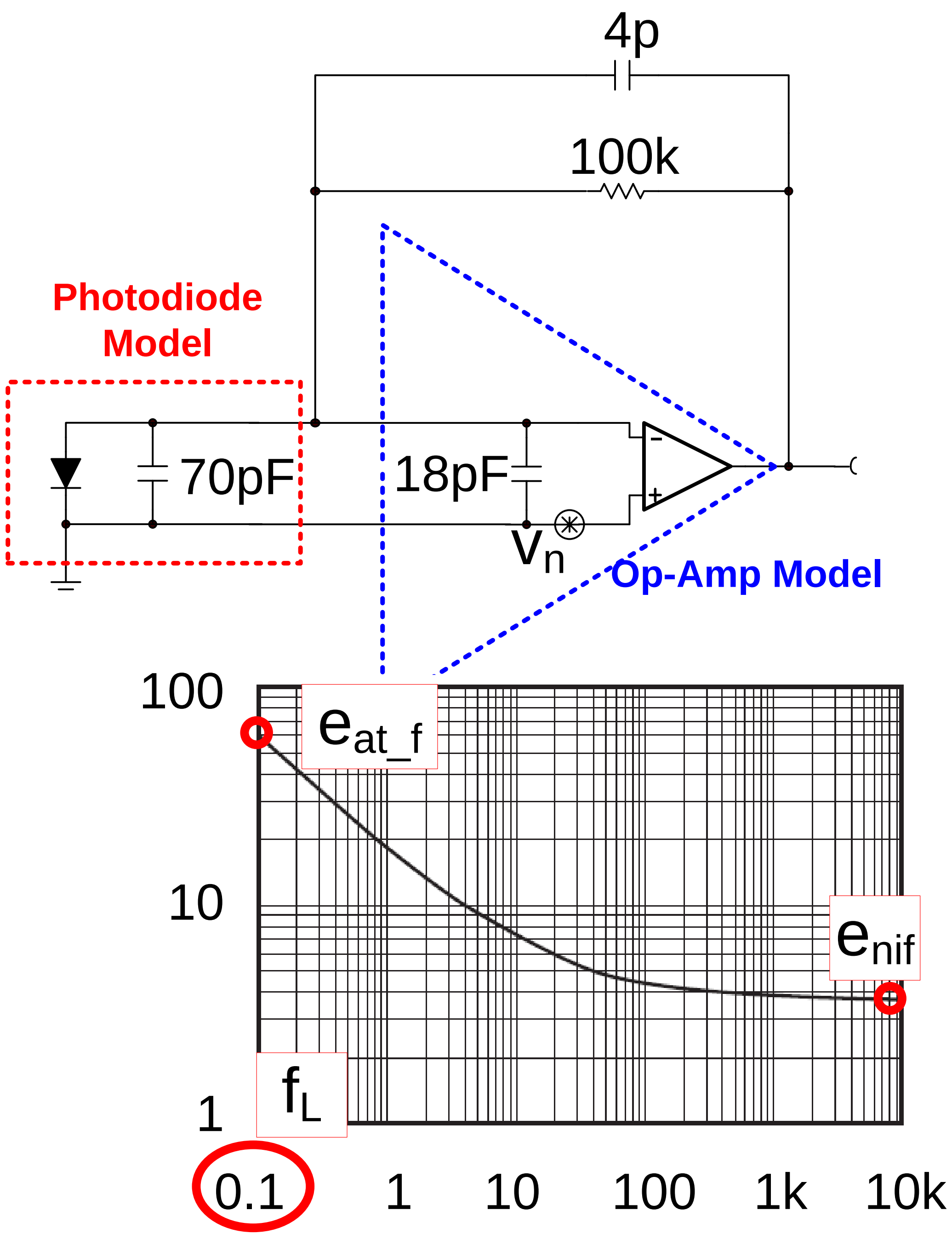




NOISE									
Input Voltage Noise:									
f = 0.1Hz to 10Hz	e_n	$V_S = \pm18V$, $V_{CM} = 0V$		250			250		nV_{PP}
Input Voltage Noise Density:									
f = 1kHz	e_n	$V_S = \pm18V$, $V_{CM} = 0V$		4			4		nV/√Hz
f = 10kHz	e_n	$V_S = \pm18V$, $V_{CM} = 0V$		3.8			3.8		nV/√Hz
Input Current Noise Density:									
f = 1kHz	i_n	$V_S = \pm18V$, $V_{CM} = 0V$		2.2			2.2		fA/√Hz
INPUT IMPEDANCE									
Differential				$10^{13} \parallel 9$			$10^{13} \parallel 9$		Ω ‖ pF
Common-Mode				$10^{13} \parallel 9$			$10^{13} \parallel 9$		Ω ‖ pF
OPEN-LOOP GAIN									
Open-Loop Voltage Gain	A_{OL}	$(V-)+3V \le V_O \le (V+)-3V$, $R_L = 1k\Omega$	120	126		120	126		dB
Over Temperature		**$(V-)+3V \le V_O \le (V+)-3V$, $R_L = 1k\Omega$**	**114**			**114**			**dB**
FREQUENCY RESPONSE									
Gain-Bandwidth Product	GBW	G = +1		22			22		MHz

OPA827 Noise Hand Calculation

$C_j := 70pF$	Photodiode Junction Capacitance (from photodiode manufacturer)
$C_{opa} := 18pF$	Opamp input capacitance (OPA827 Data Sheet)
$C_i := C_j + C_{opa}$	Total input capacitance
$f_c := 22MHz$	Unity Gain Bandwidth (OPA827 Data Sheet)
$R_f := 100k\Omega$	Feedback resistance
$C_f := 4pF$	Feedback capacitor
$e_{nif} := 3.8\frac{nV}{\sqrt{Hz}}$	Broadband Noise Spectral Density (OPA827 Data Sheet)
$f_L := .1Hz$	Lower bound on frequency (1/f region) (arbitrary lower bound of frequency)
$e_{at\_f} := 60\frac{nV}{\sqrt{Hz}}$	Flicker noise measured at f_L (OPA827 Data Sheet Noise Curve)

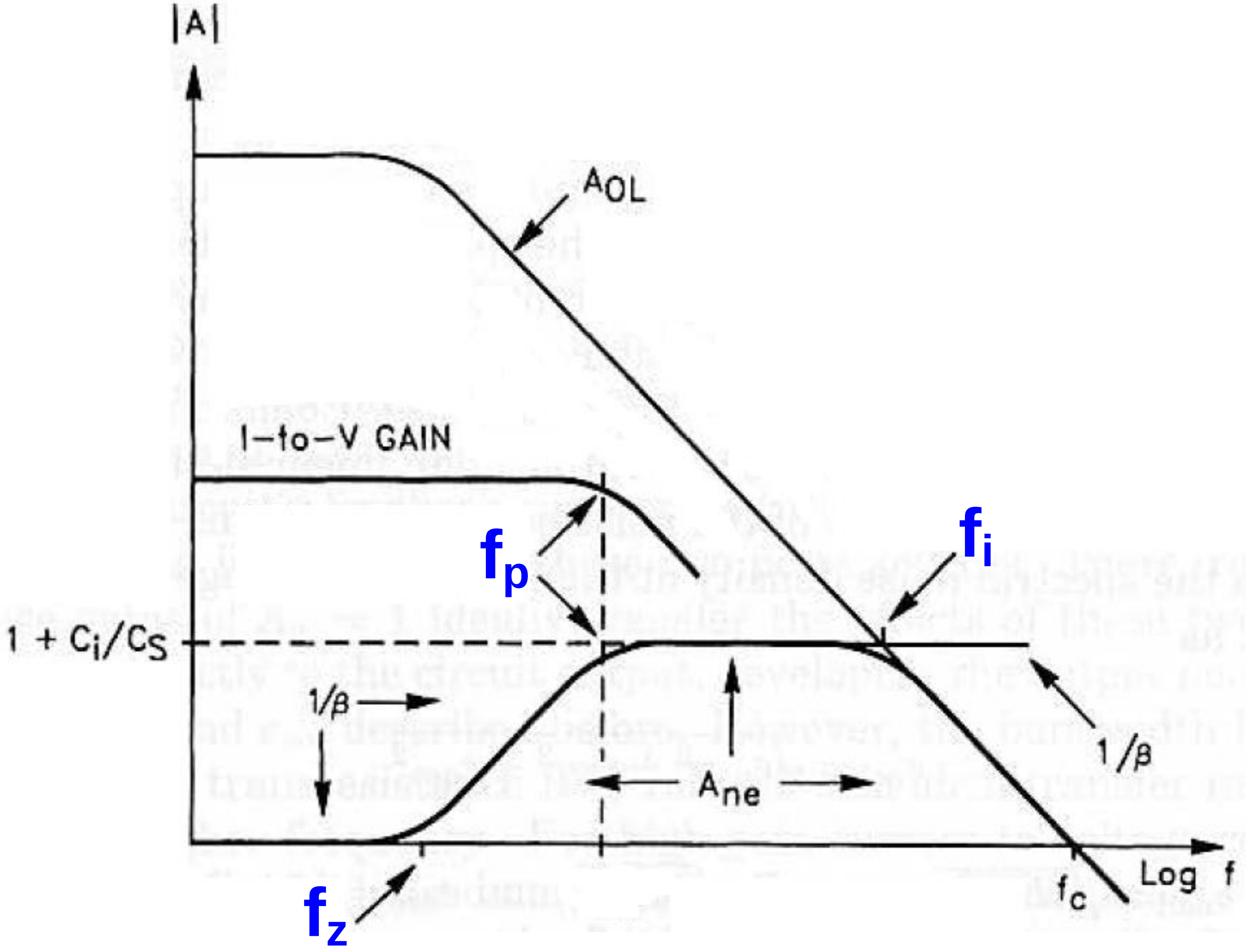


Poles and Zeros in Noise Gain Curve

$$f_z := \frac{1}{2\pi\, R_f \cdot \left(C_i + C_f\right)} = 17.299 \times 10^3\ \mathrm{Hz}$$

$$f_p := \frac{1}{2\pi\, R_f \cdot C_f} = 397.887 \times 10^3\ \mathrm{Hz}$$

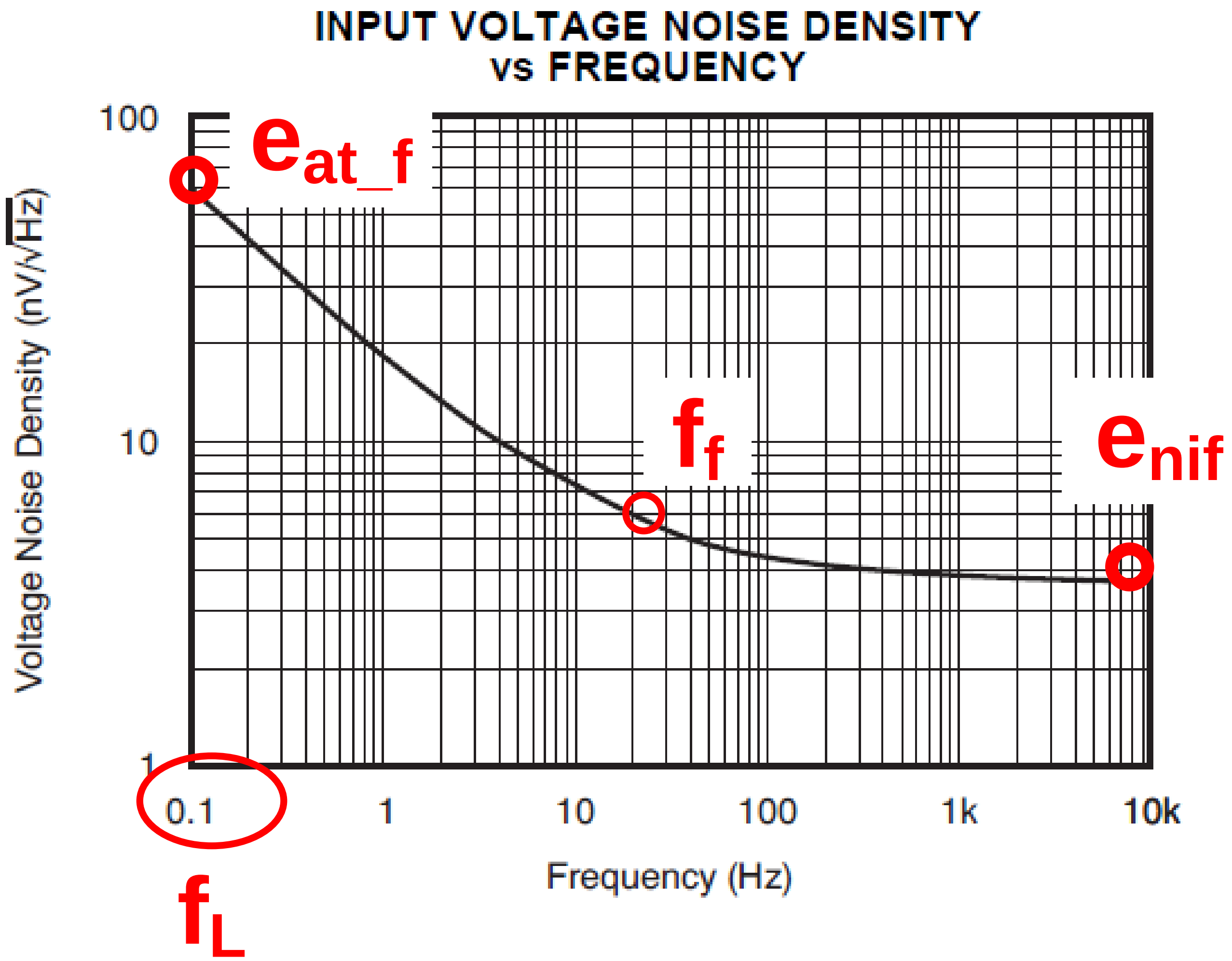
$$f_i := \frac{C_f}{C_i + C_f} \cdot f_c = 956.522 \times 10^3\ \mathrm{Hz}$$



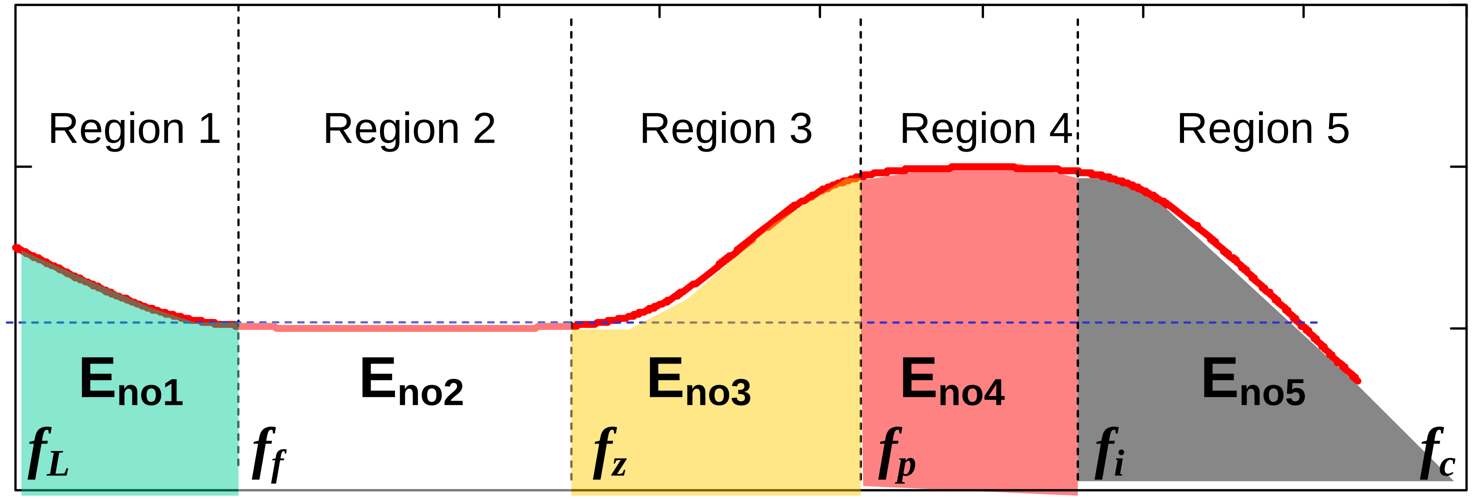
1/f (flicker) Noise Corner

$$e_{fnorm} := e_{at\_f} \cdot \sqrt{f_L} = 18.974 \times 10^{-9}\,V$$

$$f_f := \frac{e_{fnorm}^{\;2}}{e_{nif}^{\;2}} = 24.931\,Hz$$



Output Noise from OPA Noise Voltage



$$E_{noe1} := \sqrt{e_{nif}^{2} \cdot f_f \cdot \ln\left(\frac{f_f}{f_L}\right)} = 44.573 \times 10^{-9}\,V$$

$$E_{noe2} := \sqrt{e_{nif}^{2} \cdot \left(f_z - f_f\right)} = 499.444 \times 10^{-9}\,V$$

$$E_{noe3} := \sqrt{\left(\frac{e_{nif}}{f_z}\right)^{2} \cdot \frac{f_p^{3} - f_z^{3}}{3}} = 31.828 \times 10^{-6}\,V$$

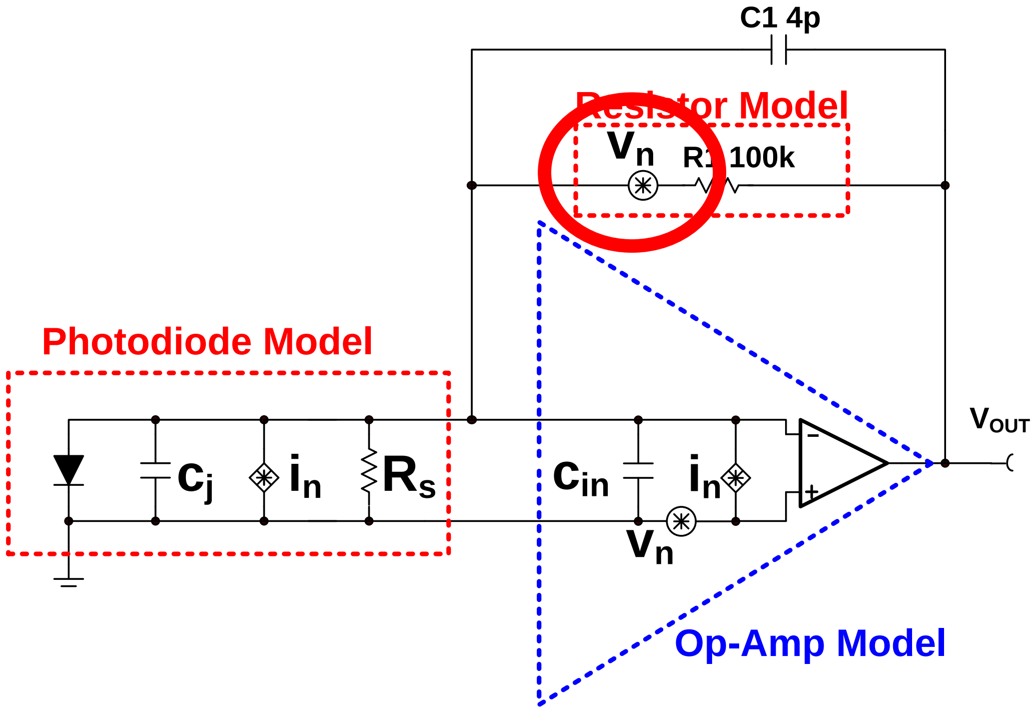
$$E_{noe4} := \sqrt{\left(e_{nif} \cdot \frac{C_i + C_f}{C_f}\right)^{2} \left(f_i - f_p\right)} = 65.324 \times 10^{-6}\,V$$

$$E_{noe5} := \sqrt{\frac{\left(e_{nif} \cdot f_c\right)^{2}}{f_i}} = 85.479 \times 10^{-6}\,V$$

$$\boxed{E_{noe} := \sqrt{E_{noe1}^{2} + E_{noe2}^{2} + E_{noe3}^{2} + E_{noe4}^{2} + E_{noe5}^{2}} = 112.193 \times 10^{-6}\,V}$$

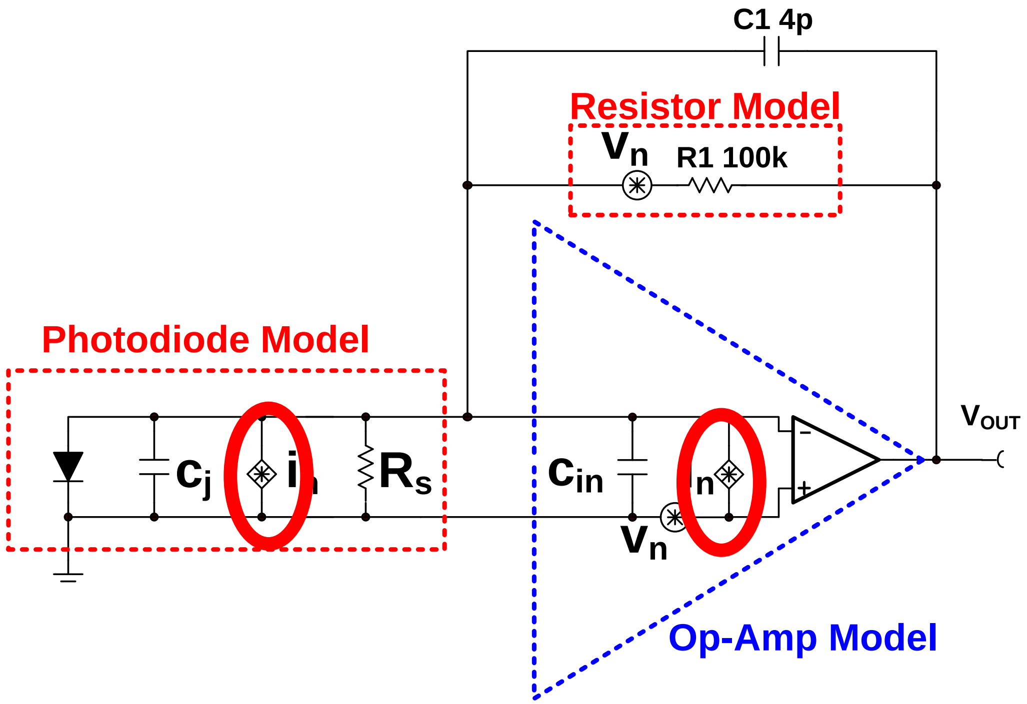
Thermal (Resistor) Noise at Output

$R_f := 100 \cdot 10^3 \Omega$	Feedback Resistance
$k_b := 1.38 \cdot 10^{-23} \frac{J}{K}$	Boltzmann constant
$T_n := 298 K$	Temperature in Kelvin (25C)
$f_p := 397.887 \times 10^3 \ Hz$	Transconductance bandwidth
$K_n := 1.57$	Noise Current from OPA827 data sheet
$BW_n := K_n \cdot f_p$	Noise bandwidth (brick wall filter)
$e_{n\_r} := \sqrt{4 k_b \cdot T_n \cdot R_f \cdot BW_n} = 32.056 \times 10^{-6} V$	Thermal noise at output



Current Noise to Voltage Noise at Output

EQUATION	DESCRIPTION
$R_f = 100 \times 10^3 \Omega$	Feedback resistance
$f_p = 397.887 \times 10^3 \text{Hz}$	Transconductance bandwidth
$i_{n\_opa} = 2.2 \times 10^{-15} \frac{A}{\sqrt{Hz}}$	Noise current from OPA827 data sheet
$i_{n\_diode} = 27.395 \times 10^{-15} \frac{A}{\sqrt{Hz}}$	Noise current from diode (calculated)
$i_{n\_total} = \sqrt{i_{n\_opa}{}^2 + i_{n\_diode}{}^2}$	Total noise current
$K_n = 1.57$	Noise bandwidth factor first order filter
$BW_n = K_n \times f_p = 624 \text{ kHz}$	Noise bandwidth (brick wall filter)
$E_{noI} = i_{n\_total} \times R_f \times \sqrt{BW_n} = 2.17\ \mu V$	Current noise at output



The Final Total Noise

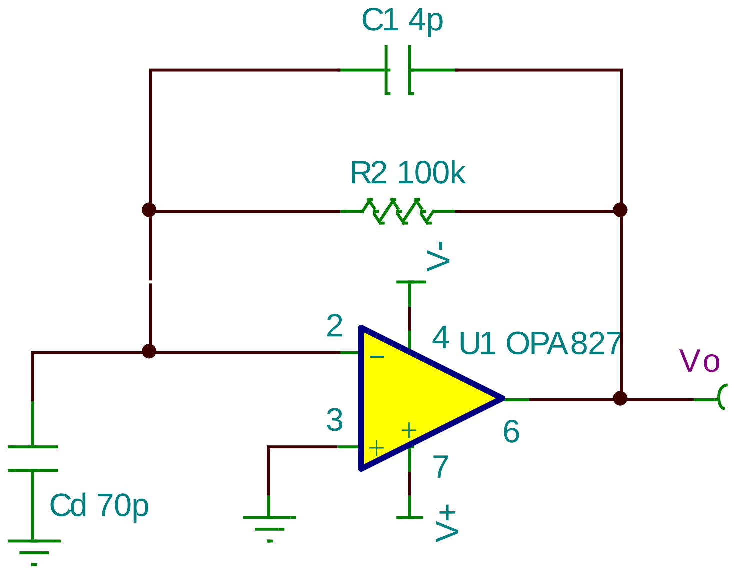
$E_{noe} := 112.193 \times 10^{-6}\ V$ Op-Amp Voltage Noise

$E_{noR} := 32.056 \times 10^{-6}\ V$ Resistor Noise

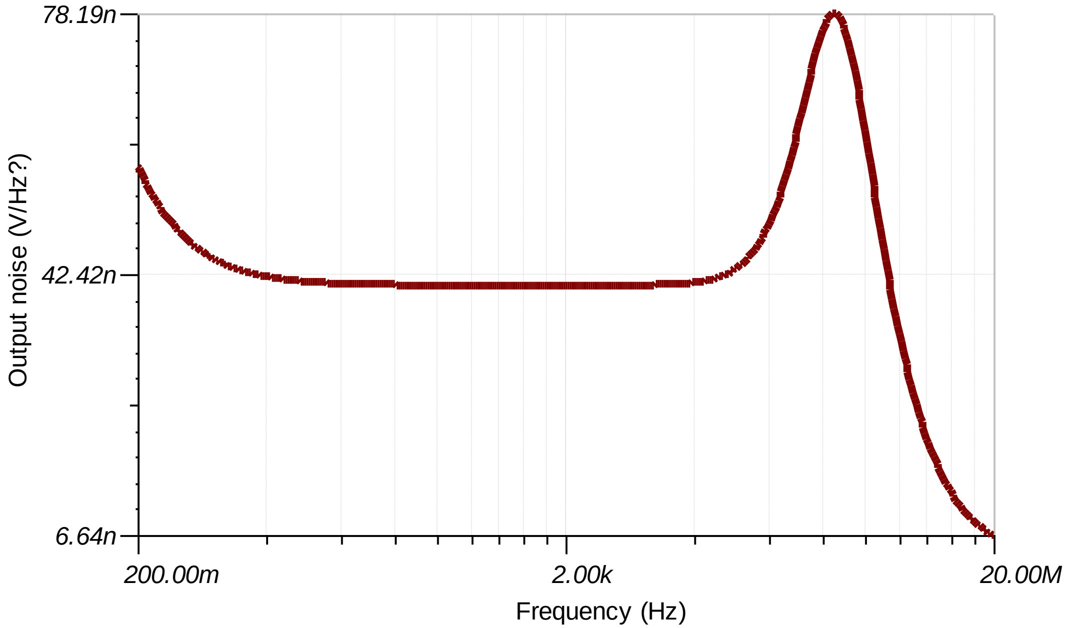
$E_{noI} := 2.172 10^{-6} V$ Op-Amp Current Noise

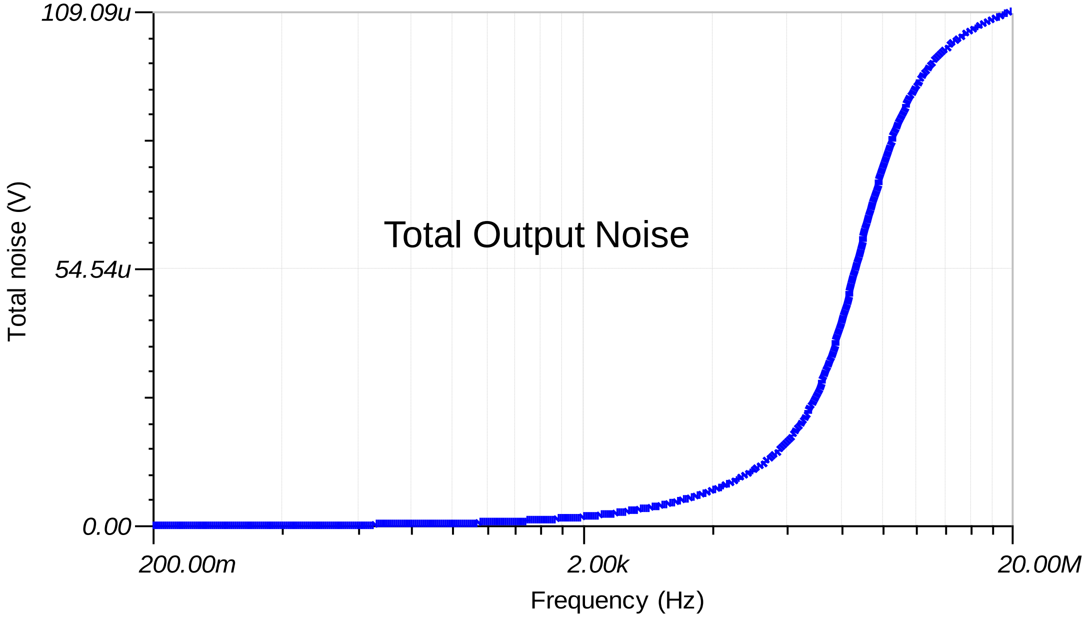
$E_{no} := \sqrt{E_{noR}^2 + E_{noI}^2 + E_{noe}^2} = 116.703 \times 10^{-6}\ V$ Total Output Noise for OPA827 Transimpedance Amp

Calculation vs Simulation

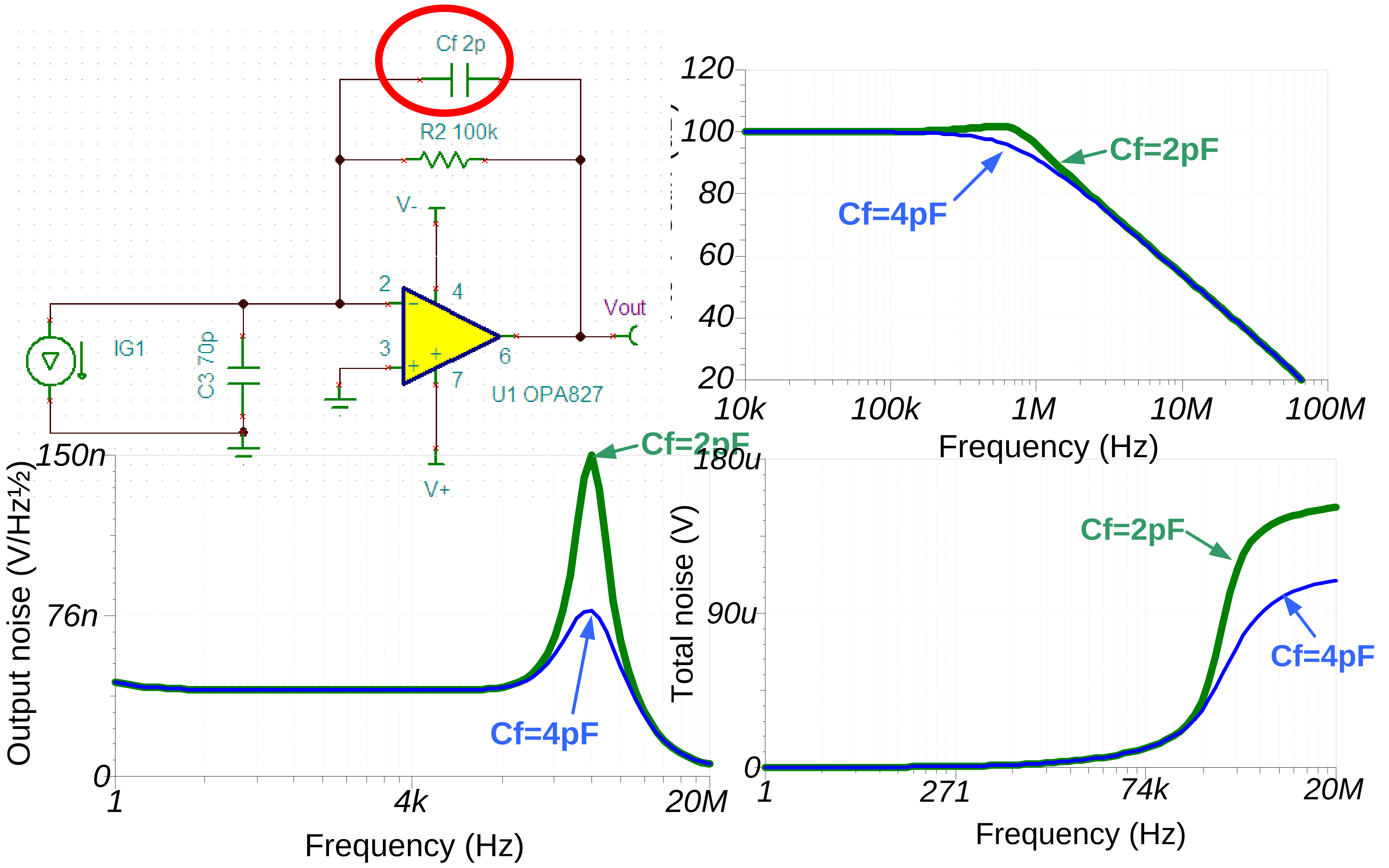


Calculated (rms)	Simulated (rms)
116.7uV	109.1uV



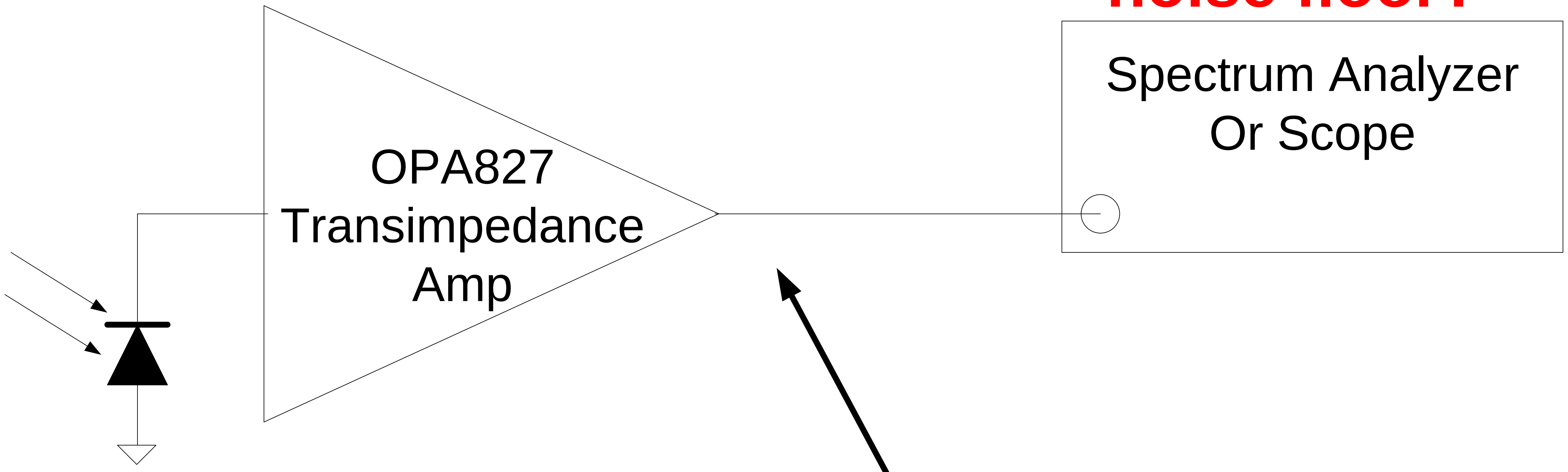


Reducing Noise (Higher Cf = Lower BW & Noise)



OPA827 Measurement Example

Validating Test Equipment Capability



What is the noise floor?

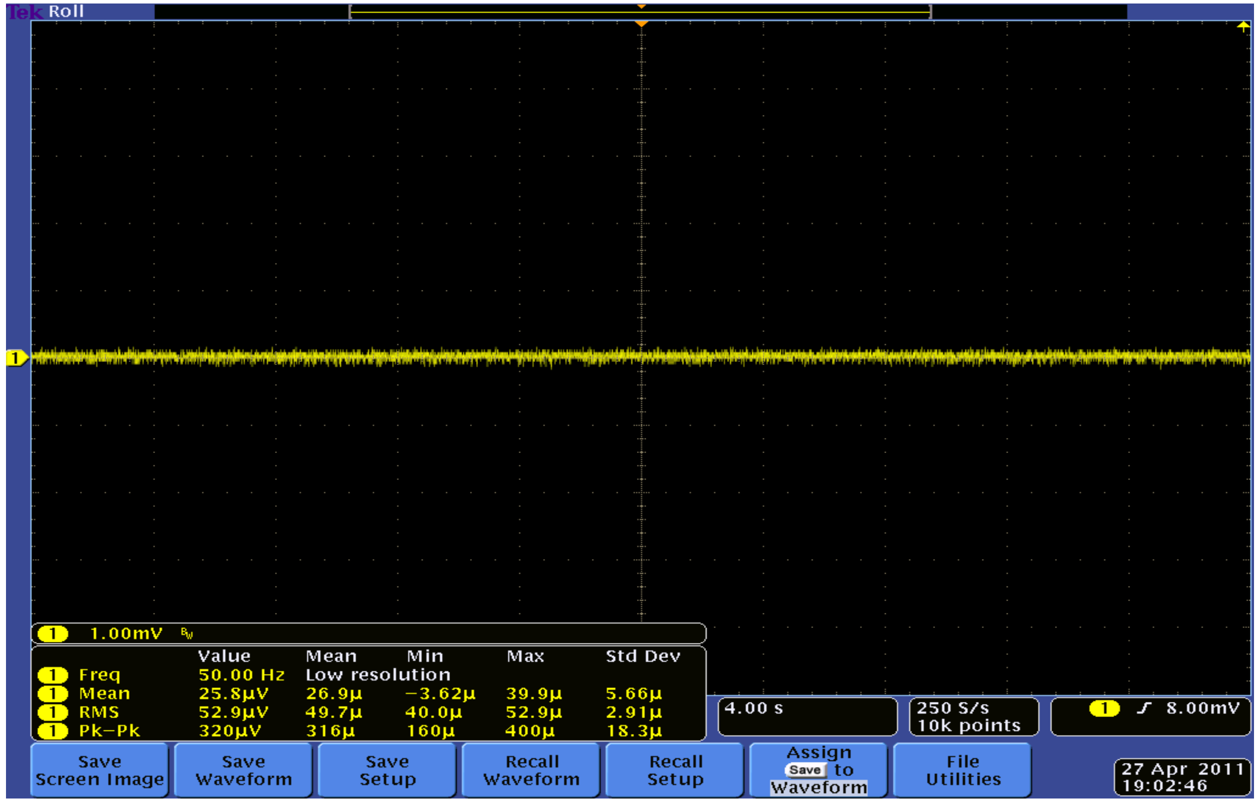
Spectrum Analyzer
Or Scope

OPA827
Transimpedance
Amp

Photodiode
(No light)

Noise Spectral Density = 3.8nV/rtHz
Total Noise = 109.1uVrms
= 654.6uVp-p

Tektronix DPO 4034 Oscilloscope



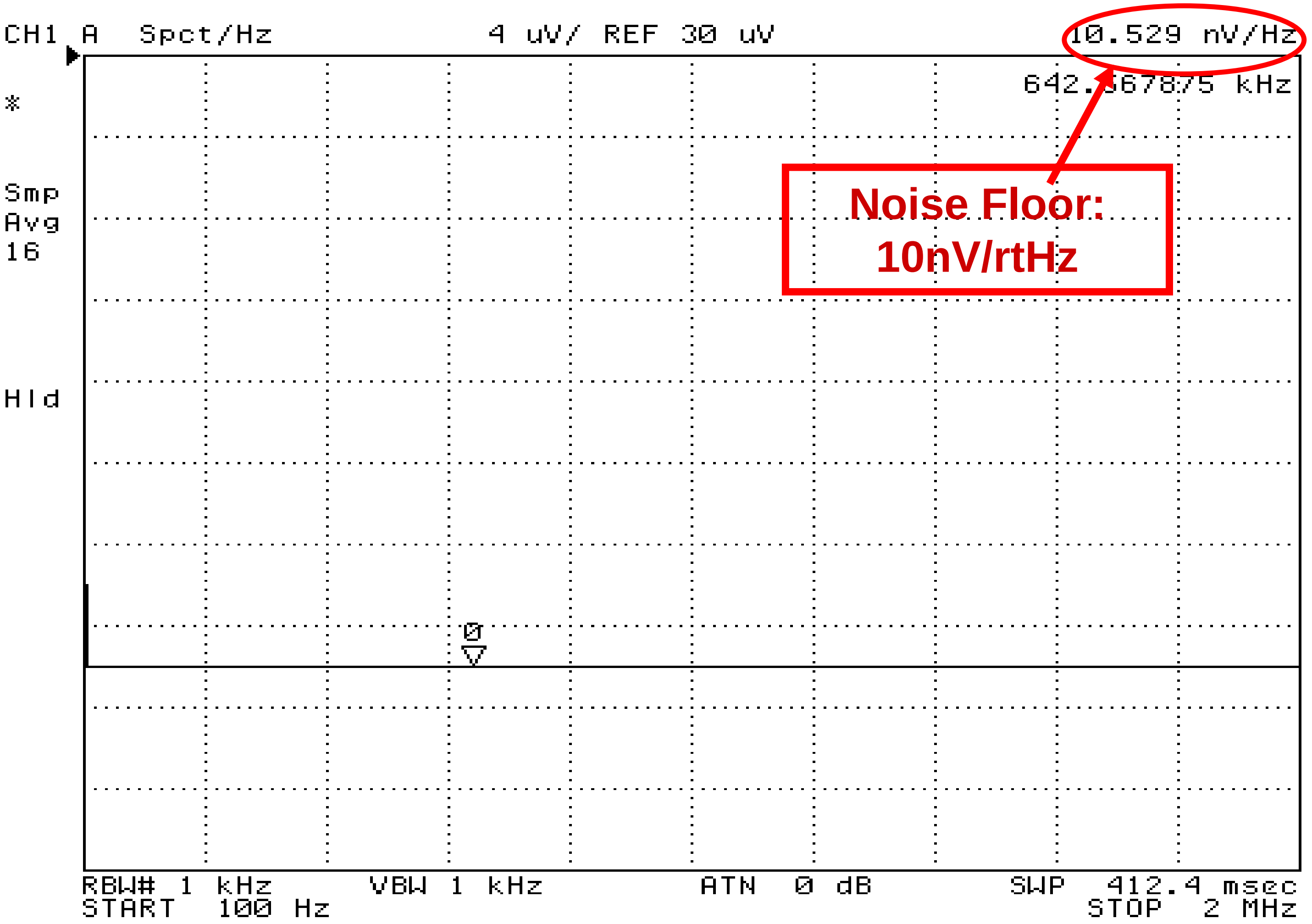
	Value	Mean	Min	Max	Std Dev
1 Freq	50.00 Hz	Low resolution			
1 Mean	25.8µV	26.9µ	−3.62µ	39.9µ	5.66µ
1 RMS	52.9µV	49.7µ	40.0µ	52.9µ	2.91µ
1 Pk−Pk	320µV	316µ	160µ	400µ	18.3µ

27 Apr 2011
19:02:46

1) Set DC couple, 20MHz bandwidth limit
2) Short input channel to measure noise floor

STDEV: 48uV (same as RMS)
P-P: 6.6*STDEV=319uV
40s P-P: 320uV

Agilent 4395A Spectrum Analyzer



Noise Floor:
10nV/rtHz

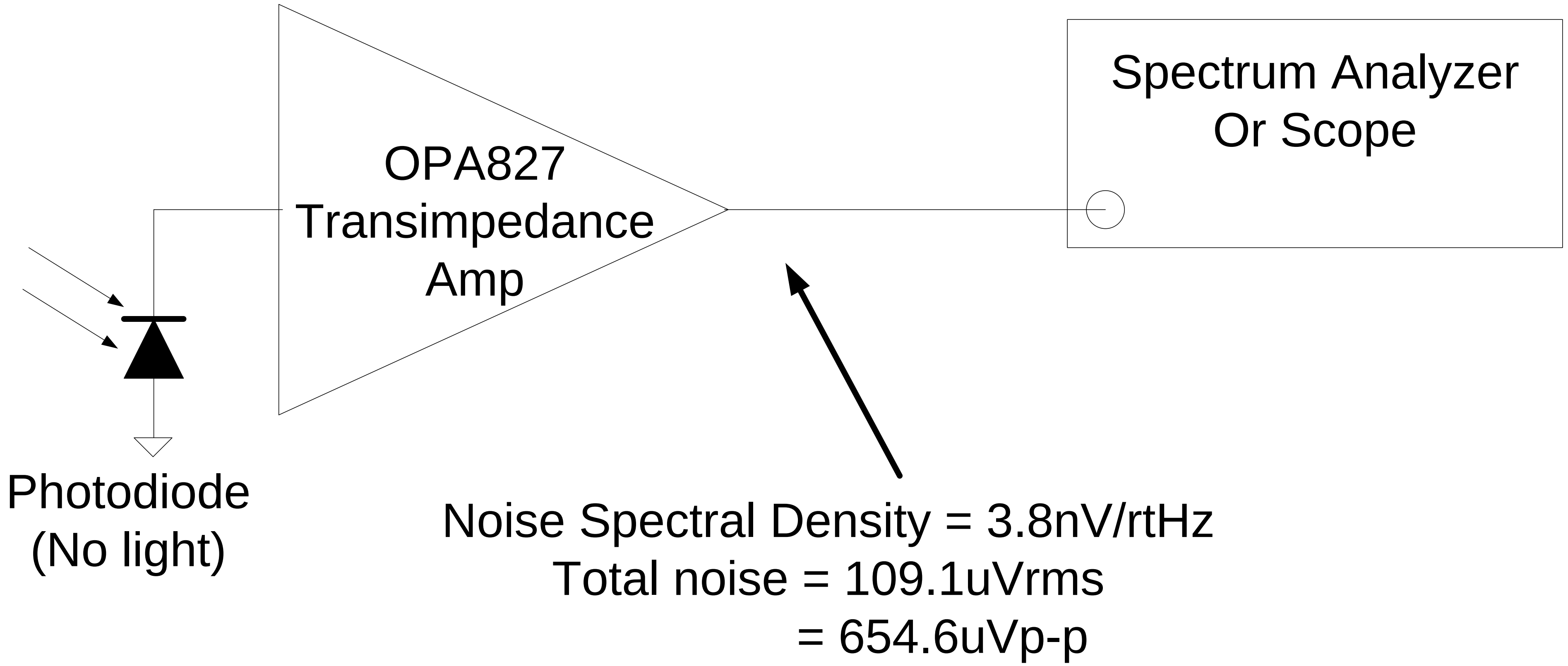
1. Frequency Range: 10Hz~500MHz
2. Noise floor: 10nV/rtHz
3. Input Impedance: 50Ω

The Noise Floors are Not Good Enough

Noise Floors:

Scope (1mV range)= 48uV rms =320uVpp

4395A Spect. Analyzer = 10nV/rtHz



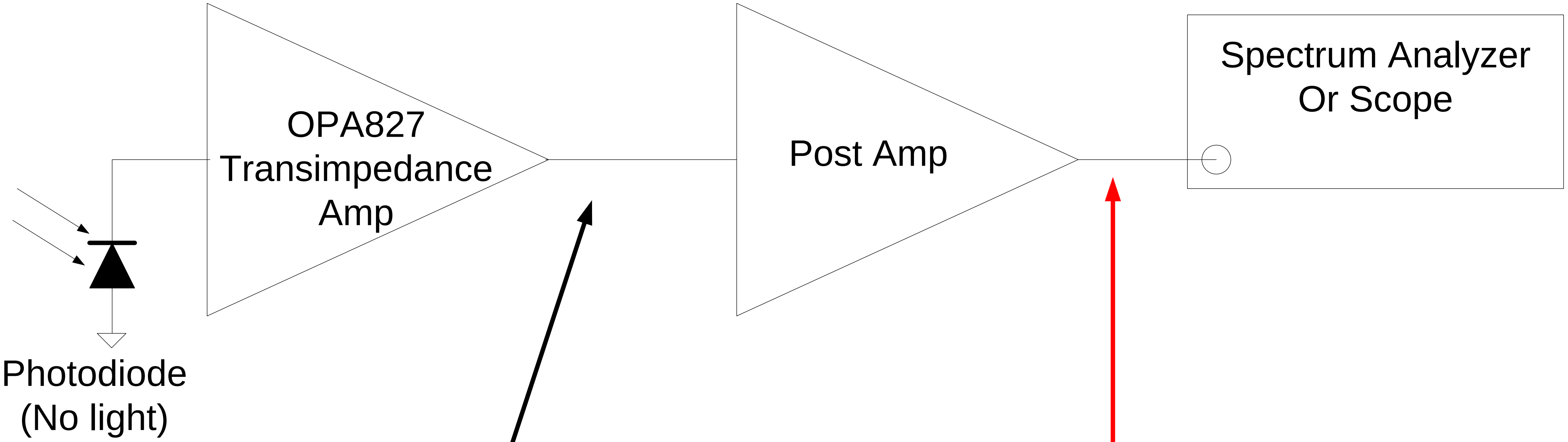
Spectrum Analyzer
Or Scope

OPA827
Transimpedance
Amp

Photodiode
(No light)

Noise Spectral Density = 3.8nV/rtHz
Total noise = 109.1uVrms
= 654.6uVp-p

Solution: Use A Post Amp, Which one?



Gain BW = 22MHz

Need
Gain > 10
BW> 22MHz (in gain)
Noise < 3.8nV/rtHz

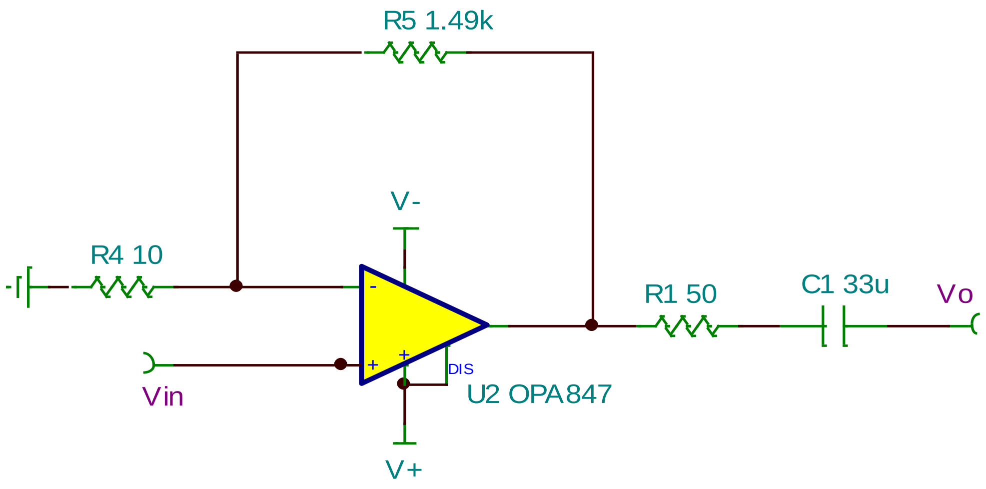
Noise Floors:
Scope (1mV range)= 48uVrms = 320uVp-p
4395A Spect. Analyzer = 10nV/rtHz

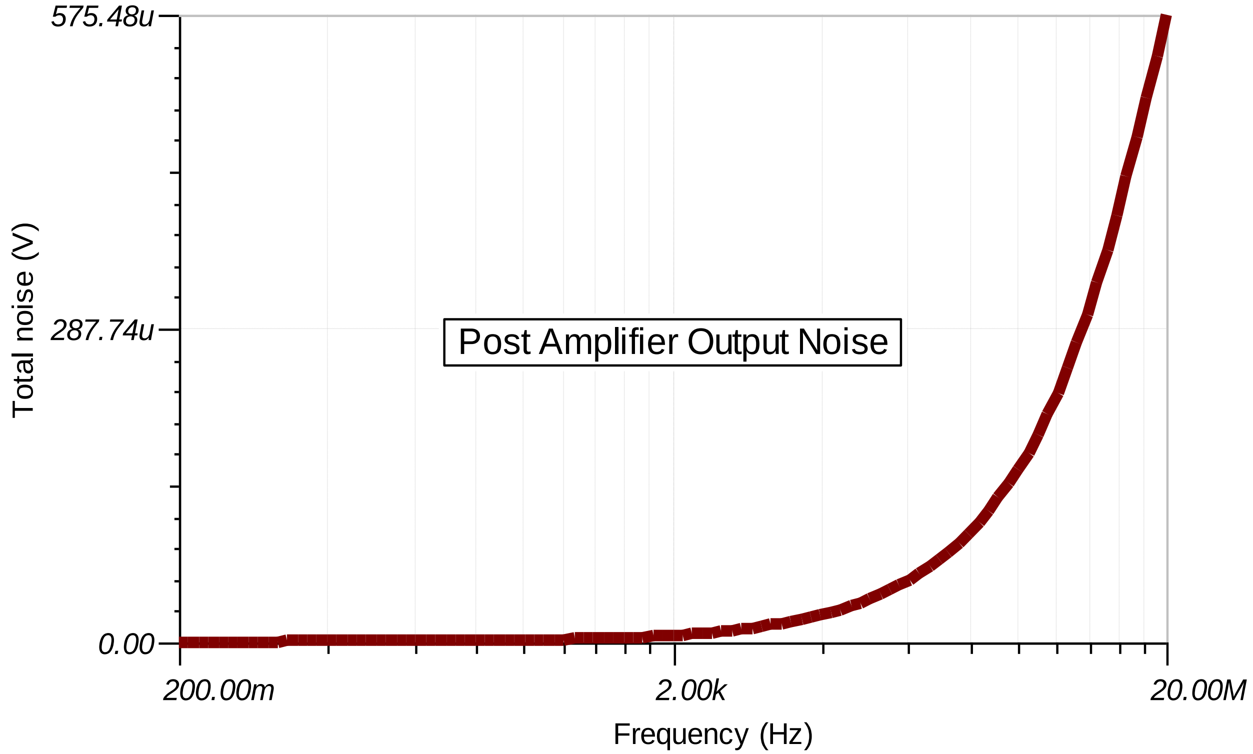
Noise Spectral Density = 3.8nV/rtHz
Total noise = 109.1uVrms
= 654.6uVp-p

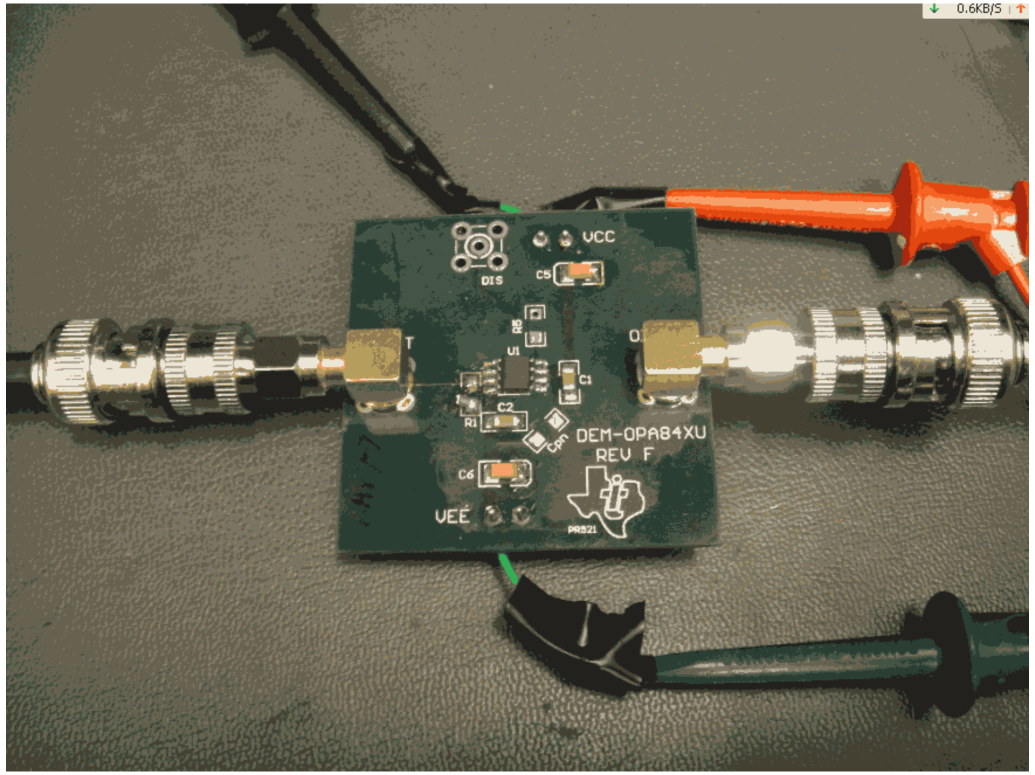
Amplified Signal from OPA827
With Negligible additional noise

Use OPA847 as post amplifier

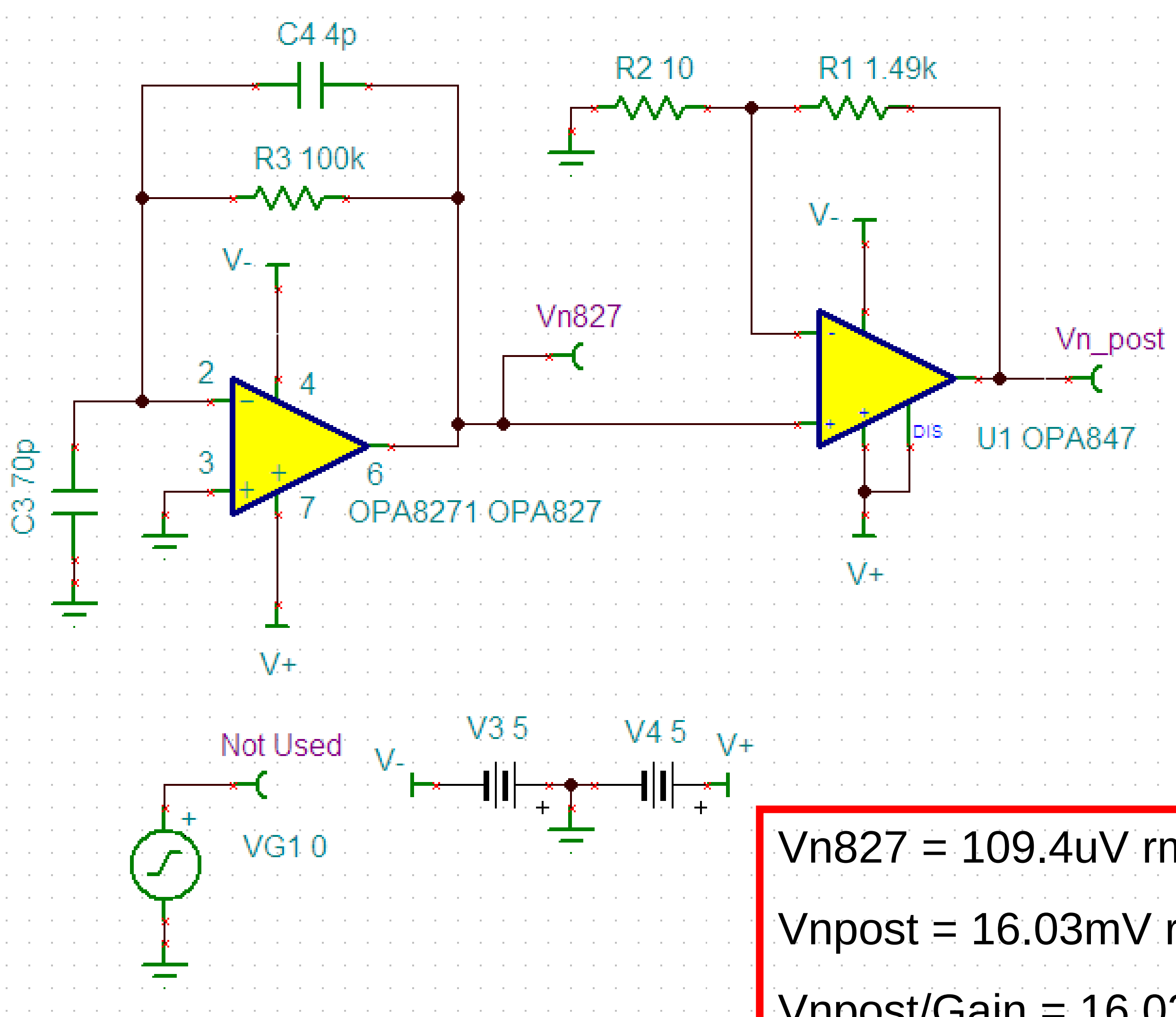
- At the gain of 150, the bandwidth is 26MHz
- 0.85nV/√Hz Input Voltage Noise
- 2.5pA/√Hz Input Current Noise
- ±100uV Input Offset Voltage (Typical))







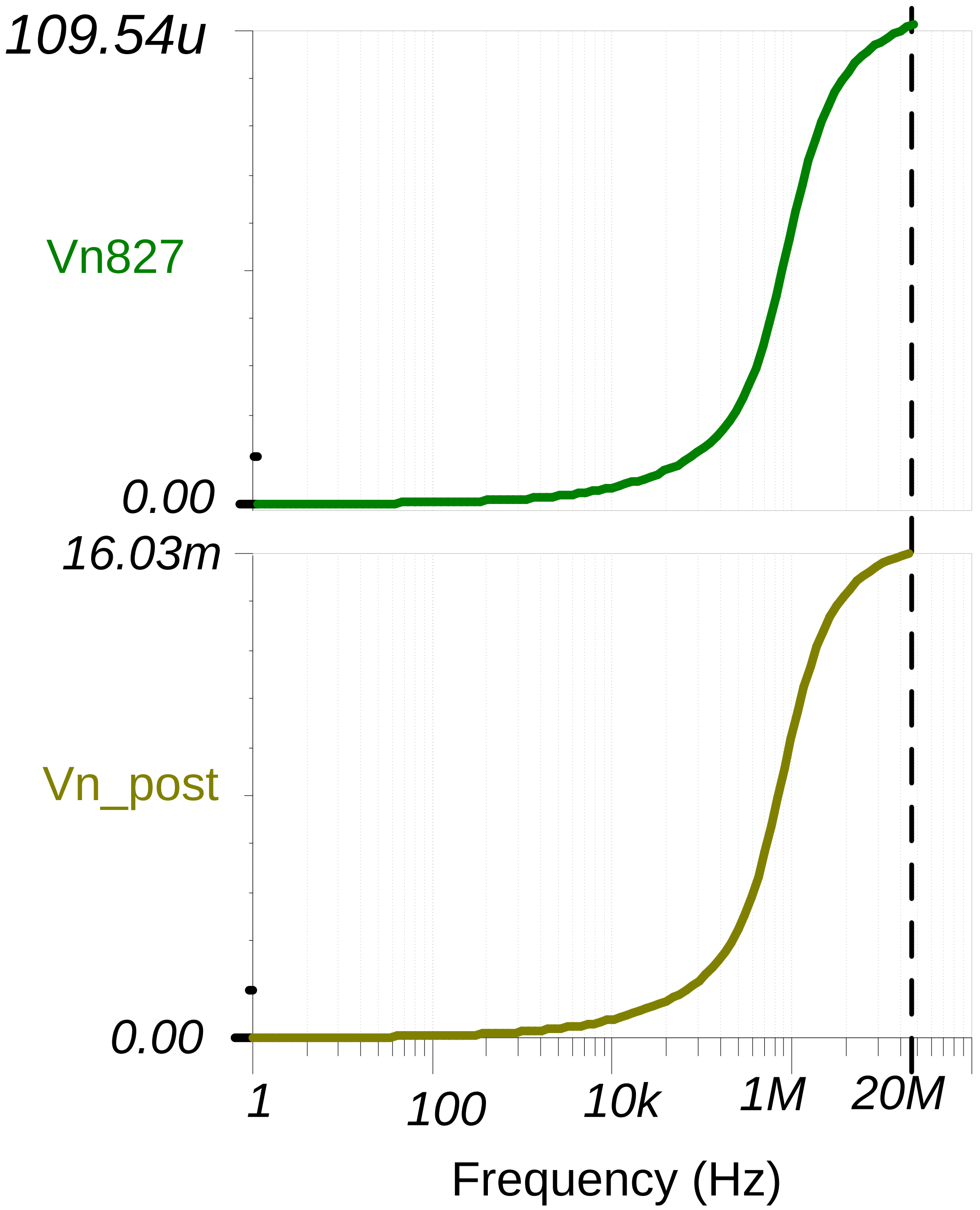
Post Amplifier adds relatively small error!



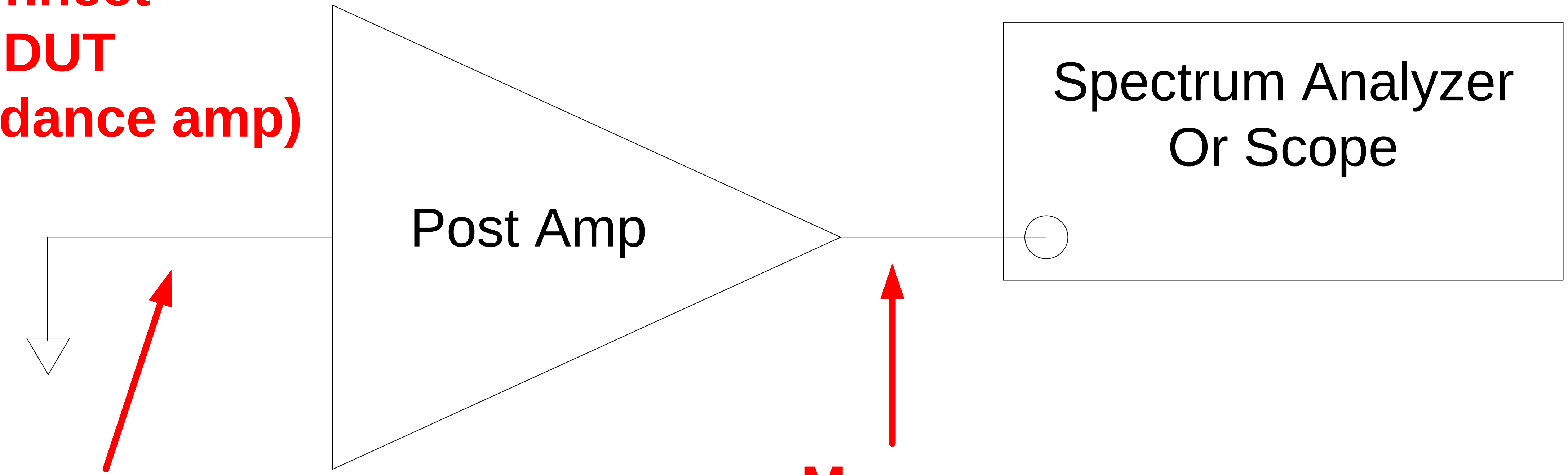
Vn827 = 109.4uV rms

Vnpost = 16.03mV rms

Vnpost/Gain = 16.03mV/150 = 106.8uV rms



Test the Noise Floor of post amp + instrument

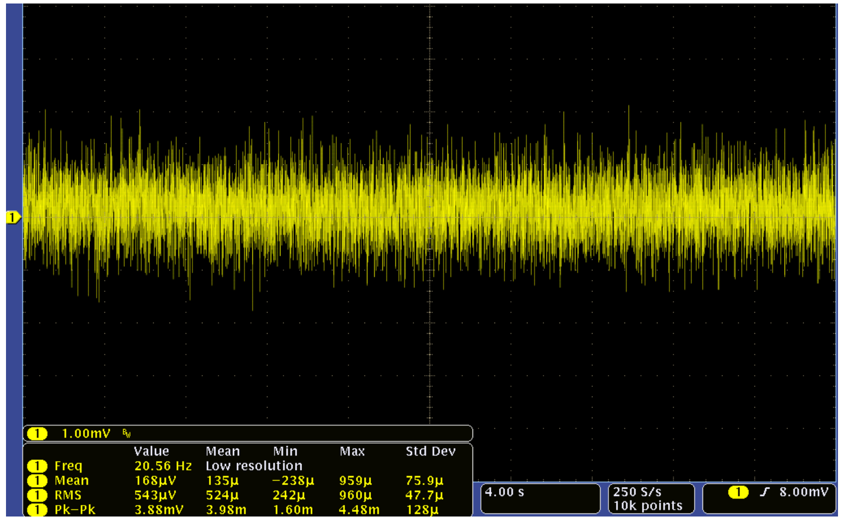


Disconnect
The DUT
(Transimpedance amp)

GND input

Measure
Noise

Test The Noise Floor – Post Amp Noise Scope



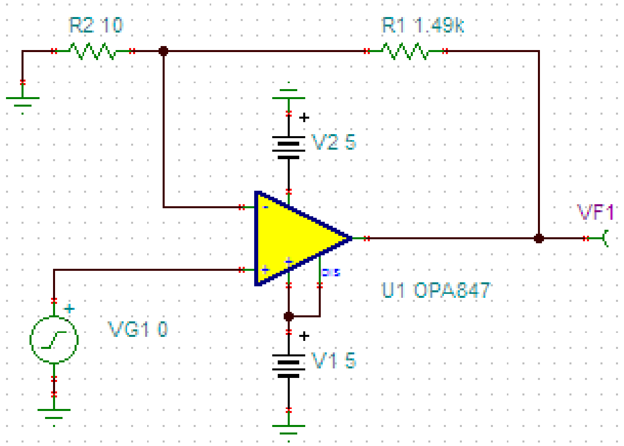
	Value	Mean	Min	Max	Std Dev
1 Freq	20.56 Hz	Low resolution			
1 Mean	168µV	135µ	−238µ	959µ	75.9µ
1 RMS	543µV	524µ	242µ	960µ	47.7µ
1 Pk–Pk	3.88mV	3.98m	1.60m	4.48m	128µ

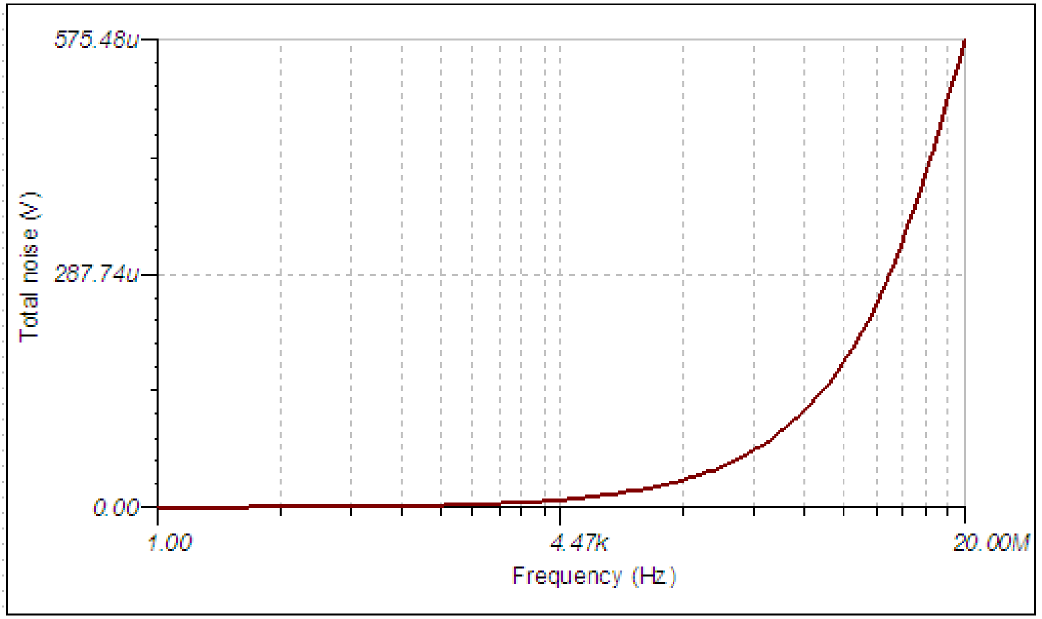
Simulated (rms)	Measured (rms)
575uV	518uV

STDEV: 518uV

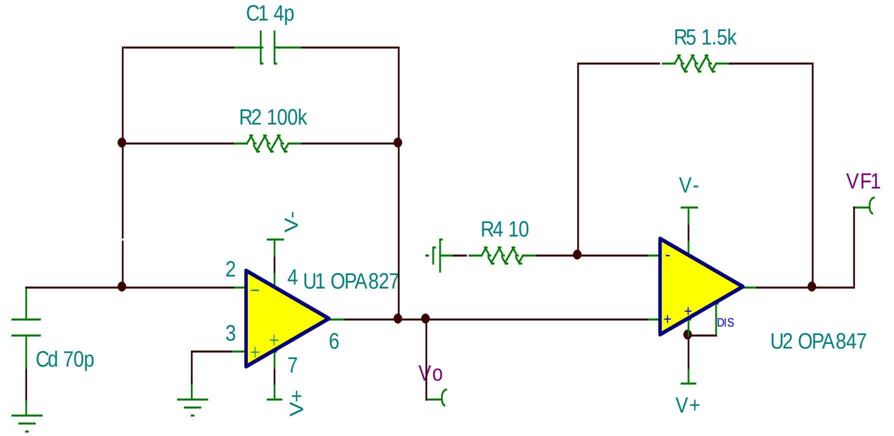
P-P: 6.6*STDEV=3.4mV

40s P-P: 3.88mV

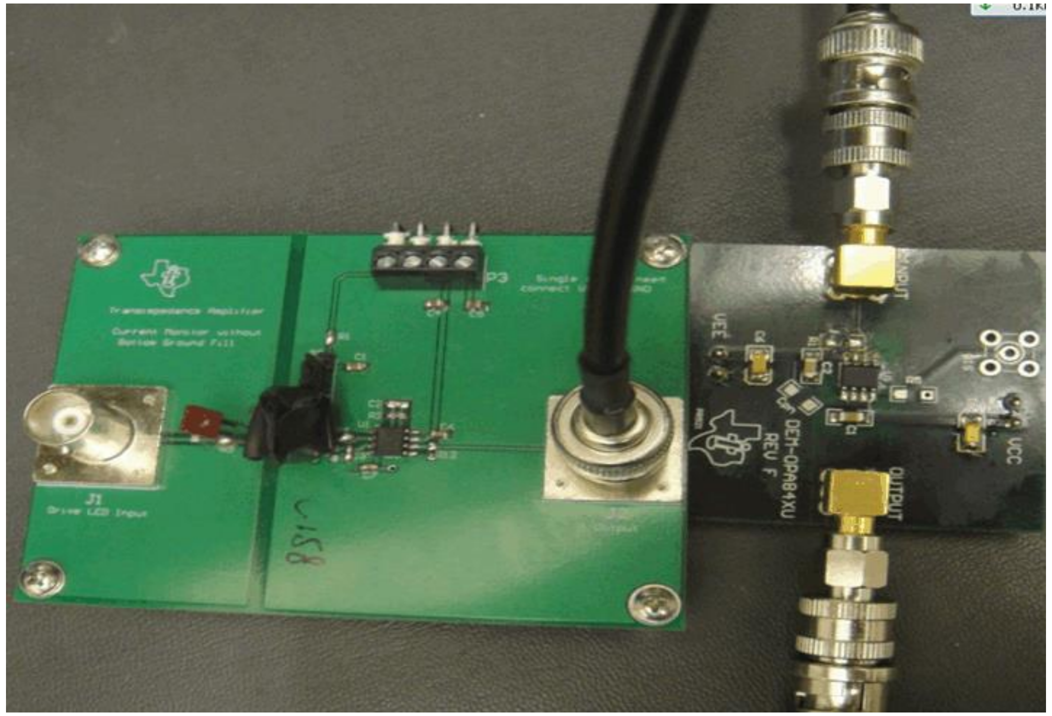


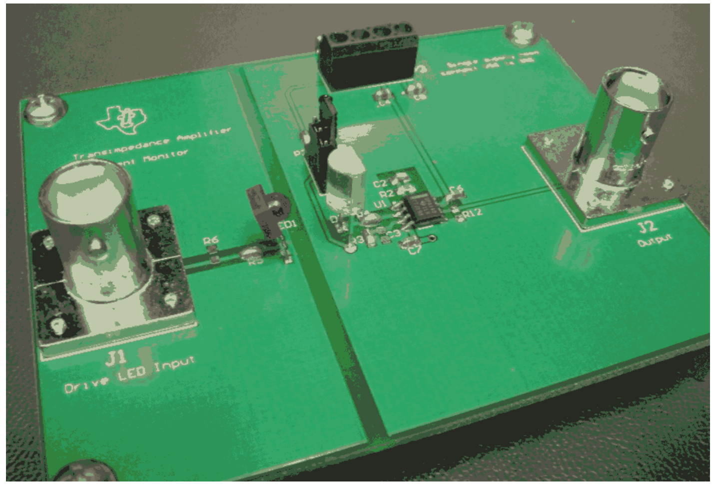


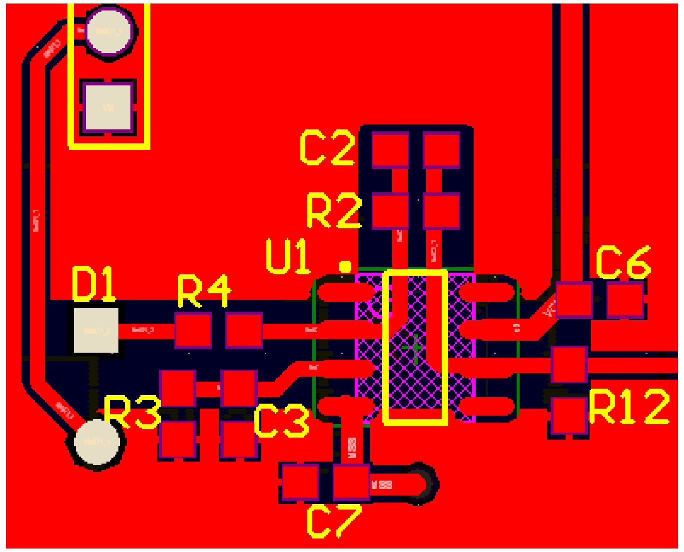
Hardware Connections



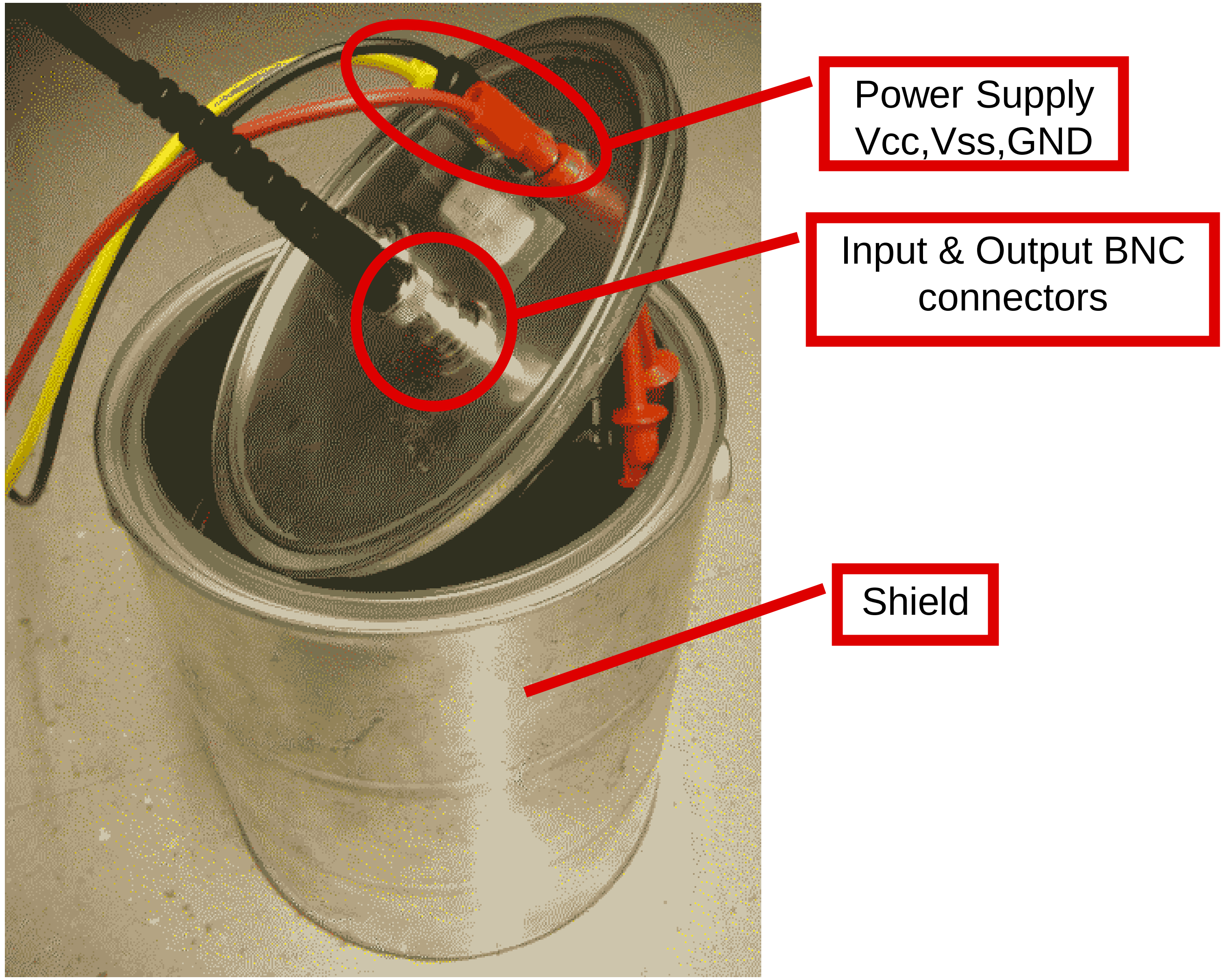
1. PDB-C158-ND photodiode
2. 70pF junction capacitance at Vr=0 V
3. 100dB I-V gain
4. 4pF compensation capacitor
5. ±5V power supply.







Shield the Circuit

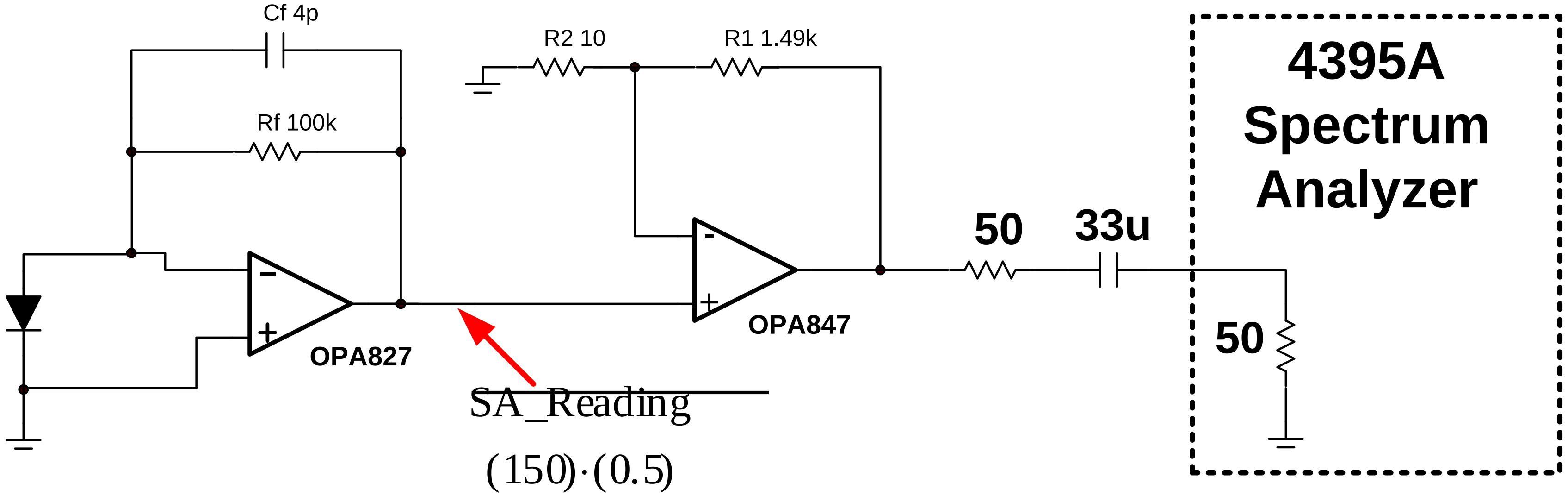


Power Supply
Vcc,Vss,GND

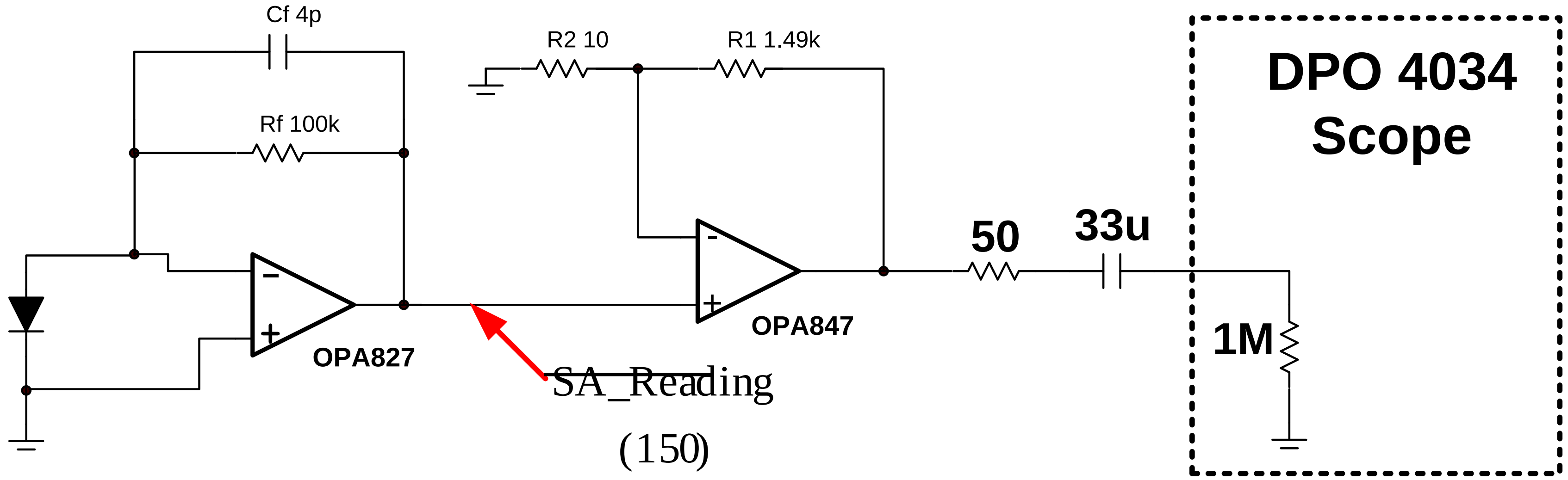
Input & Output BNC connectors

Shield

Divide by post amp gain for OPA827 Output Noise

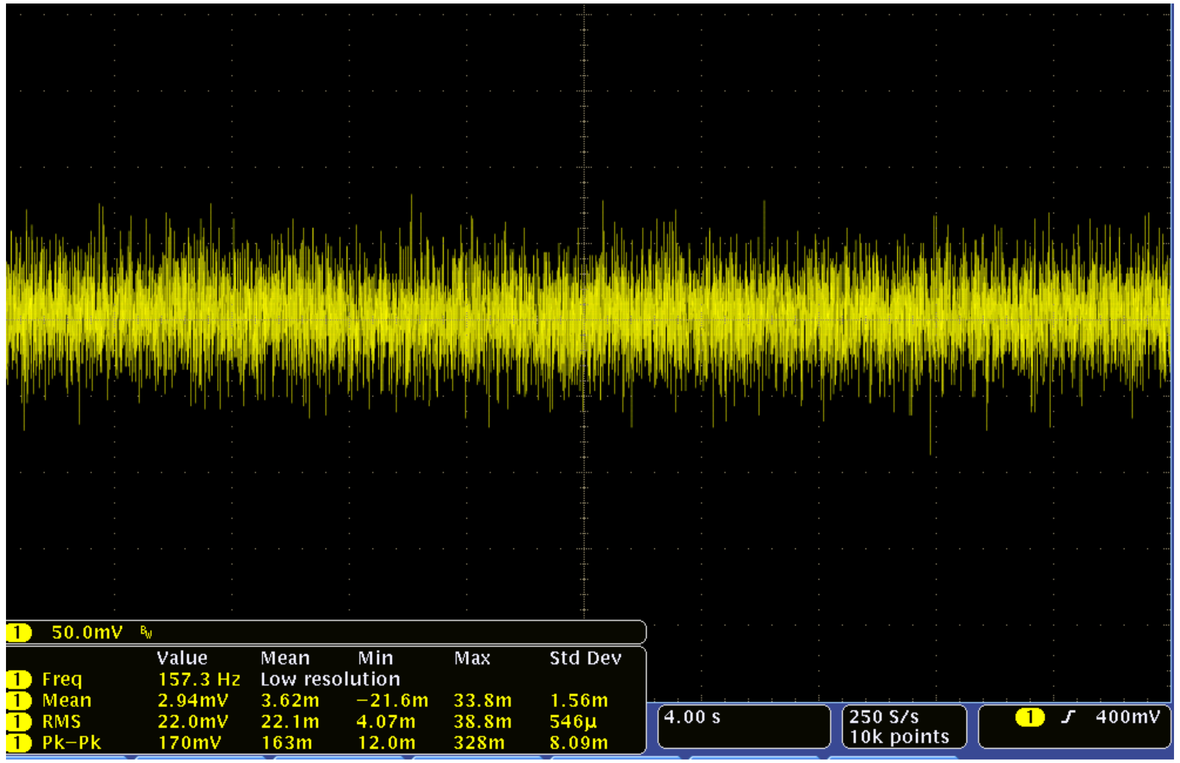


$$\frac{SA\_Reading}{(150)\cdot(0.5)}$$



$$\frac{SA\_Reading}{(150)}$$

Measured vs Simulated (DPO 4034 Scope)



	Value	Mean	Min	Max	Std Dev
1 Freq	157.3 Hz	Low resolution			
1 Mean	2.94mV	3.62m	−21.6m	33.8m	1.56m
1 RMS	22.0mV	22.1m	4.07m	38.8m	546µ
1 Pk−Pk	170mV	163m	12.0m	328m	8.09m

OPA847 Measured at Scope:

STDEV: 21.7mV

P-P: 6.6*STDEV=143mV

40s P-P: 170mV

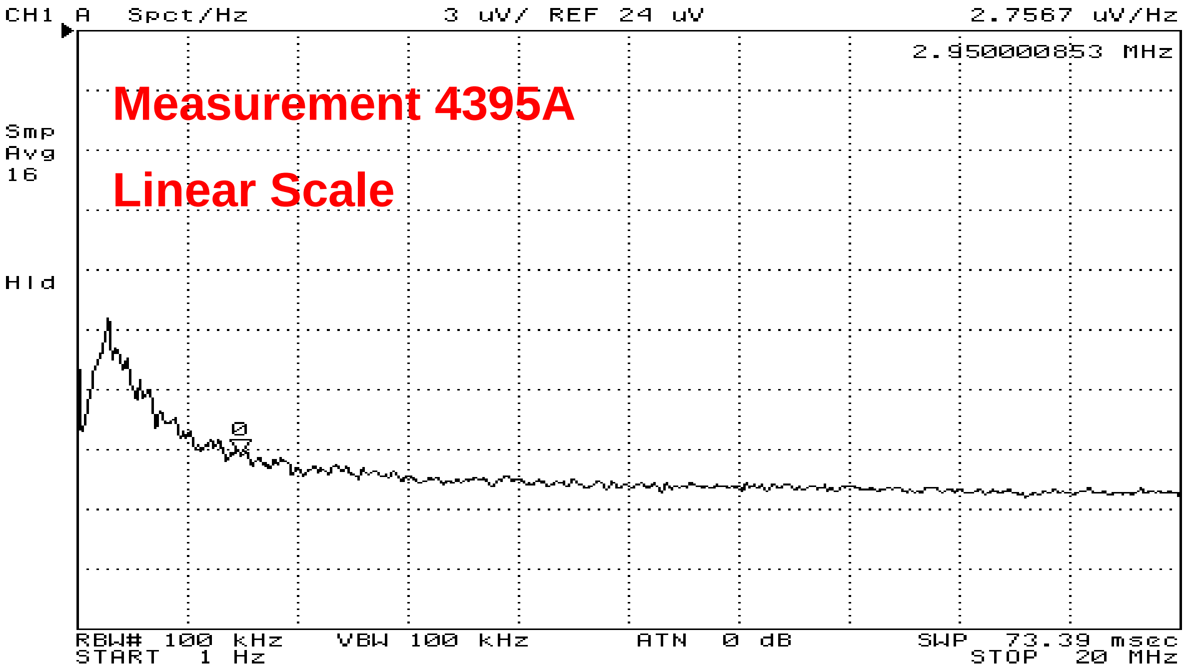
At OPA827 (DUT) Output:

Divide by gain

Vn827 = 21.7mV/150 = 144.6uV

Calculated (rms)	Simulated (rms)	Measured (rms)
116.7uV	109.1uV	144.6uV

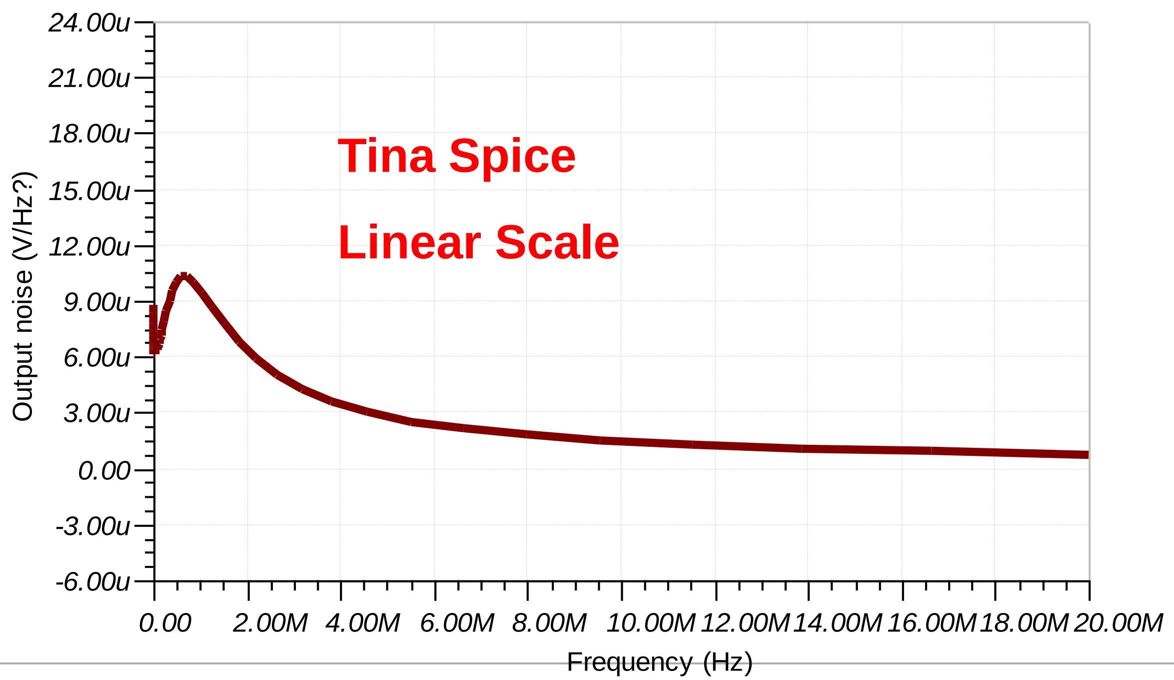
Measured Spectral Density Spectrum Analyzer



Measurement 4395A

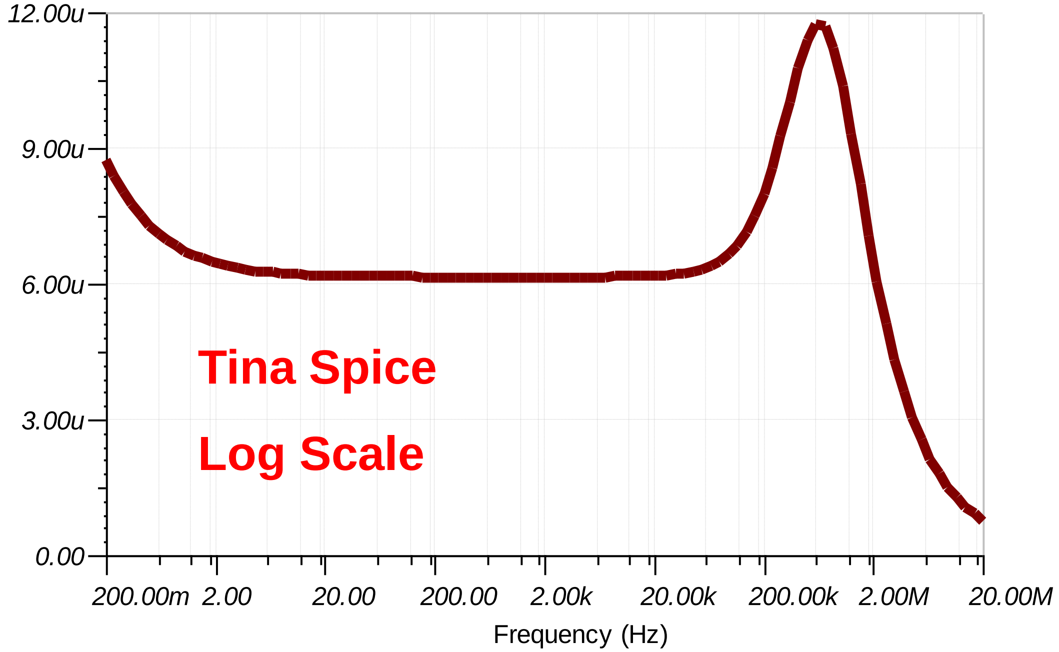
Linear Scale

1. Agilent 4395A Spectrum Analyzer test 1Hz~20MHz span, 3uV/div, REF=24uV.
2. The tested noise density curve shape is the same as simulation.



Tina Spice

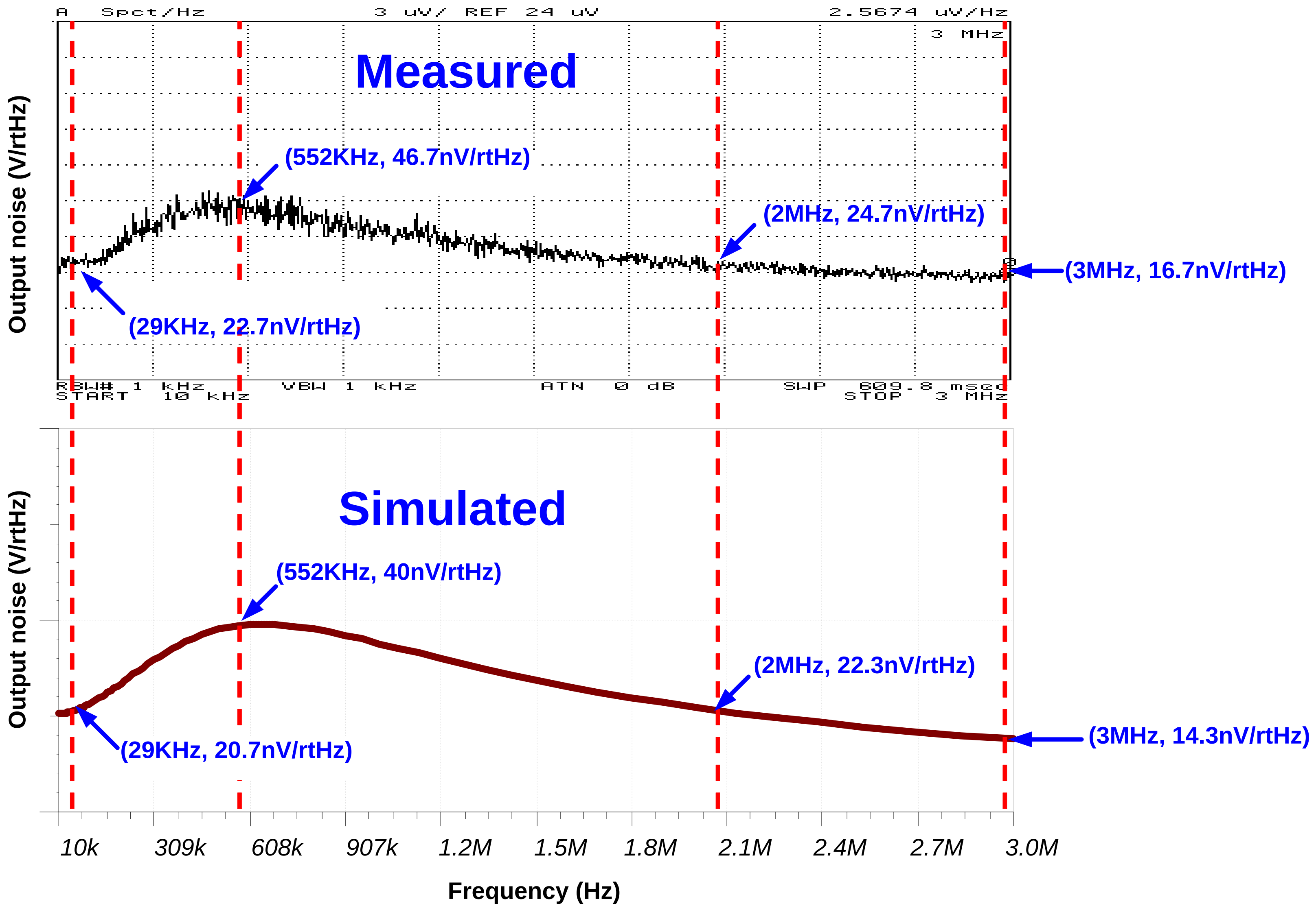
Linear Scale



Tina Spice

Log Scale

Measured vs Simulated



Measured

(552KHz, 46.7nV/rtHz)

(2MHz, 24.7nV/rtHz)

(3MHz, 16.7nV/rtHz)

(29KHz, 22.7nV/rtHz)

Simulated

(552KHz, 40nV/rtHz)

(2MHz, 22.3nV/rtHz)

(3MHz, 14.3nV/rtHz)

(29KHz, 20.7nV/rtHz)

Thanks for your time

- Bryan Zhao (赵 伟)
- Matt Hann
- Collin Wells, Peter Semig, Curtis Mayberry

References

- Jerald Graeme <Photodiode Amplifiers>
- Art Kay < Op-Amp Noise Calculation and Measurement >
- HAMAMATSU <Photodiode Technical Information>
- Tim Green <Operational amp stability>