

Analog Active Audio Filters

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ABSTRACT

Analog active audio filters can be used to compensate frequency response problems in a variety of systems. Their responses are examined here to simplify the filter design task for design engineers.

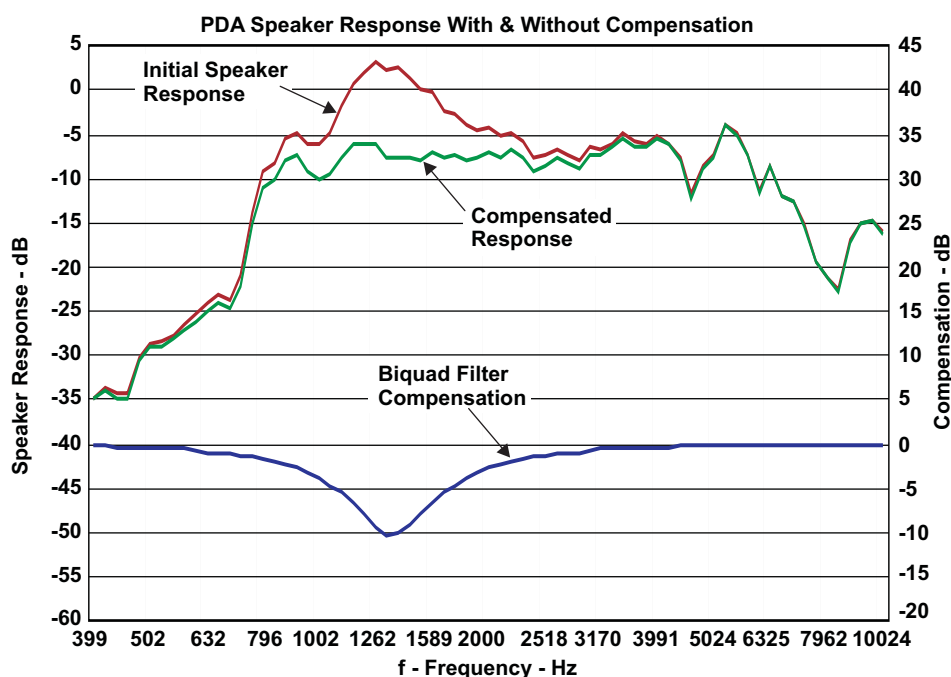
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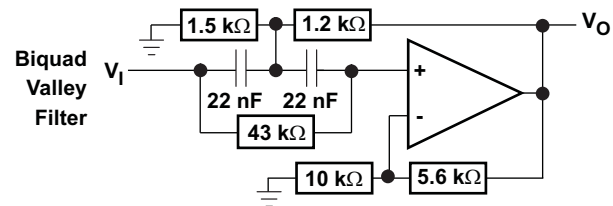
1 Introduction

Active audio filters may be used to compensate problems in frequency response of audio systems and loudspeakers. This paper deals with analog filters. These filters can produce a response that is approximately the inverse of a system response or a loudspeaker acoustic response so that when the two are summed the result is nearly flat. They may also be used to produce the difference of a target response minus the system or loudspeaker response, so that when the two are summed the result is nearly the target. In either case the final response is more accurate or more pleasing.

Cell phone and PDA speaker responses like the one shown below often have annoying peaks that reduce intelligibility. This is compensated with the response of a biquad filter, also shown. The sum, the compensated response, is much more pleasing and far more intelligible than the original.



The schematic for the biquad filter is at right. This and other filters are discussed in detail in this paper.



This paper begins with the relationship between filter singularities and their responses. Then it considers a number of possible filter implementations. (It's typically easier to adjust a set of filter singularities to achieve the best result than to adjust the numerous component values in a circuit. Once the singularities are decided the filter usually can be implemented relatively easily.) It provides equations for responses and the parameters in them and discusses optimizing component choices.

Audio filters may be first, second or higher order. First and second order analog filters are generally well understood and their audio uses are somewhat limited, so they are examined briefly. The paper also examines biquadratic filters, or biquads, in more depth because they are more powerful tools for response compensation or EQ. Bridged-T filters will be added in a later version.

2 First Order Filters

First-order filters implement responses with single poles as their denominators. They have limited response bands, either low-pass or high-pass, which are described below.

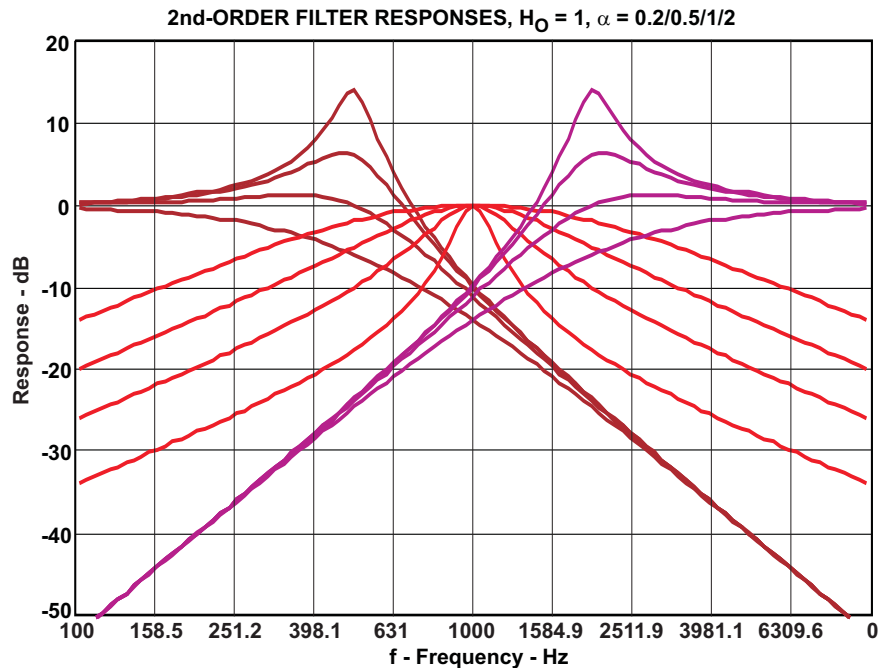
- Low pass: $H(s) = H_o \omega_o / (s + \omega_o)$.
For small s , this is H_o , flat low-frequency response; for large s , it is $H_o \omega_o / s$, a first-order rolloff.
- High pass: $H(s) = H_o s / (s + \omega_o)$.
For large s , this is H_o , flat high-frequency response; for small s , it is $H_o s / \omega_o$, a first-order rolloff.

3 Second Order Filters

Second-order filters implement responses with quadratic terms as their denominators. They have limited response bands, either low-pass, band-pass or high-pass, which are described below.⁽¹⁾

- Low pass: $H(s) = H_o \omega_o^2 / (s^2 + \alpha \omega_o s + \omega_o^2)$.
For small s , this is H_o , flat low-frequency response; for large s , it is $H_o \omega_o^2 / s^2$, a second-order rolloff.
Response magnitude is $G(\omega) = \sqrt{H_o^2 \omega_o^4 / (\omega^4 + \omega^2 \omega_o^2 (\alpha^2 - 2) + \omega_o^4)}$.
Phase is $\Phi(\omega) = \pi/2 - \arctan(\alpha \omega_o \omega / (\omega_o^2 - \omega^2))$.
- Band pass: $H(s) = H_o \alpha \omega_o s / (s^2 + \alpha \omega_o s + \omega_o^2)$.
For small s , this is $H_o \alpha s / \omega_o$, an increasing first-order response or rolloff; for large s , it is $H_o \alpha \omega_o / s$, a decreasing first-order response or rolloff. At $s = j\omega_o$, it is H_o , band-center response.
Response magnitude is $G(\omega) = \sqrt{H_o^2 \alpha^2 \omega_o^2 / (\omega^4 + \omega^2 \omega_o^2 (\alpha^2 - 2) + \omega_o^4)}$.
Phase is $\Phi(\omega) = \arctan(\alpha \omega_o \omega / (\omega_o^2 - \omega^2))$.
- High pass: $H(s) = H_o s^2 / (s^2 + \alpha \omega_o s + \omega_o^2)$.
For large s , this is H_o , flat high-frequency response; for small s , it is $H_o s^2 / \omega_o^2$, a second-order rolloff.
Response magnitude is $G(\omega) = \sqrt{H_o^2 \omega^4 / (\omega^4 + \omega^2 \omega_o^2 (\alpha^2 - 2) + \omega_o^4)}$.
Phase is $\Phi(\omega) = \pi - \arctan(\alpha \omega_o \omega / (\omega_o^2 - \omega^2))$.

H_o scales response magnitude, while ω_o sets the characteristic frequency, the frequency at which the filter operates. The variable α sets the sharpness of the peak the filter produces, which varies inversely with α (sharper peak with smaller α).



⁽¹⁾ Reference: Operational Amplifiers, Design and Applications, Graeme, Tobey and Huelsman, Burr-Brown, McGraw-Hill Book Company, 1971, ISBN 07-064917-0, pages 284-286.

4 Biquadratic Filters

Biquadratic or biquad filters implement responses with quadratic terms for both their numerators and their denominators. They have wide responses, with a single peak or valley or with low or high frequency boost with or without peaking. Their responses have the following form.

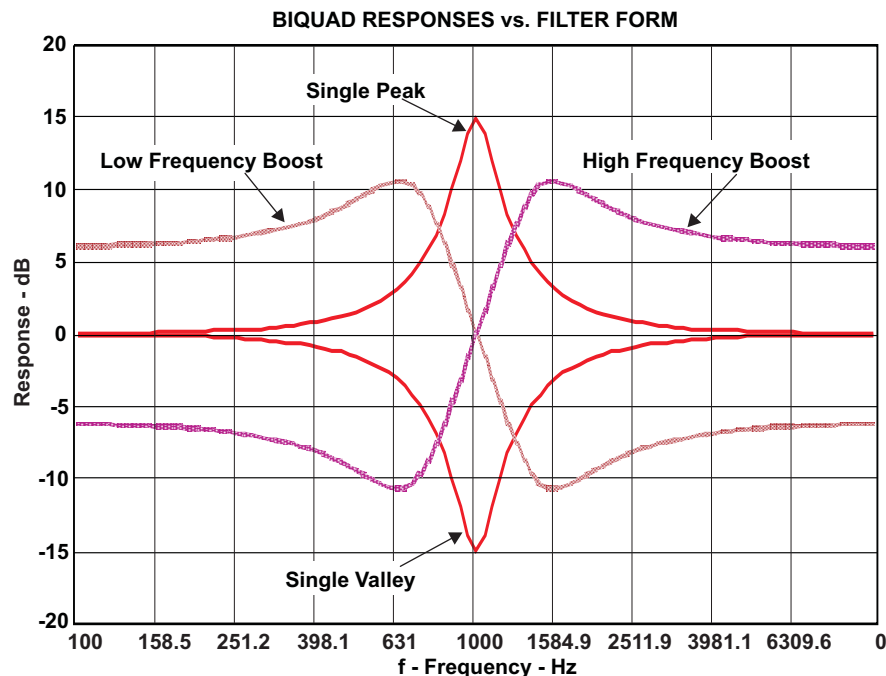
$$H(s) = H_0 (s^2 + \alpha_z \omega_z s + \omega_z^2) / (s^2 + \alpha_p \omega_p s + \omega_p^2).$$

Filter response is influenced by relationships between ω_z and ω_p and between α_z and α_p .

- Single peak or valley. The filter produces this response when ω_z and ω_p are equal.
If α_z is greater than α_p the filter produces a peak. If α_p is greater than α_z the filter produces a valley.
- Low frequency boost. The filter produces this response when ω_z is greater than ω_p .
If α_z is less than about 1, the response includes a valley above the boost frequency. If α_p is less than about 1, the response includes a peak below the boost frequency.
- High frequency boost. The filter produces this response when ω_p is greater than ω_z .
If α_z is less than about 1, the response includes a valley below the boost frequency. If α_p is less than about 1, the response includes a peak above the boost frequency.

Response magnitude is $G(\omega) = \sqrt{(\omega^4 + \omega^2 \omega_z^2 (\alpha_z^2 - 2) + \omega_z^4) / (\omega^4 + \omega^2 \omega_p^2 (\alpha_p^2 - 2) + \omega_p^4)}$.

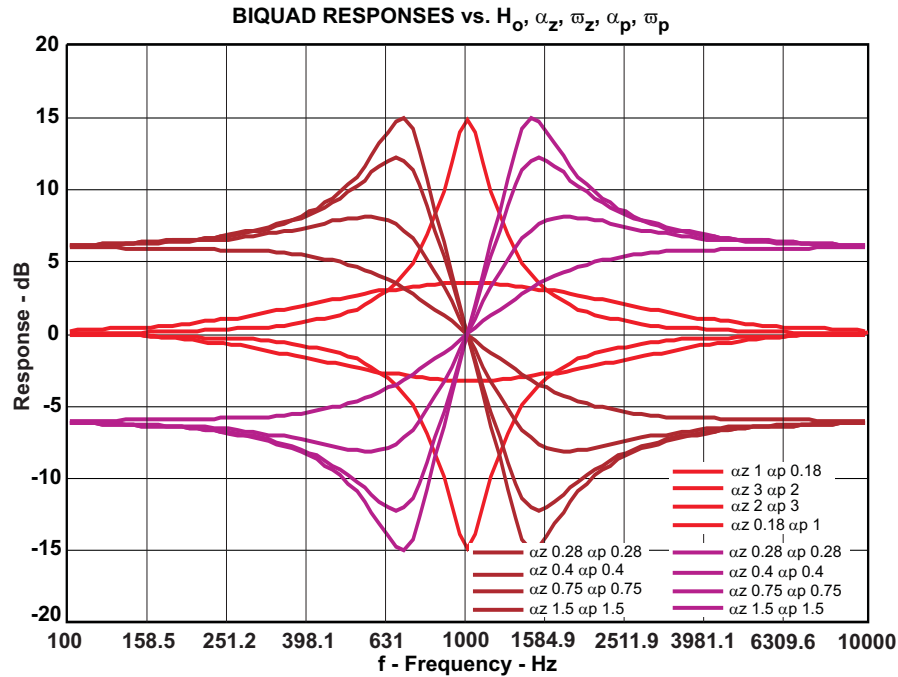
Phase is $\Phi(\omega) = \arctan(\alpha_z \omega_z \omega / (\omega_z^2 - \omega^2)) - \arctan(\alpha_p \omega_p \omega / (\omega_p^2 - \omega^2))$.



Traces in the graph that follows illustrate something of the range of responses a biquad filter can generate. The responses are arranged as follows for clarity.

- Single peak and valley responses are presented in the order of decreasing peaks, or decreasing α_z with respect to α_p . For all these responses H_0 is 1 and ω_z and ω_p are 1 kHz.
- Low and high frequency boost responses are presented in the order of decreasing peaks and valleys, or increasing α_z and α_p . Also, α_z and α_p are made equal, ω_z and ω_p are placed symmetrically around 1kHz, and H_0 is set to 0.5 for low frequency boost and 2 for high frequency boost, to make the responses symmetrical around zero dB and 1kHz.

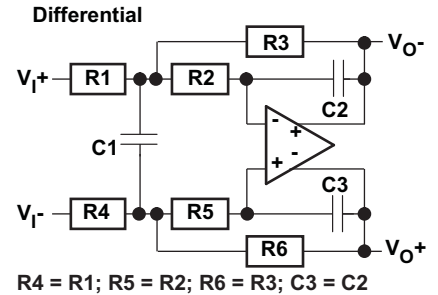
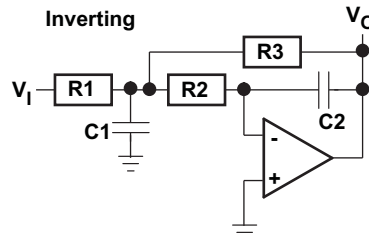
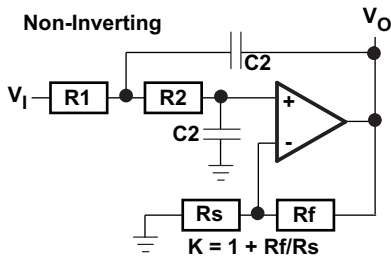
Note that α_z and α_p do not have to be equal! Varying α_z and α_p can create responses that range among and beyond the extremes in the graph.



5 Analog Filter Implementations: Second-Order Filters.

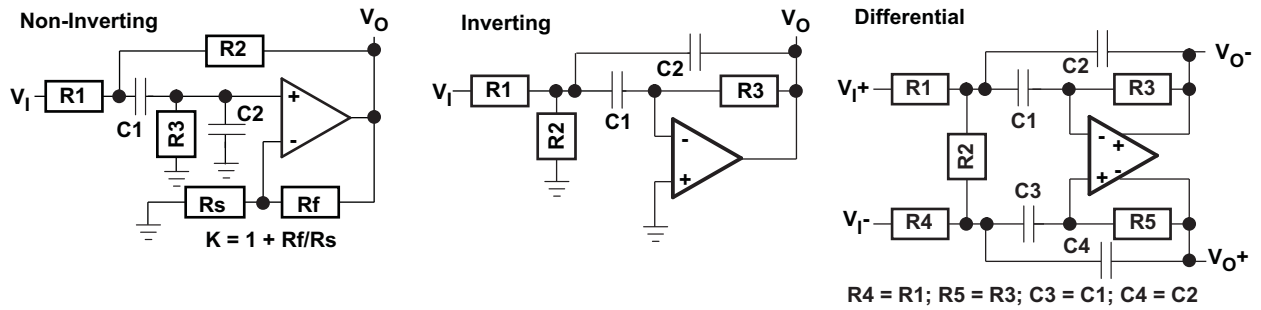
Second-order filters may be non-inverting or inverting. The schematics below show single-ended forms, both non-inverting and inverting, and an inverting, differential form, with equations for their H_o , α and ω_o .

Low-Pass Filters



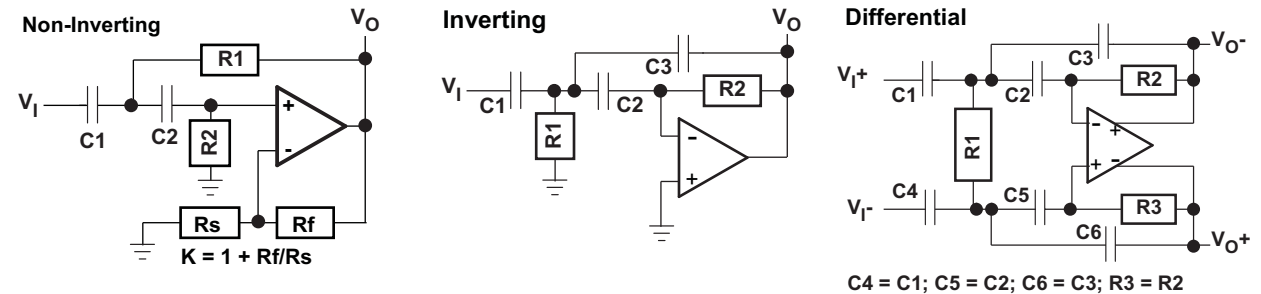
	Non-Inverting	Inverting	Differential Inverting
H_o	$K \left(= 1 + \frac{R_f}{R_s} \right)$	$\frac{R3}{R1}$	$\frac{R3}{R1}$
ω_o	$\frac{1}{\sqrt{(R1 \times R2 \times C1 \times C2)}}$	$\frac{1}{\sqrt{(R2 \times R3 \times C1 \times C2)}}$	$\frac{1}{\sqrt{(R2 \times R3 \times (C1 \times 2) \times C2)}}$
α	$(1-K) \times \sqrt{\frac{R1 \times C1}{R2 \times C2}} + \sqrt{\frac{R2 \times C2}{R1 \times C1}} + \sqrt{\frac{R1 \times C2}{R2 \times C1}}$	$\sqrt{\frac{C2}{C1}} \times \left[\sqrt{\frac{R2}{R3}} + \sqrt{\frac{R3}{R2}} + \frac{(\sqrt{(R2 \times R3)})}{R1} \right]$	$\sqrt{\frac{C2}{C1 \times 2}} \times \left[\sqrt{\frac{R2}{R3}} + \sqrt{\frac{R3}{R2}} + \frac{(\sqrt{(R2 \times R3)})}{R1} \right]$

Band-Pass Filters



	Non-Inverting	Inverting	Differential Inverting
H_o	$K \left(= 1 + \frac{R_f}{R_s} \right)$	$\frac{R3 / R1}{\left(1 + \frac{C2}{C1} \right)}$	$\frac{R3 / R1}{\left(1 + \frac{C2}{C1} \right)}$
ω_o	$\frac{1}{\sqrt{R1 \times R2 \times C1 \times C2}}$	$\frac{1}{\sqrt{\left(\frac{R3 \times C1 \times C2 \times R1 \times R2}{R1 + R2} \right)}}$	$\frac{1}{\sqrt{\left(\frac{R3 \times C1 \times C2 \times R1 \times R2 / 2}{(R1 + R2 / 2)} \right)}}$
α	$(1-K) \times \sqrt{\frac{R1 \times C1}{R2 \times C2}} + \sqrt{\frac{R2 \times C2}{R1 \times C1}} + \sqrt{\frac{R1 \times C2}{R2 \times C1}}$	$\left[\sqrt{\frac{C1}{C2}} + \sqrt{\frac{C2}{C1}} \right] \times \sqrt{\left(\frac{R1 \times R2}{((R1 + R2) R3)} \right)}$	$\left[\sqrt{\frac{C1}{C2}} + \sqrt{\frac{C2}{C1}} \right] \times \sqrt{\left(\frac{R1 \times R2 / 2}{((R1 + R2) R3)} \right)}$

High-Pass Filters

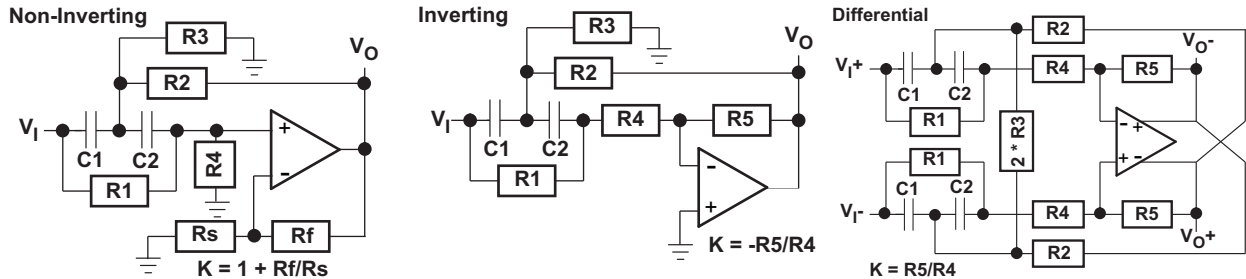


	Non-Inverting	Inverting	Differential Inverting
H_o	$K \left(= 1 + \frac{R_f}{R_s} \right)$	$\frac{C1}{C3}$	$\frac{R3 / R1}{\left(1 + \frac{C2}{C1} \right)}$
ω_o	$\frac{1}{\sqrt{(R1 \times R2 \times C1 \times C2)}}$	$\frac{1}{\sqrt{(R1 \times R2 \times C2 \times C3)}}$	$\frac{1}{\sqrt{\left(\frac{R1}{2} \right) \times R2 \times C2 \times C3}}$
α	$(1-K) \times \sqrt{\frac{R1 \times C1}{(R2 \times C2)}} + \sqrt{\frac{R2 \times C2}{(R1 \times C1)}} + \sqrt{\frac{R1 \times C2}{(R2 \times C1)}}$	$\left[\frac{C1}{\sqrt{C2 \times C3}} + \sqrt{\frac{C2}{C3}} + \sqrt{\frac{C3}{C2}} \right] \times \sqrt{\frac{R1}{R2}}$	$\left[\frac{C1}{\sqrt{C2 \times C3}} + \sqrt{\frac{C2}{C3}} + \sqrt{\frac{C3}{C2}} \right] \times \sqrt{\left(\frac{R1 / 2}{R2} \right)}$

6 Analog Filter Implementations: Biquadratic Filters.

Biquadratic filters also may be non-inverting, inverting, or differential inverting. The schematics below show all these forms with equations for their H_o , α and ω_o . The equations begin with a factor K, the gain of the inner opamp circuit, for each of the filter forms. The remaining quantities, D, H_o , ω_z , α_z , ω_p and α_p , are common to all the filter forms. D is a multiplier used to simplify the following equations.

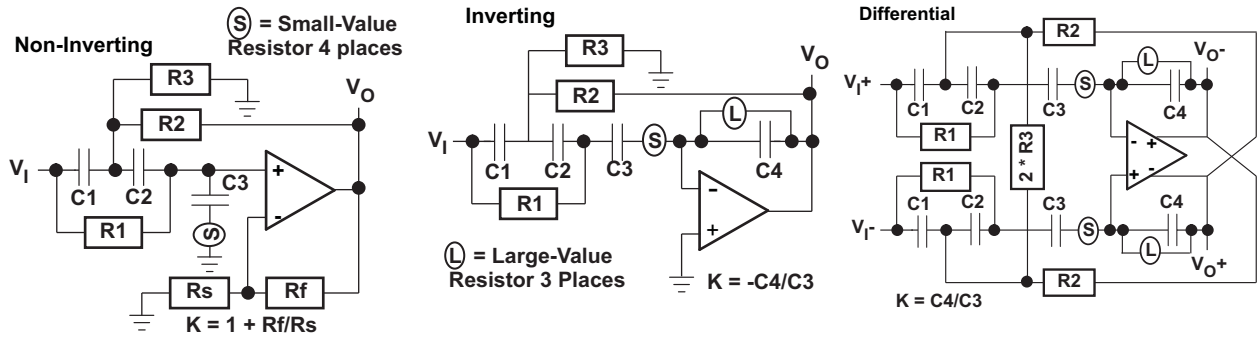
Biquad Filters – High Frequency Boost



	Non-Inverting	Inverting	Differential Inverting
K	$1 + \frac{R_f}{R_s}$	$-\frac{R_5}{R_4}$ (NEGATIVE)	$\frac{R_5}{R_4}$
D	$\frac{R_3}{(R_2 + R_3)}$	Multiplier for R2 and K	
H_o	K	Response at high frequency	
ω_z	$\frac{1}{\sqrt{(R_1 D R_2 C_1 C_2)}}$	ω_p	$\frac{1}{\sqrt{\left(\frac{R_1 D R_2 R_4 C_1 C_2}{R_1 + R_4} \right)}}$
α_z	$\frac{(C_1 + C_2) \sqrt{D R_2}}{\sqrt{(R_1 C_1 C_2)}}$	α_p	$\frac{(R_1 + R_4) D R_2 (C_1 + C_2) + R_1 R_4 C_2 (1 - K D)}{\sqrt{(R_1 D R_2 R_4 C_1 C_2 \times (R_1 + R_4))}}$

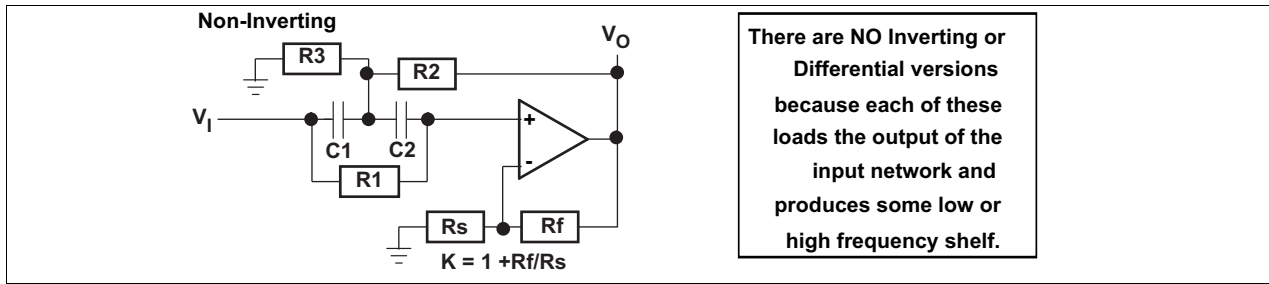
Schematics for low frequency boost filters follow. The input circuit in each filter includes a series chain of capacitors to ground or virtual ground. This load could destabilize either the opamp in the filter or an opamp driving the input, so a small value resistor, maybe 100Ω, is added in series with the final cap in the chain. Also, feedback elements in the inverting and differential filters are capacitors. These provide no path for DC bias, so a large value resistor, maybe 1 to 10 M, is added in parallel with the feedback cap. Of course, these resistances will have a small effect on filter responses, but they should not degrade them significantly.

Biquad Filters – Low Frequency Boost



	Non-Inverting	Inverting	Differential Inverting
K	$1 + \frac{R_f}{R_s}$	$-\frac{C_4}{C_3}$ (NEGATIVE)	$\frac{C_4}{C_3}$
D	$\frac{R_3}{(R_2 + R_3)}$	Multiplier for R2 and K	
H_o	$\frac{K \times C_1 C_2}{(C_1 C_2 + C_1 C_3 + C_1 C_3)}$	Response at low frequency	
ω_z	$\frac{1}{\sqrt{(R_1 D R_2 C_1 C_2)}}$	ω_p	$\frac{1}{\sqrt{(R_1 D R_2 \times (C_1 C_2 + C_1 C_3 + C_1 C_3))}}$
α_z	$\frac{(C_1 + C_2) \sqrt{(D R_2)}}{\sqrt{(R_1 C_1 C_2)}}$	α_p	$\frac{D R_2 \times (C_1 + C_2) + R_1 C_3 + R_1 C_2 \times (1 - K D)}{\sqrt{(R_1 D R_2 \times (C_1 C_2 + C_1 C_3 + C_1 C_3))}}$

Biquad Filters – Single Peak or Valley



	Non-Inverting		
K	$1 + \frac{R_f}{R_s}$		
D	$\frac{R_3}{(R_2 + R_3)}$	Multiplier for R2 and K	
H _o	K	Response at very low and high frequencies	
ω_z	$\frac{1}{\sqrt{(R_1 D R_2 C_1 C_2)}}$	$\omega_p (= \omega_z)$	$\frac{1}{\sqrt{(R_1 D R_2 C_1 C_2)}}$
α_z	$\frac{(C_1 + C_2) \times \sqrt{D R_2}}{\sqrt{(R_1 C_1 C_2)}}$	α_p	$\frac{D R_2 \times (C_1 + C_2) + R_1 C_2 \times (1 - K D)}{\sqrt{(R_1 D R_2 C_1 C_2)}}$

The most complicated quantities are α_z and α_p , so we will look at these in some detail. As we will see, it is more difficult to achieve low values of α_z and α_p than high, so we will concentrate on reducing these quantities. We will consider how to produce values as small as about 0.5, a value that provides significant peaking.

α_z is the same for all 3 filter configurations. The ratio $(C_1 + C_2) / \sqrt{(C_1 C_2)}$ in α_z ranges from about 3.5 to 2 to about 3.5 again as (C_1/C_2) ranges from 0.1 to 1 to 10, so α_z is reduced by making C1 and C2 different in value. So it is typically best to make C1 and C2 similar in value. α_z can be controlled by varying the ratio $\sqrt{(D R_2 / R_1)}$. If $R_1 = 20 \times D R_2$ and $C_1 = C_2$, $\alpha_z = 2 / \sqrt{(20)} = 0.45$, probably close to the lowest value needed.

For α_p we face similar constraints with the first term or two in the numerators, but we have the advantage of the last term, which is negative in non-inverting and differential filters for any KD product greater than 1. So we can use this term to reduce α_p if we need to do so. (Beware, however: if KD is made large enough, the last term will cancel the rest of the numerator, α_p will equal zero and the filter will oscillate at ω_p !)

Note that we do NOT have this advantage in inverting biquads! In those, since K is negative, the sum $(1 - KD)$ is always greater than 1. As a result, it is likely to be very difficult to achieve low values of α_p . For this reason inverting biquads are not likely to be generally useful in EQ work.

Single peak or valley filters are a special case. In these, $\alpha_p = \alpha_z + R_1 C_2 \times (1 - KD) / \sqrt{(R_1 D R_2 C_1 C_2)}$. If C1 and C2 are similar in value, $\alpha_p \approx \alpha_z + (1 - KD) \sqrt{(R_1 / D R_2)} \approx \alpha_z + 2 \times (1 - KD) / \alpha_z$. If KD is less than 1, $(1 - KD)$ is positive, α_p is greater than α_z and the filter creates a valley. If KD equals 1, α_p equals α_z and there is no peak or valley. If KD is greater than 1, the second term is negative, α_p is smaller than α_z and the filter creates a peak.

Note that, for single peak or valley filters, reducing α_z increases the magnitude of the second term in α_p . So in valley filters, with $KD < 1$, reducing α_z tends to increase α_p . This tends to make the resulting valley broad and deep. Make $(1 - KD)$ smaller to narrow or reduce the valley. In peak filters, with $KD > 1$, reducing α_z tends to reduce α_p by making the last term in α_p more negative. This tends to make the resulting peak narrow and high. Make $(1 - KD)$ less negative to broaden or reduce the peak.

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