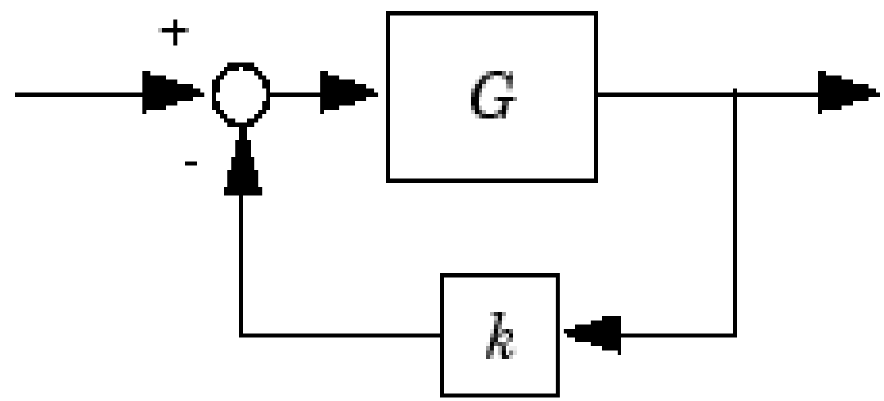
Stability, Gain Margins, Phase Margins

Stability Assessment for feedback

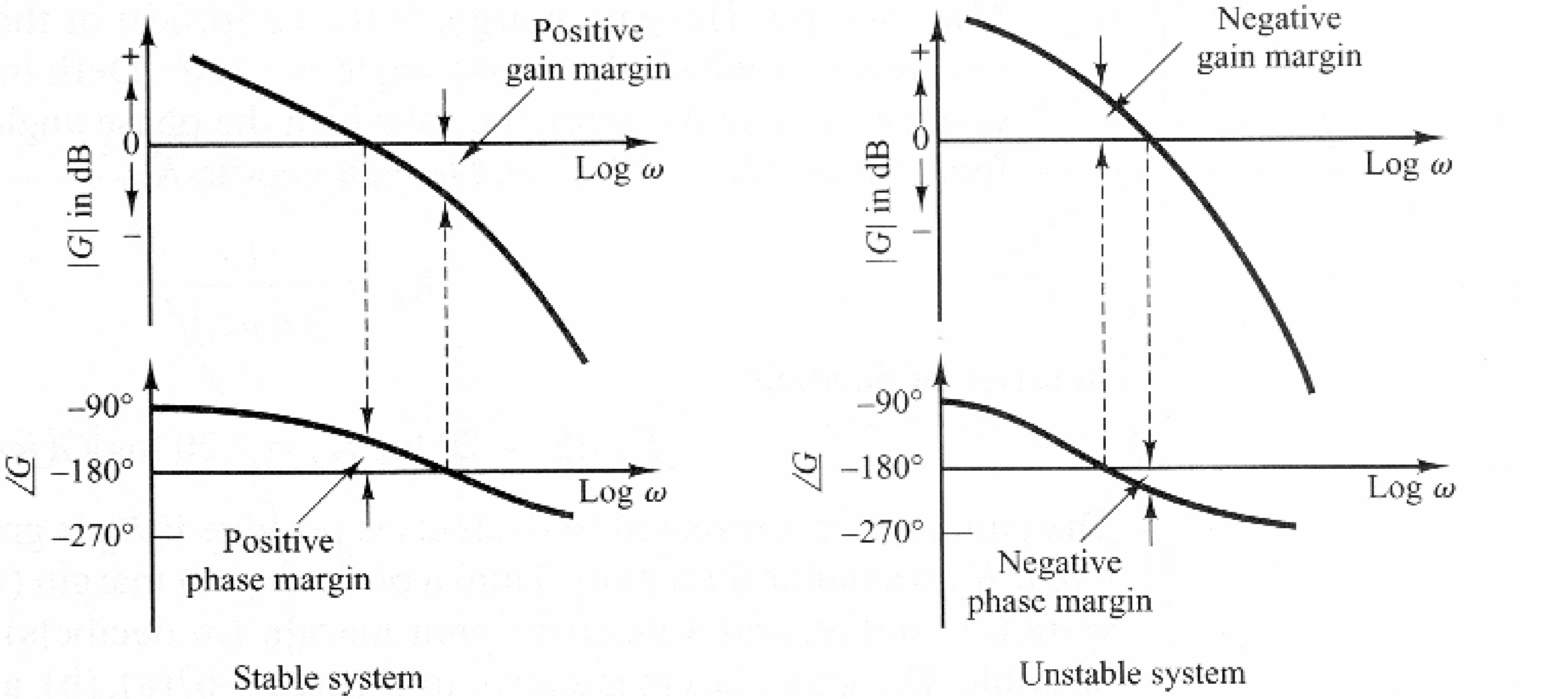
- Using Bode plots of the open-loop system, G(s)
- Characteristic equations 1+kG(s)=0
- Based on Nyquist stability criteria



2 Criteria for feed back stability: Phase & Gain Margins

- kG(s)=-1
- Magnitude : 1 = 0 dB
- Phase: +/- 180 degrees
- Looking at the Bode plots of the open-loop system, G(s)
- Cross-over frequencies

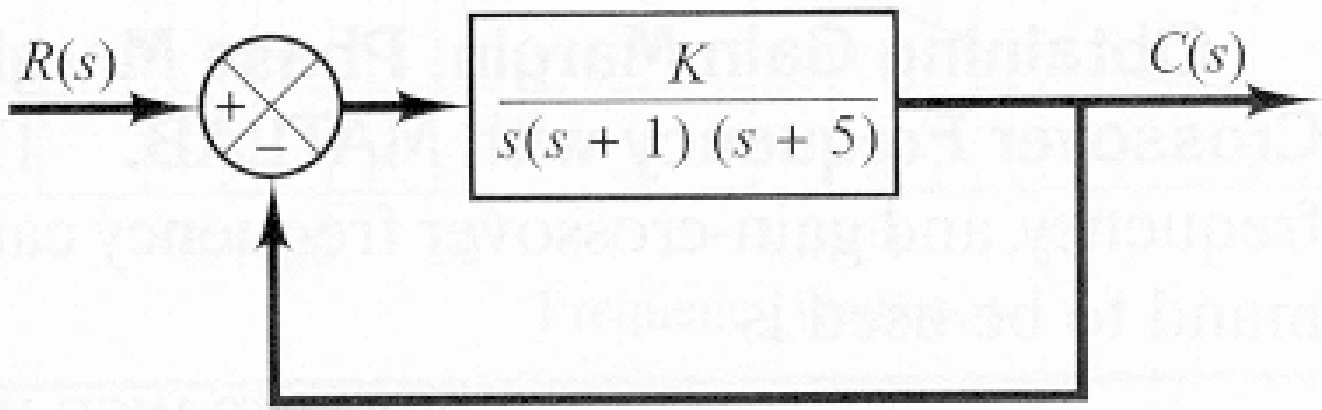
Comparison of two

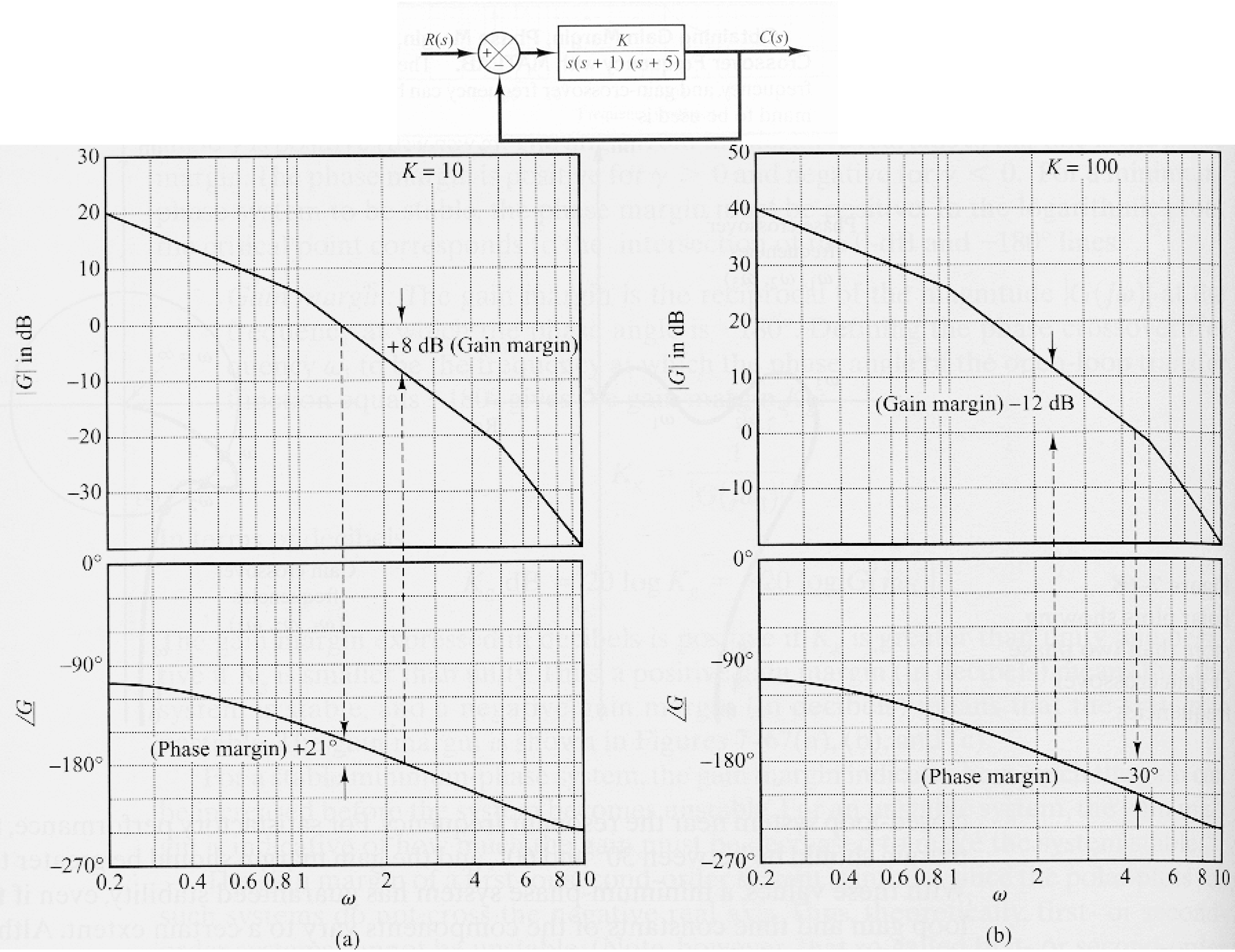


(a)

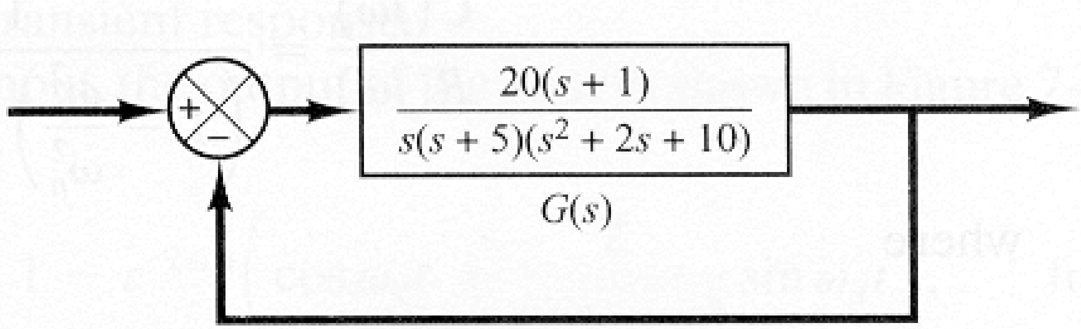
Example

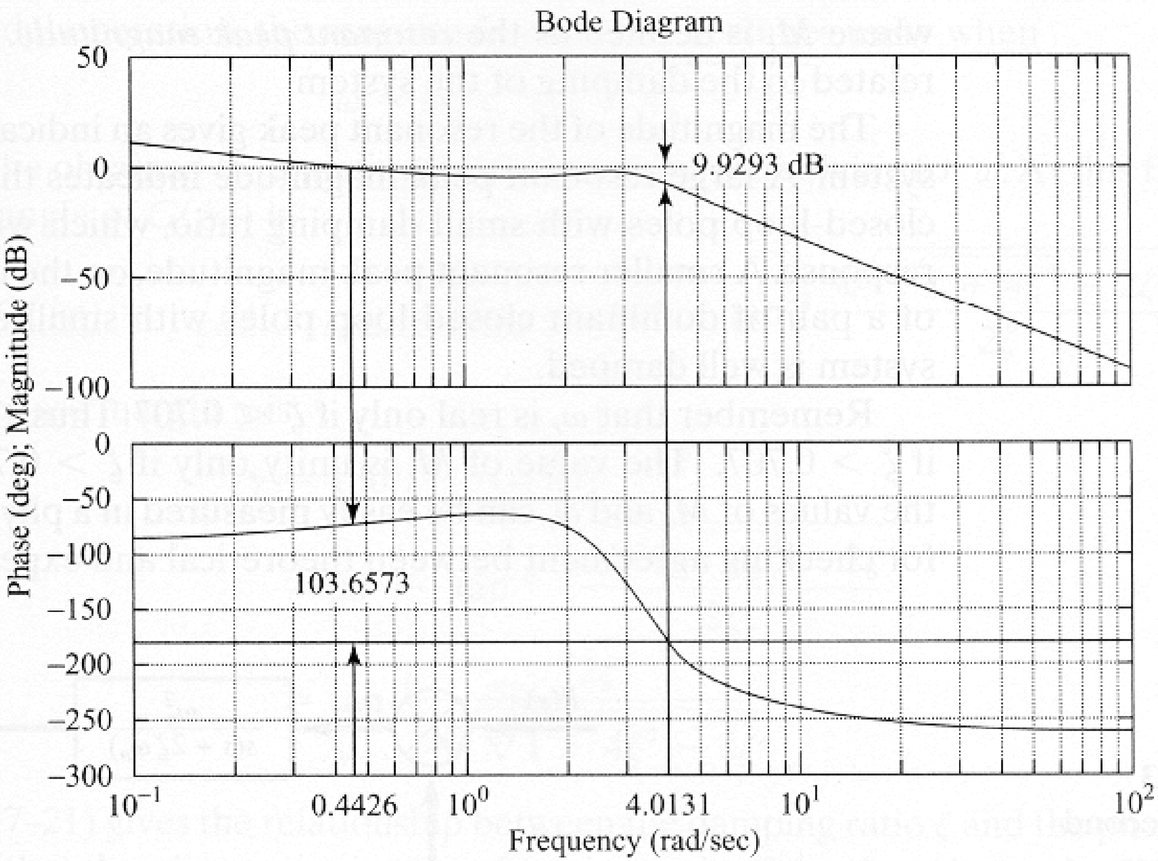
$K = 10$ and $K = 100$.





R(s)
C(s)
K
s(s + 1) (s + 5)
K = 10
+8 dB (Gain margin)
|G| in dB
(Phase margin) +21°
/G
ω
(a)
K = 100
(Gain margin) −12 dB
|G| in dB
(Phase margin)
−30°
/G
ω
(b)


+
−
20(s + 1)
s(s + 5)(s2 + 2s + 10)
G(s)


Bode Diagram
Phase (deg); Magnitude (dB)
50
0
−50
−100
9.9293 dB
0
−50
−100
−150
−200
−250
−300
103.6573
10−1
0.4426
100
4.0131
101
102
Frequency (rad/sec)

MATLAB : margin

MARGIN Gain and phase margins and crossover frequencies.

[Gm,Pm,Wcg,Wcp] = MARGIN(SYS) computes the gain margin Gm, the phase margin Pm, and the associated frequencies Wcg and Wcp, for the SISO open-loop model SYS (continuous or discrete).

The gain margin Gm is defined as 1/G where G is the gain at the -180 phase crossing.

The gain margin in dB is derived by   Gm_dB = 20*log10(Gm)

The phase margin Pm is in degrees.

The loop gain at Wcg can increase or decrease by this many dBs before losing stability.

Matlab

```
num = [20  20];
den = conv([1  5  0],[1  2  10]);
sys = tf(num,den);
w = logspace(-1,2,100);
bode(sys,w)
[Gm,pm,wcp,wcg] = margin(sys);
GmdB = 20*log10(Gm);
[GmdB  pm  wcp  wcg]

ans =

   9.9293  103.6573  4.0131  0.4426
```

End