Op-Amp Noise Calculation and Measurement

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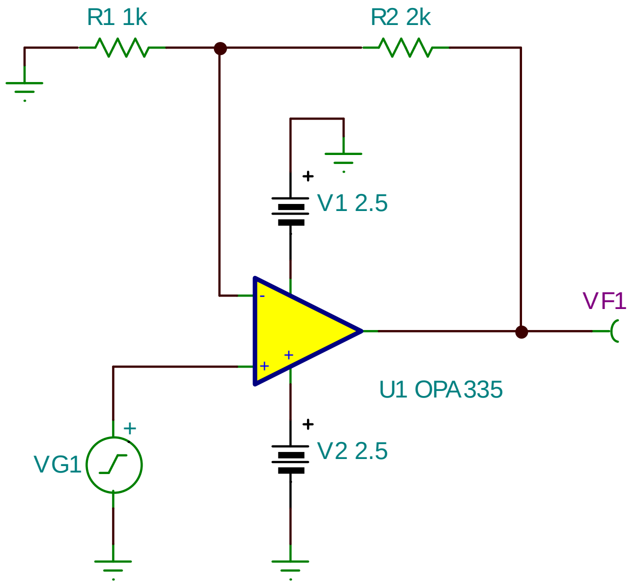
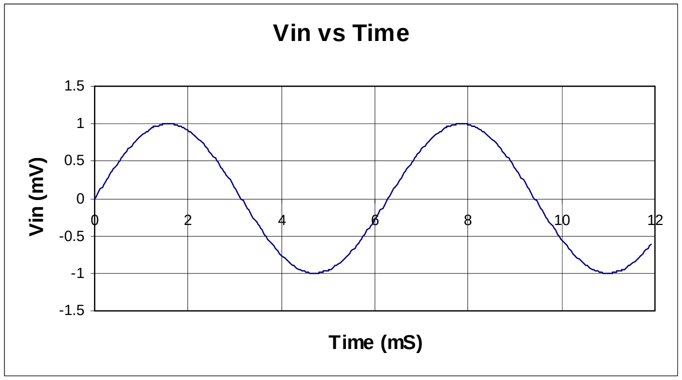
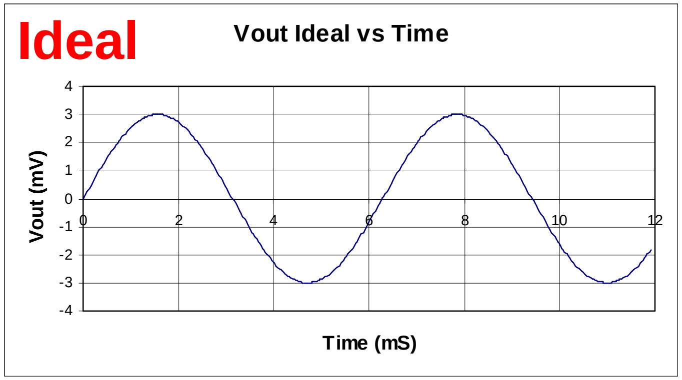
Noise Presentation Contents

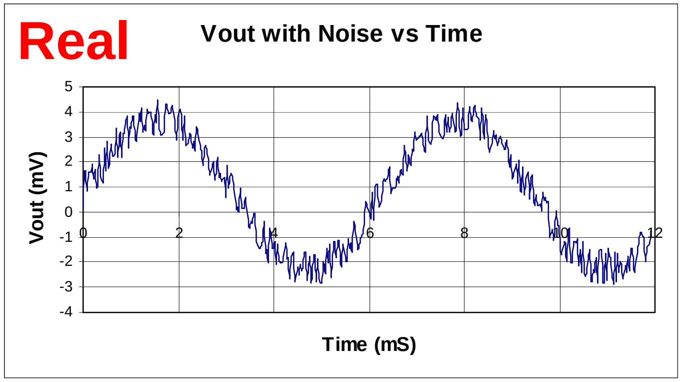
- Review of white noise and 1/f noise
- Noise Hand Calculations
- Tina Spice Noise Analysis
- Noise Measurement
- Appendix 2 – Analysis Details

What is Intrinsic Noise
Why do I Care?





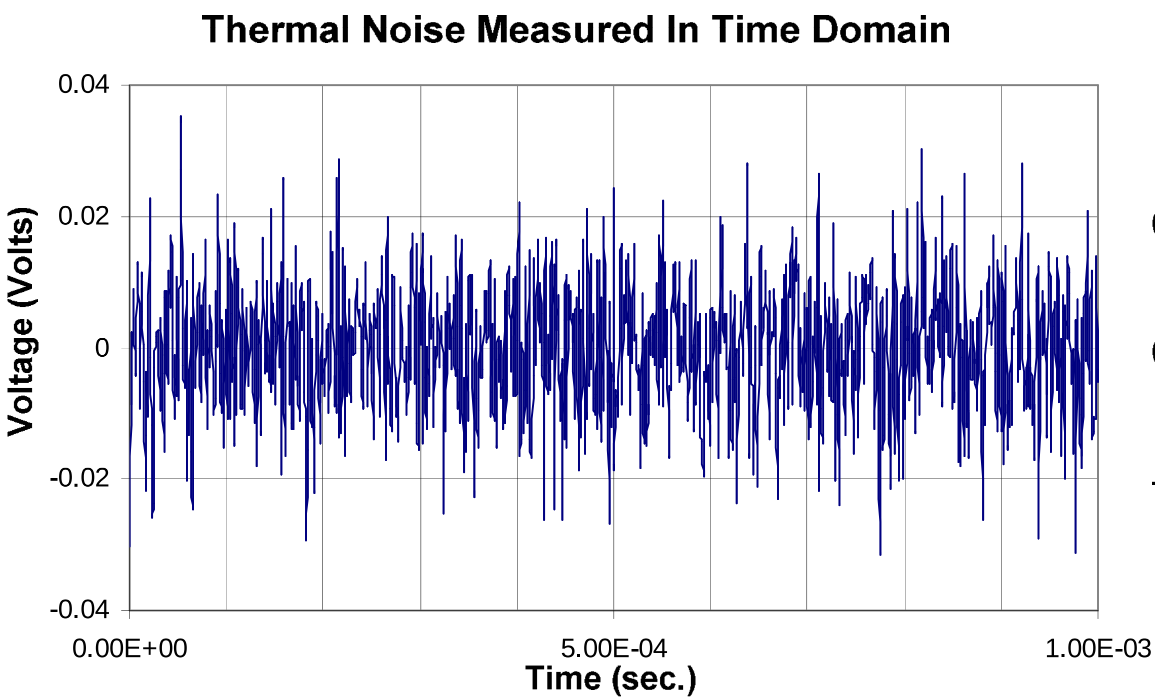


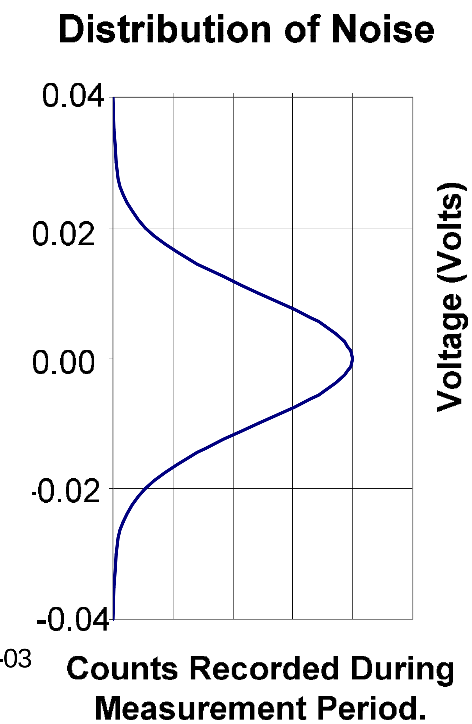


- The op-amp itself generates noise
- Noise acts as an error—it corrupts the signal
- Calculate, simulate, and measure this noise
- Reduce Noise!

White noise or broadband noise

normal distribution

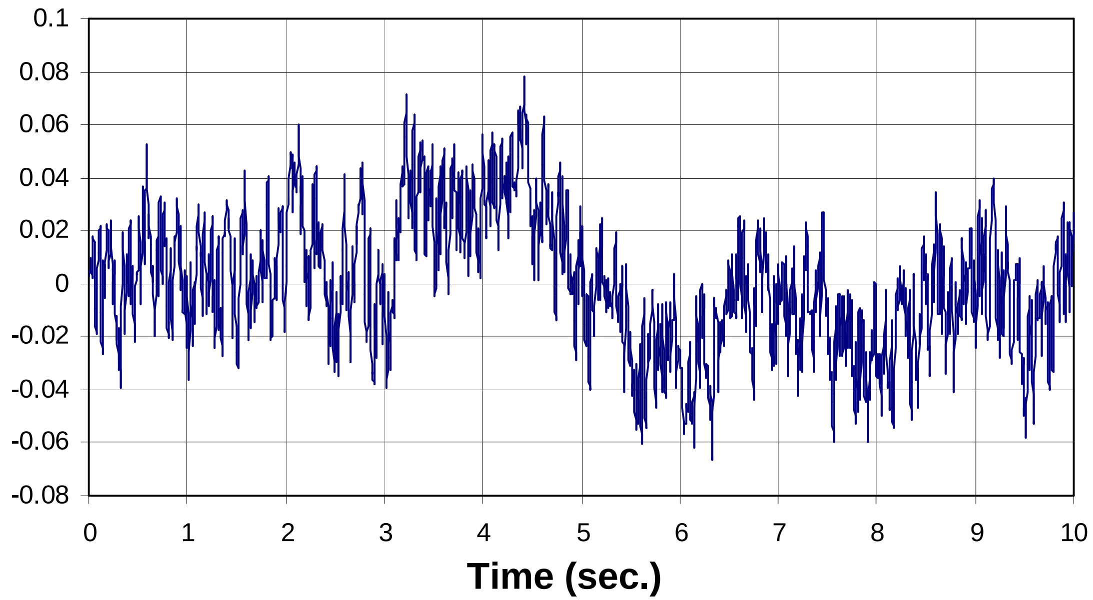




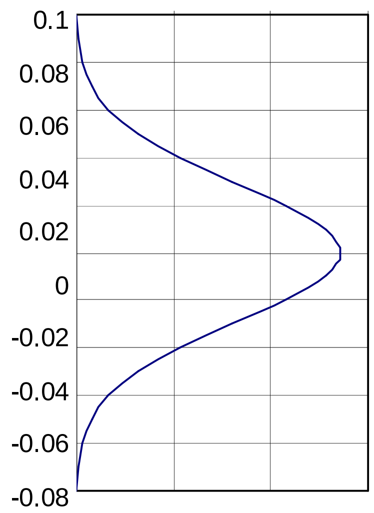
1/f or pink noise

normal distribution

1/f Noise Measured in Time Domain

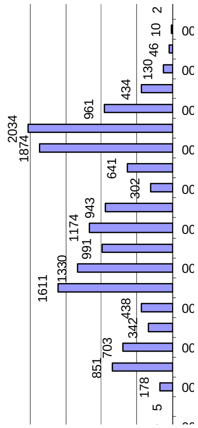


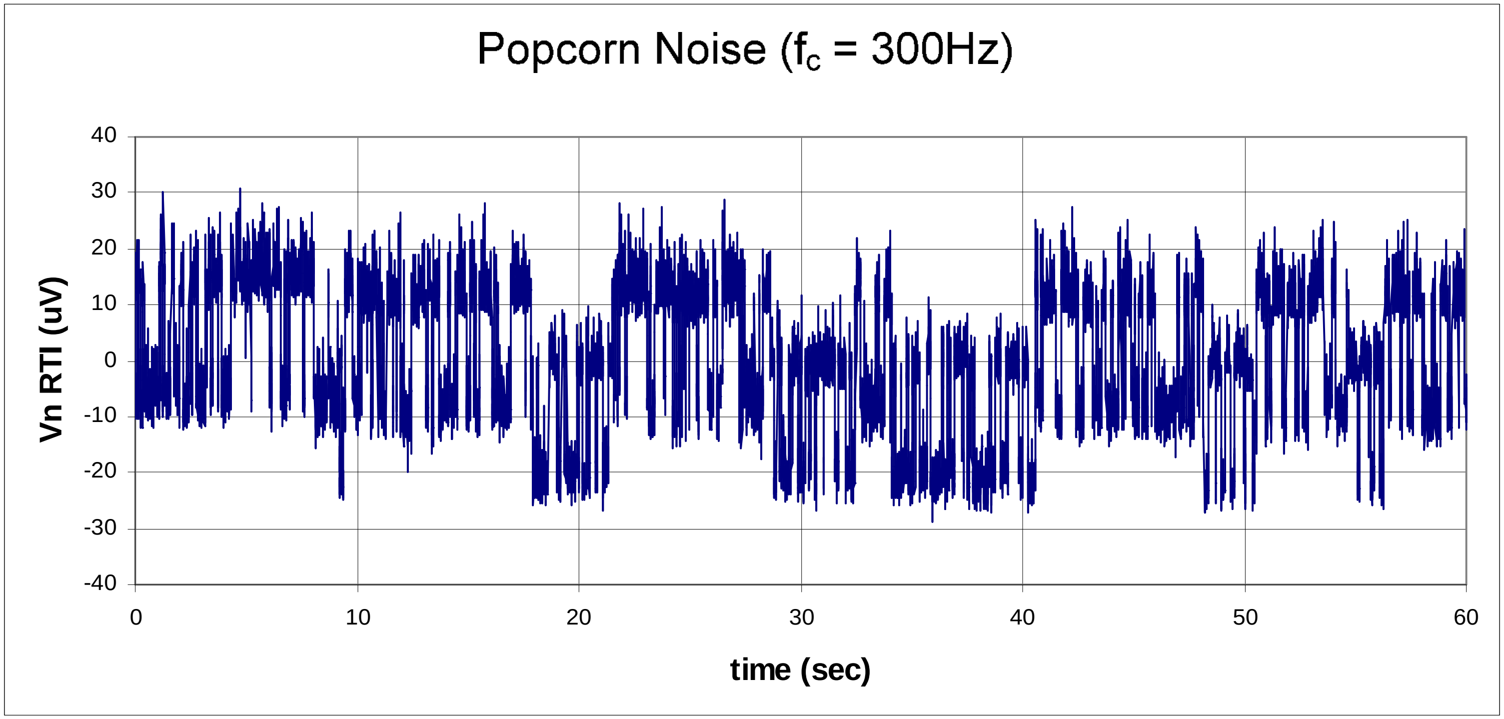
Distribution of Noise



(Burst) Popcorn Noise

Bimodal (or multi-modal) distribution





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Synonyms

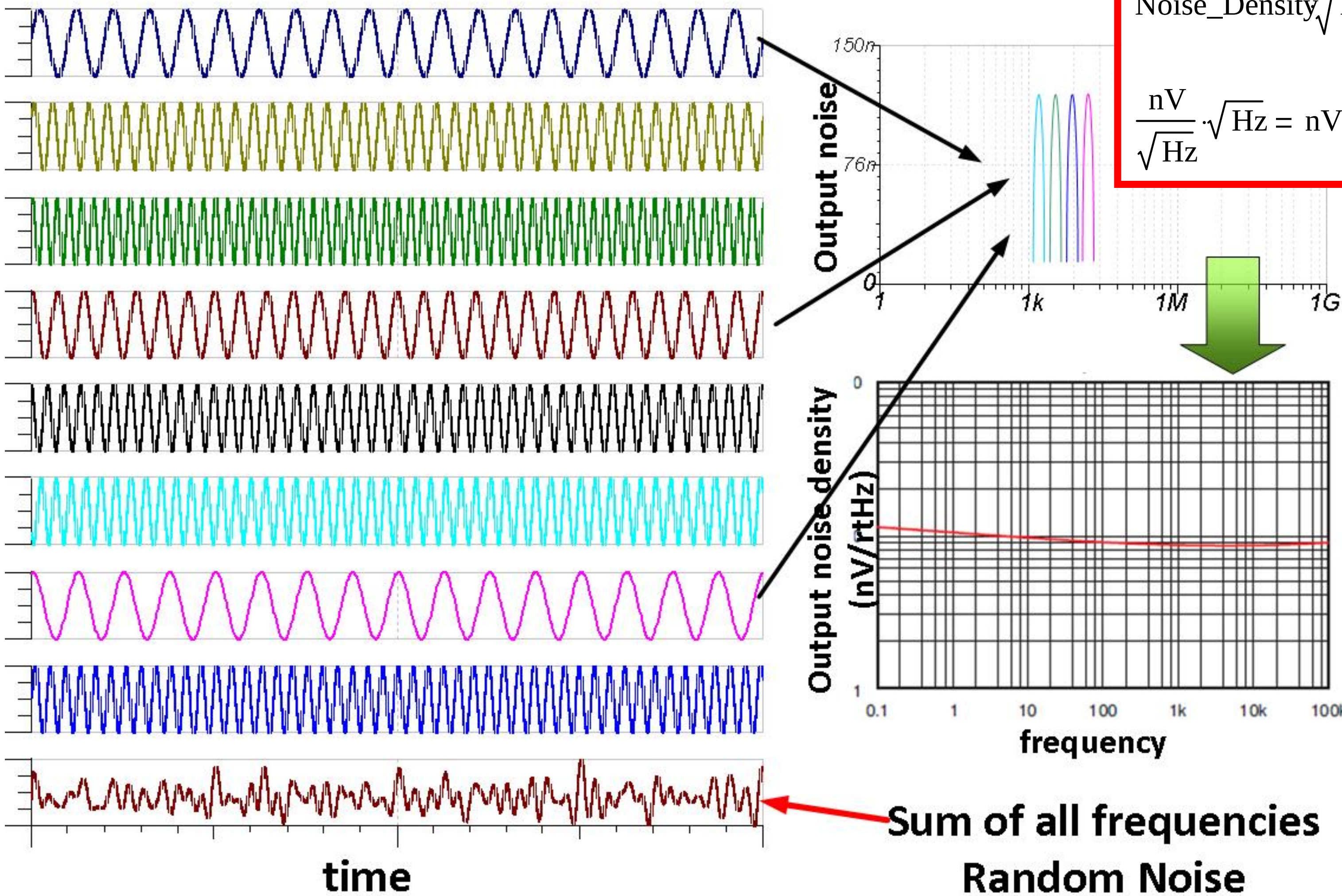
- **Broadband Noise** – White Noise, Johnson Noise, Thermal Noise, Resistor Noise
- **1/f Noise** – Pink Noise, Flicker Noise, Low Frequency Noise, Excess Noise
- **Burst Noise** – Popcorn Noise, Red Noise random telegraph signals (RTS).

  Strictly speaking, these terms are not 100% synonymous. For example, broadband noise on an op-amp may be a combination of thermal noise and shot noise.

What is Spectral Density?

$$\text{Noise\_Density}\sqrt{BW_n} = \text{RMS\_Noise}$$

$$\frac{nV}{\sqrt{Hz}}\cdot\sqrt{Hz} = nV$$

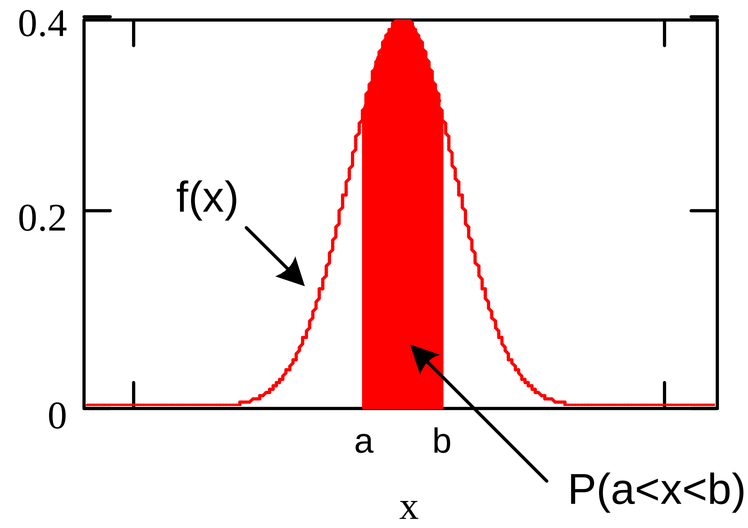


Statistics Review – PDF

Probability **Density** function for Normal (Gaussian) distribution

Outline of Gaussian Curve

$$f(x) = \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{\left[\frac{-(x-\mu)^2}{2\sigma^2}\right]}$$



Probability **Distribution** function for Normal (Gaussian) distribution

$$P(a < x < b) = \int_a^b f(x)\,dx = \int_a^b \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{\left[\frac{-(x-\mu)^2}{2\sigma^2}\right]} dx$$

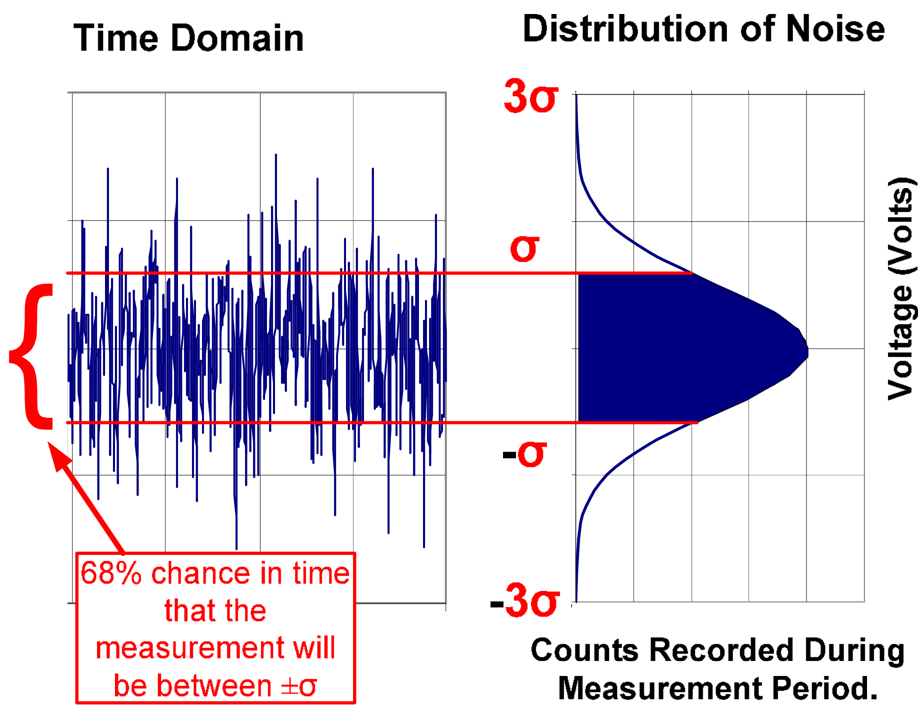
Where
$P(a < x < b)$ -- the probability that x will be in the interval (a, b)
x-- the random variable. In this case noise voltage.
μ -- the mean value
σ -- the standard deviation

Probability an event will occur within interval

For example, if P(-1<x<+1) = 0.3 then there is a 30% chance that x is between -1 and 1.

STDEV Relationship to Peak-to-Peak for a Gaussian PDF

+/-3 STD Deviations = 6 sigma à 99.7%



The Probability Distribution Function P(a<x<b) gives the probability that an event happens between a and b.

$$P(a < x < b) = \int_a^b f(x)\, dx$$

Gaussian PDF

$$f(x) = \frac{1}{\sigma\cdot\sqrt{2\pi}}\cdot e^{\left[\frac{-(x-\mu)^2}{2\sigma^2}\right]}$$

Let $\mu = 0$ because noise has no mean value (dc component).

$$P(-\sigma < x < \sigma) = \int_a^b f(x)\, dx$$

$$P(-\sigma < x < \sigma) = \int_{-\sigma}^{\sigma} \frac{1}{\sigma\cdot\sqrt{2\pi}}\cdot e^{\left[\frac{-(x)^2}{2\sigma^2}\right]}\, dx$$

$$\int_{-\sigma}^{\sigma} \frac{1}{\sigma\cdot\sqrt{2\pi}}\cdot e^{\left[\frac{-(x)^2}{2\sigma^2}\right]}\, dx = 0.683$$

STDEV Relationship to Peak-to-Peak

Number of Standard Deviations	Percent chance of measuring voltage
2σ (same as ±σ)	68.3%
3σ (same as ±1.5σ)	86.6%
4σ (same as ±2σ)	95.4%
5σ (same as ±2.5σ)	98.8%
6σ (same as ±3σ)	99.7%
6.6σ (same as ±3.3σ)	99.9%

Is standard deviation the same as RMS?

RMS vs STDEV

Stdev = RMS when the Mean is zero (No DC component). For all the noise analysis we do this will be the case. The noise signals we consider are Gaussian signals with zero mean. Note that the two formulas are equal to each other if you set $\mu = 0$ (zero average). See further information in appendix.

RMS

$$\mathrm{RMS} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}$$

Where

x_i – data samples

n – number of samples

Standard deviation

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} \left(x_i - \mu\right)^2}$$

Where

x_i – data samples

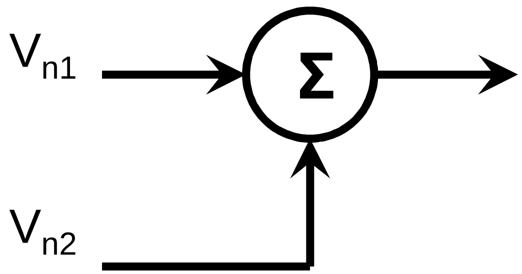
μ – average of all samples

n – number of samples

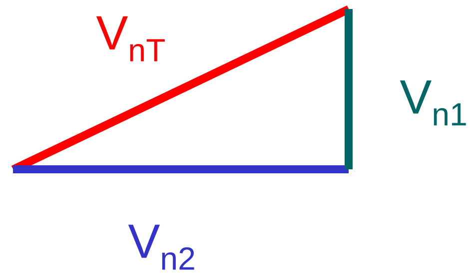
DC component will create σ reading error
$\theta \neq$ RMS

Add Noise As Vectors (RMS Sum)

Sum of two Random Uncorrelated Noise Sources



$$V_{nT} = \sqrt{V_{n1}^2 + V_{n2}^2}$$



Example

$V_{n1} = 3mVrms$

$V_{n2} = 5mVrms$

$$V_{nT} = \sqrt{(3mVrms)^2 + (5 \cdot mVrms)^2} = 5.83mVrms$$

Thermal Noise

The mean- square open- circuit voltage (e) across a resistor (R) is:

$e_n = \sqrt{(4kT_K R\Delta f)}$ where:

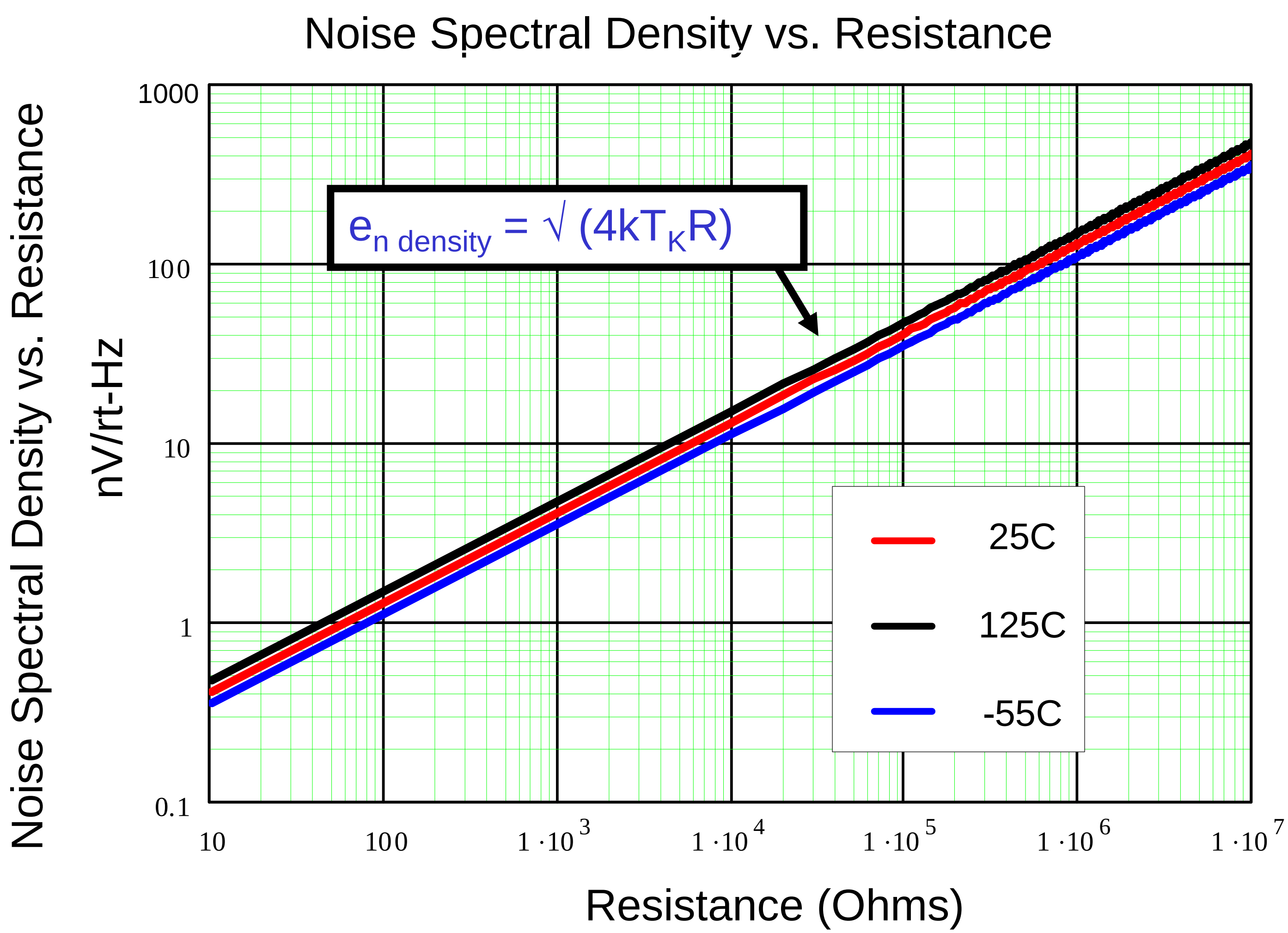
- T_K is Temperature (^oK)
- R is Resistance (Ω)
- f is frequency (Hz)
- k is Boltzmann's constant (1.381E-23 joule/^oK)
- e_n is volts (V_{RMS})

To convert Temperature Kelvin to

$T_K = 273.15^oC + T_C$

Random motion of charges generate noise

Thermal Noise

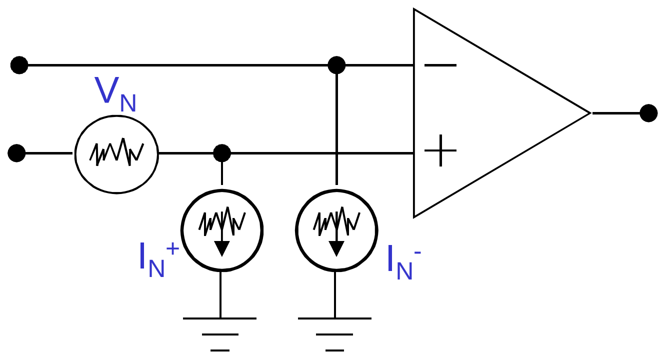


Resistance versus Spectral Density

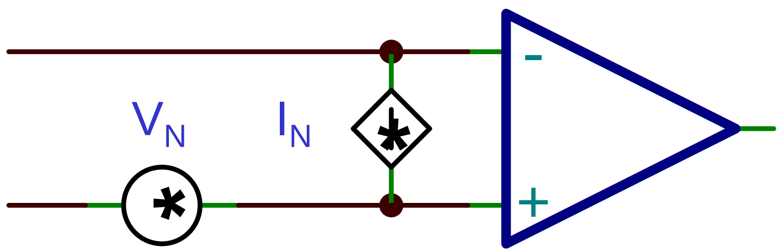
Op-Amp Noise Model

Noise Model

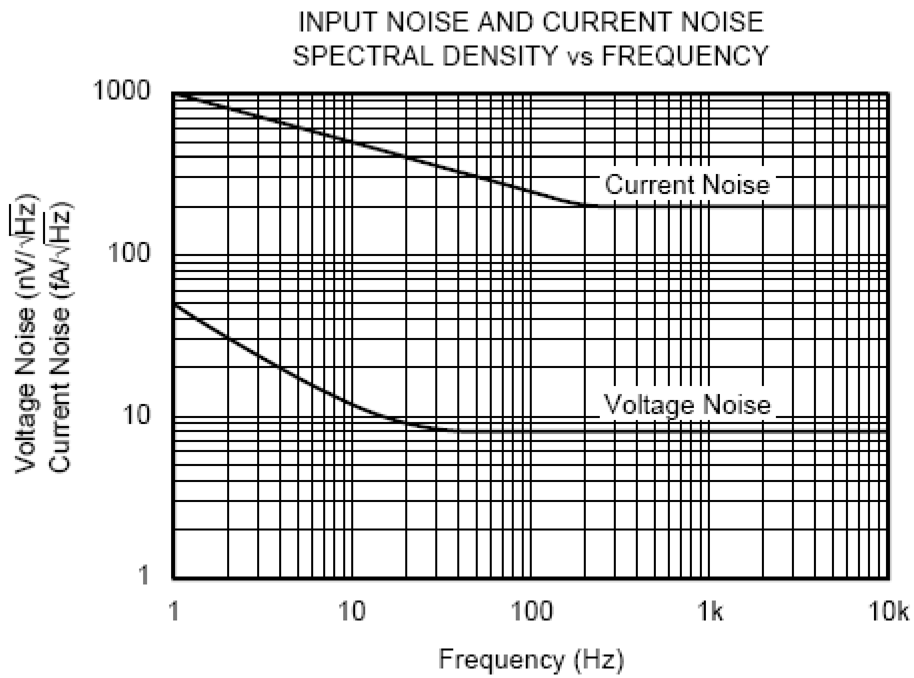
(I_N+ and I_N- are not correlated)



Tina Simplified Model

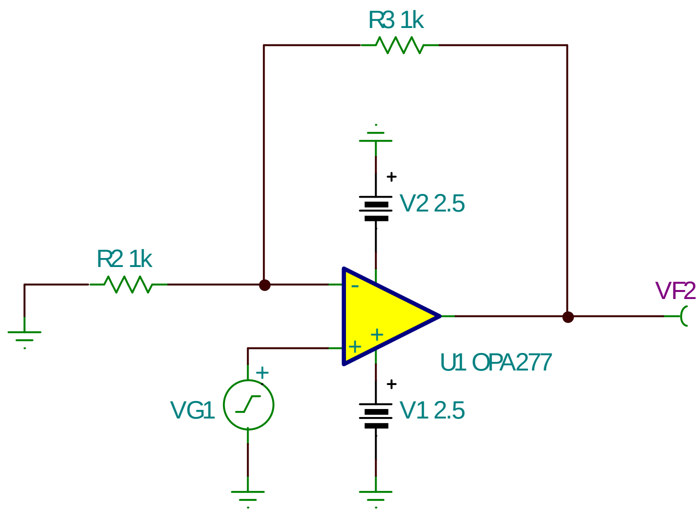


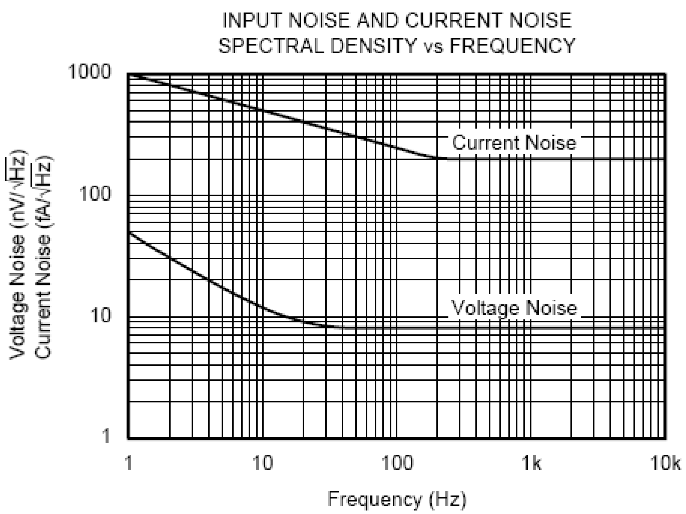
OPA277 Data



Hand Calculation Technique

Noise Analysis for Simple Op-Amp Circuit





Noise Sources

Op-Amp Voltage Noise Source

Op-Amp Current Noise Sources

Resistor Noise Sources

Calculation Considerations

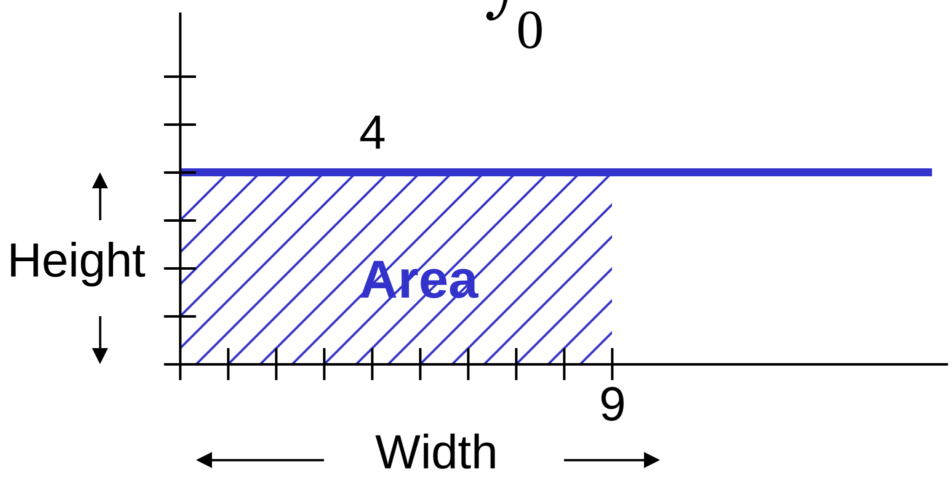
Convert Noise Spectrum to Noise Voltage
- External Filter Bandwidth Limit
- Op-Amp Closed Loop Bandwidth

Noise Gain

Calculus Reminder

$$\int_0^9 4\,dx = 4 \cdot 9 = 36$$

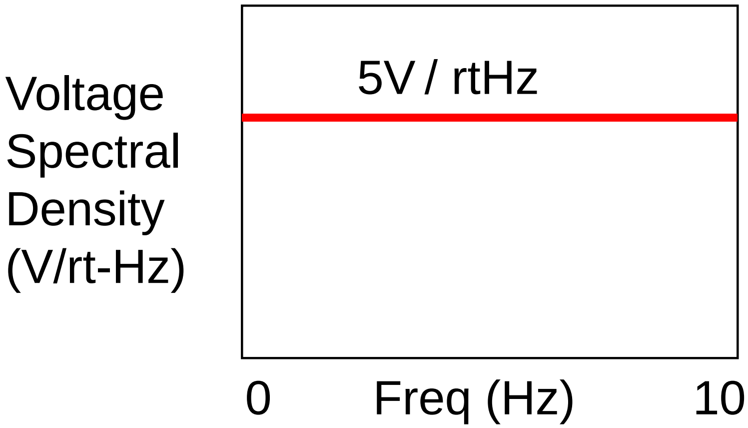
Height x Width



Integral = Area under the curve

Convert Noise Spectrum to Noise Voltage

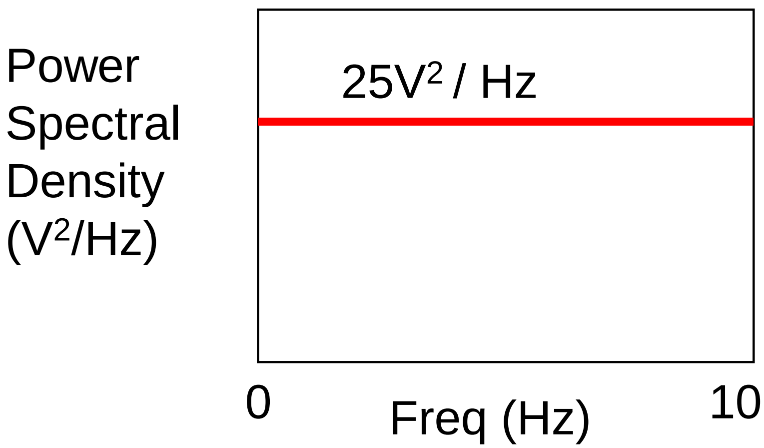
(Broadband Only – Simple Case)



You can't integrate the **Voltage** spectral density curve to get noise

$$\int_0^{10} V\_spec\_dens\ df = 5\cdot\frac{V}{\sqrt{Hz}}\cdot 10\cdot Hz = 50\cdot\frac{V\cdot Hz}{\sqrt{Hz}}$$

Wrong



You integrate the **Power** spectral density curve to get noise

$$NoisePower = \int_0^{10} (V\_spec\_dens)^2\ df = 25\cdot\frac{V^2}{Hz}\cdot 10\cdot Hz = 250\cdot V^2$$

$$NoiseVoltage = \sqrt{NoisePower} = \sqrt{250\cdot V^2} = 15.811V \quad \text{RMS}$$

Correct

Noise Gain for Voltage Noise Source

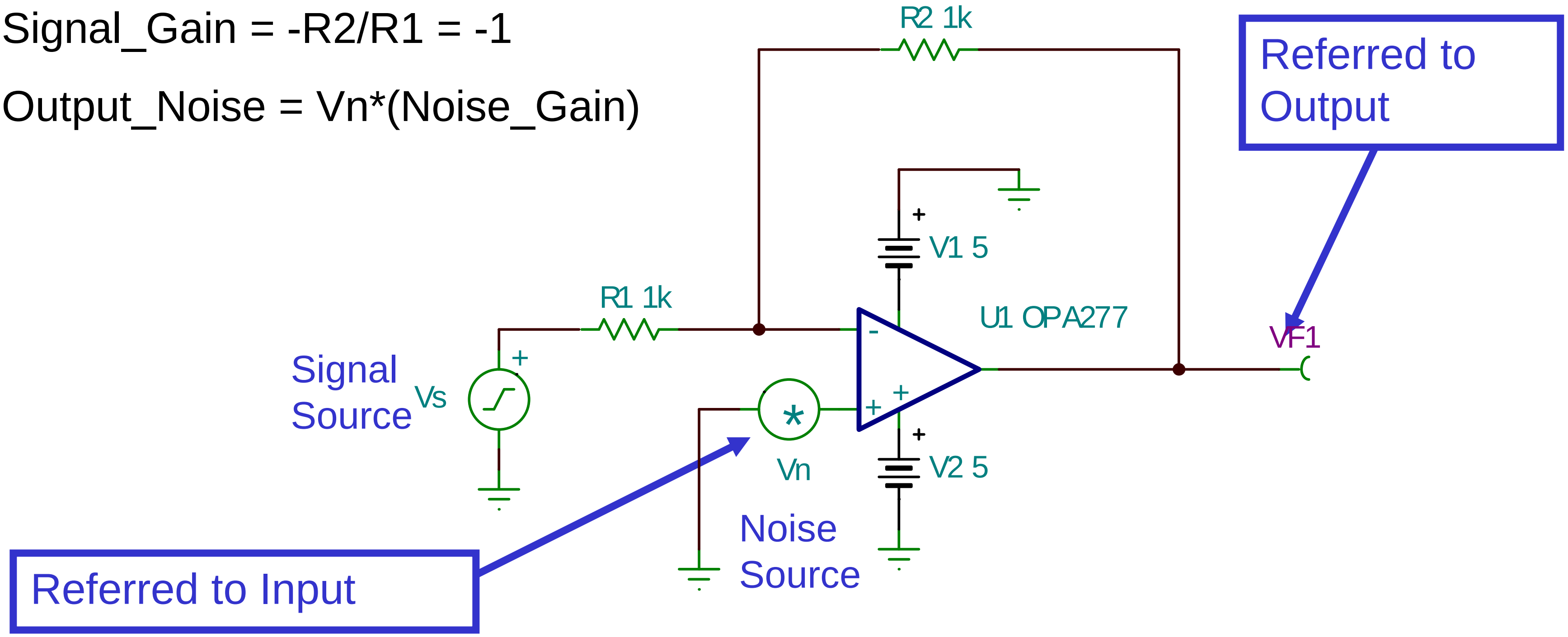
Noise Gain – Gain seen by the noise source.

Example:

Noise_Gain = (R2/R1) + 1 = 2

Signal_Gain = -R2/R1 = -1

Output_Noise = Vn*(Noise_Gain)



Understanding The Spectrum:
Total Noise Equation (Current or Voltage)

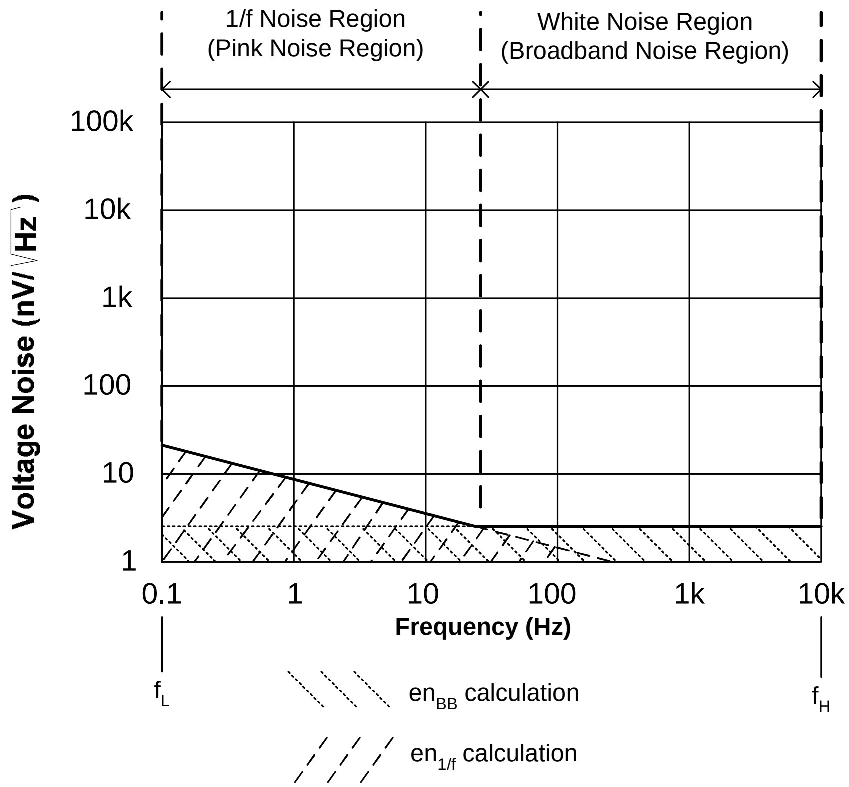
$$e_{nT} = \sqrt{[(e_{n1/f})^2 + (e_{nBB})^2]}$$

where:

e_{nT} = Total rms Voltage Noise in volts rms

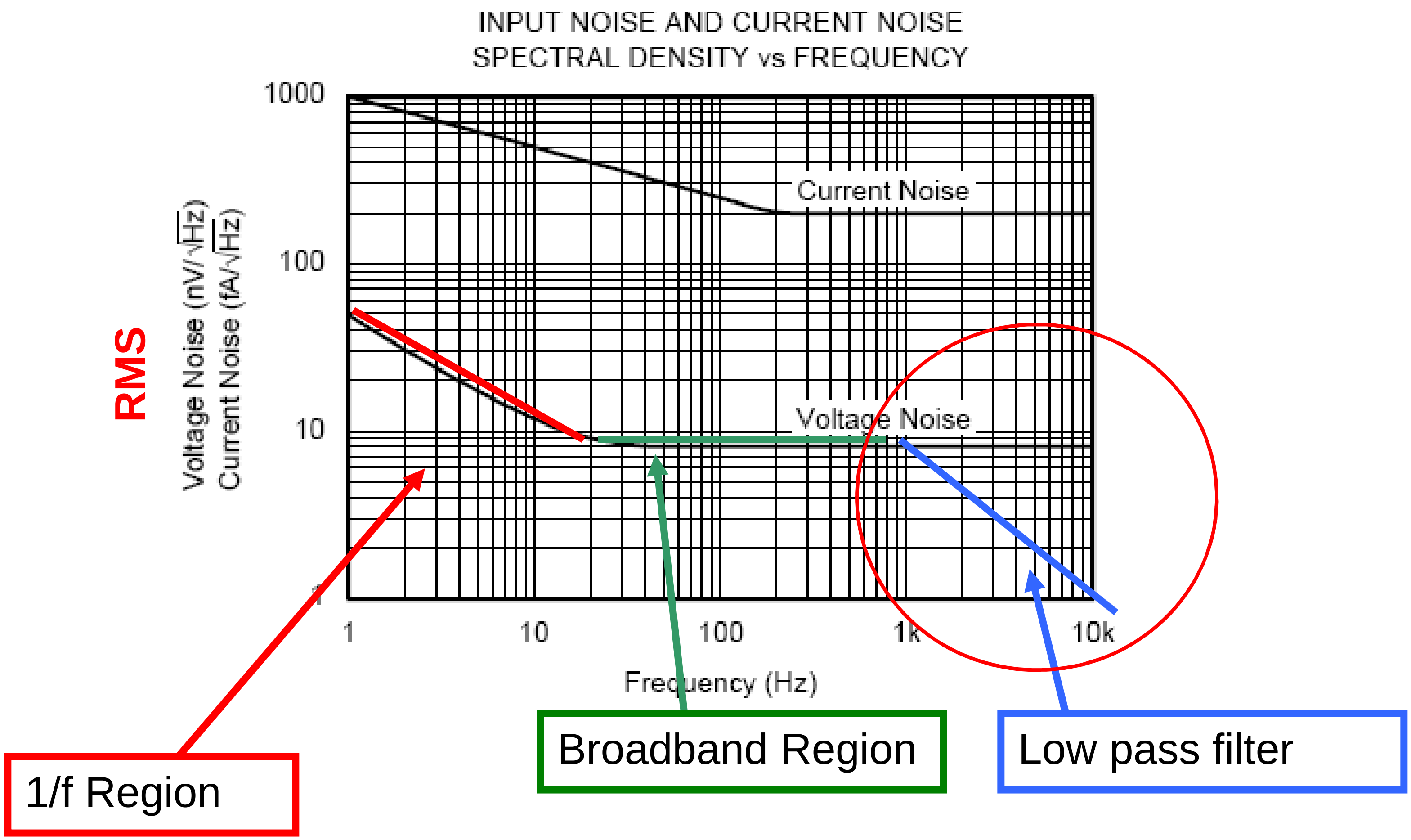
$e_{n1/f}$ = 1/f voltage noise in volts rms

e_{nBB} = Broadband voltage noise in volts rms

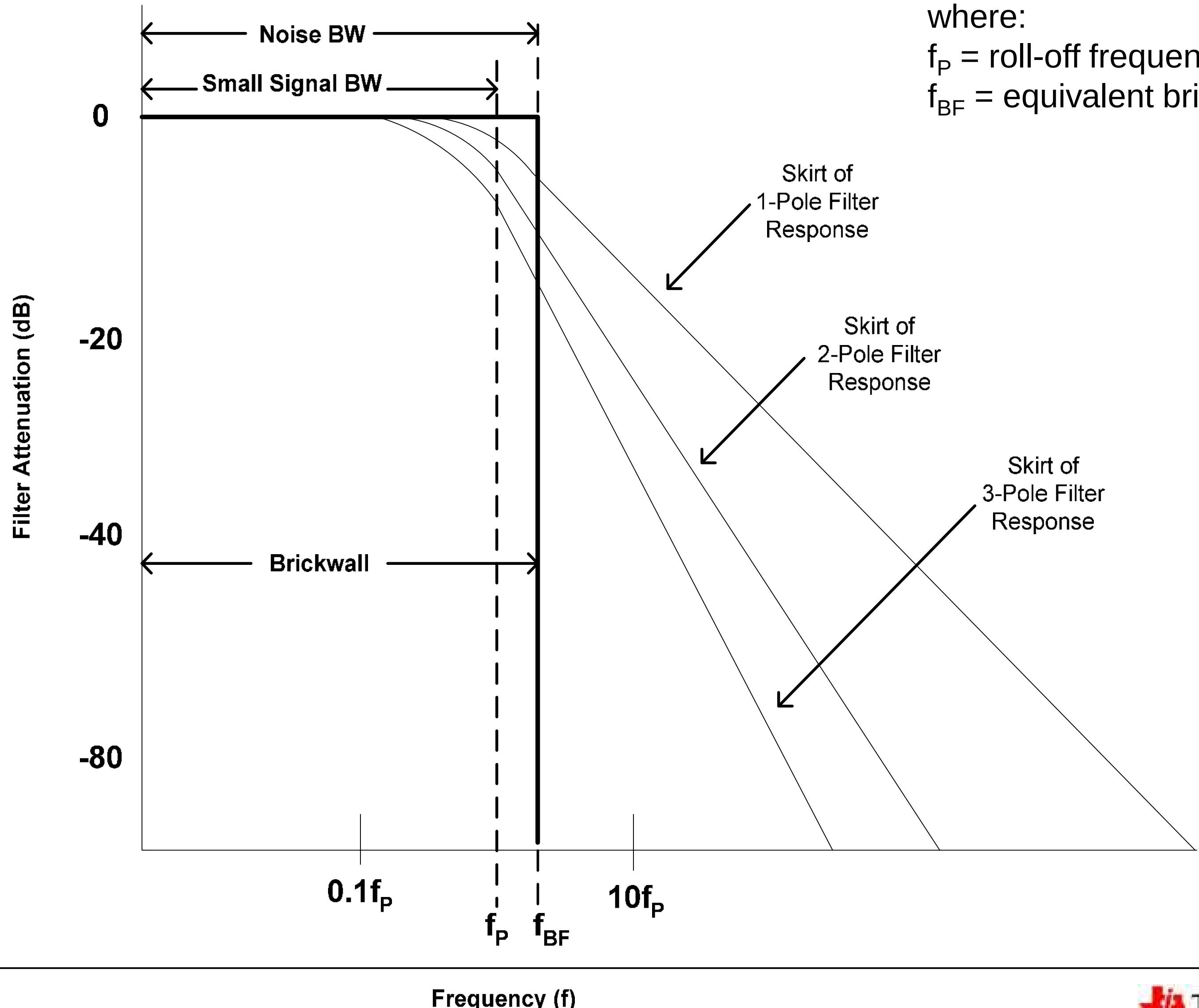


Low Pass Filter Shapes the Spectrum

How do we convert this plot to noise?



Real Filter Correction vs Brickwall Filter



where:
f_P = roll-off frequency of pole or poles
f_{BF} = equivalent brickwall filter frequency

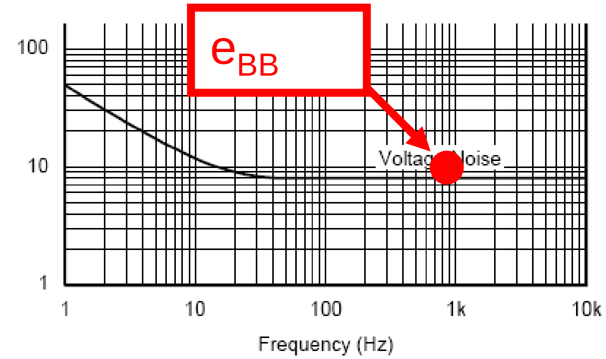
AC Noise Bandwidth Ratios for n^{th} Order Low-Pass Filters

$BW_n = (f_H)(K_n)$ Effective Noise Bandwidth

Real Filter Correction vs Brickwall Filter

Number of Poles in Filter	Kn AC Noise Bandwidth Ratio
1	1.57
2	1.22
3	1.16
4	1.13
5	1.12

Broadband Noise Equation



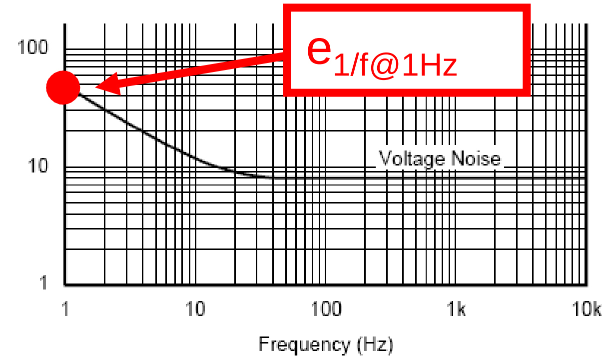
$$BW_n = (f_H)(K_n)$$

where:
BW_n = noise bandwidth for a given system
f_H = upper frequency of frequency range of operation
K_n = "Brickwall" filter multiplier to include the "skirt" effects of a low pass filter

$$en_{BB} = (e_{BB})(\sqrt{[BW_n]})$$

where:
en_{BB} = Broadband voltage noise in volts rms
e_{BB} = Broadband voltage noise density ; usually in nV/√Hz
BW_n = Noise bandwidth for a given system

1/f Noise Equation
(see appendix for derivation)



$$e_{1/f@1Hz} = (e_{1/f@f})(\sqrt{[f]})$$

where:

$e_{1/f@1Hz}$ = normalized noise at 1Hz (usually in nV)

$e_{1/f@f}$ = voltage noise density at f ; (usually in nV/√Hz)

f = a frequency in the 1/f region where noise voltage density is known

$$en_{1/f} = (e_{1/f@1Hz})(\sqrt{[\ln(f_H/f_L)]})$$

where:

$en_{1/f}$ = 1/f voltage noise in volts rms over frequency range of operation

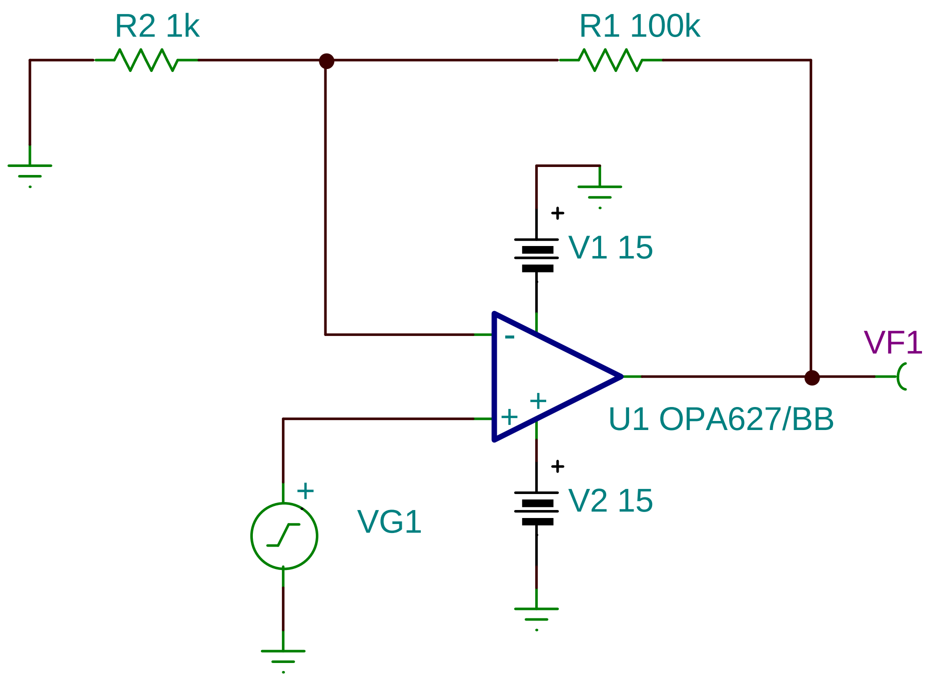
$e_{1/f@1Hz}$ = voltage noise density at 1Hz; (usually in nV)

f_H = upper frequency of frequency range of operation

(Use BW_n as an approximation for f_H)

f_L = lower frequency of frequency range of operation

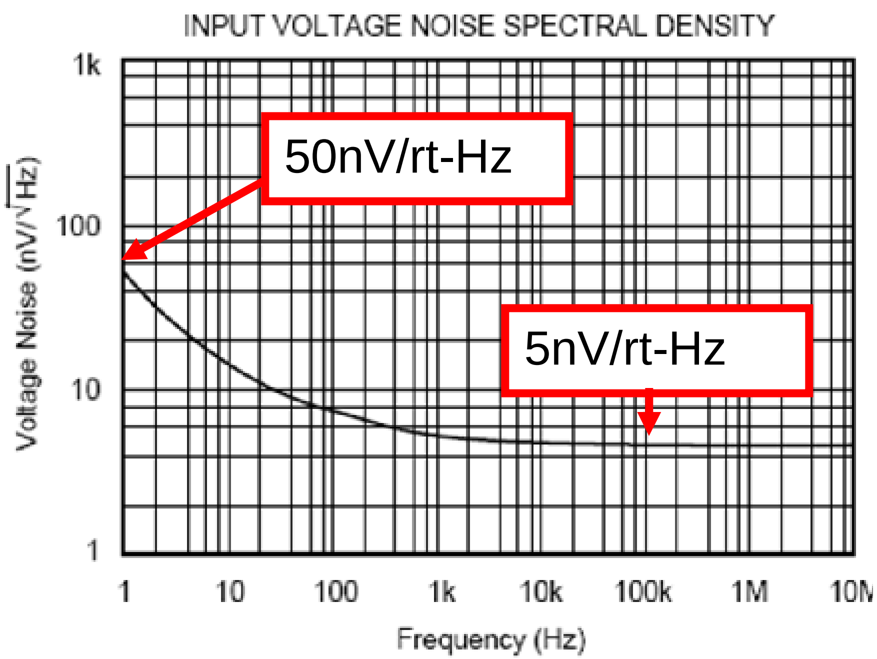
Example Noise Calculation

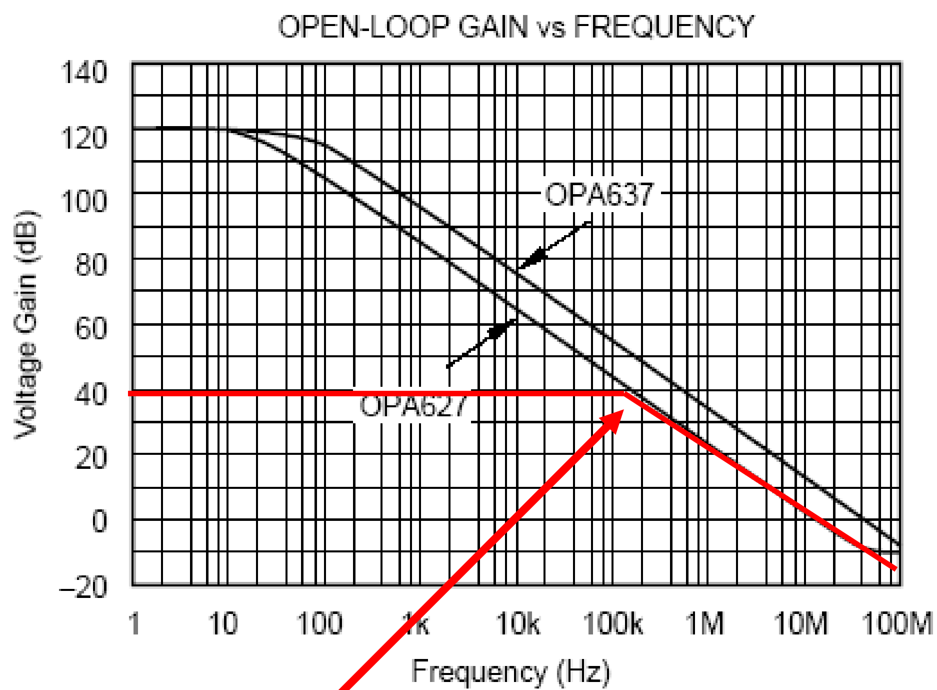


Given:
OPA627
Noise Gain of 101

Find (RTI, RTO):
Voltage Noise
Current Noise
Resistor Noise

Voltage Noise Spectrum and Noise Bandwidth





Unity Gain Bandwidth = 16MHz

Closed Loop Bandwidth = 16MHz / 101 = 158kHz

Example Voltage Noise Calculation

Voltage Noise Calculation:

Broadband Voltage Noise Component:

$\mathbf{BW_n \approx (f_H)(K_n)}$ (note Kn = 1.57 for single pole)

$BW_n \approx (158kHz)(1.57) = 248kHz$

$\mathbf{en_{BB} = (e_{BB})(\sqrt{BW_n})}$

$en_{BB} = (5nV/\sqrt{Hz})(\sqrt{248kHz}) = 2490nV\ rms$

1/f Voltage Noise Component:

$\mathbf{e_{1/f@1Hz} = (e_{1/f@f})(\sqrt{f})}$

$e_{1/f@1Hz} = (50nV/\sqrt{Hz})(\sqrt{1Hz}) = 50nV$

$\mathbf{en_{1/f} = (e_{1/f@1Hz})(\sqrt{[\ln(f_H/f_L)]})}$ **Use $f_H = BW_n$**

$en_{1/f} = (50nV)(\sqrt{[\ln(248kHz/1Hz)]}) = 176nV\ rms$

Total Voltage Noise (referred to the input of the amplifier):

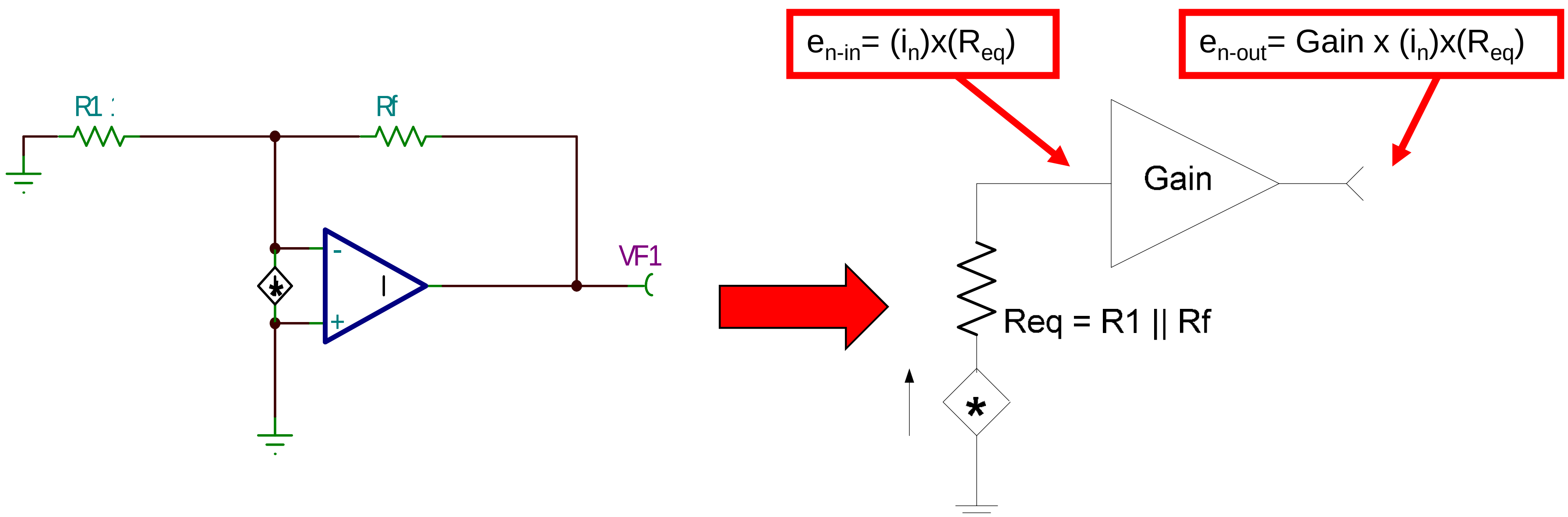
$\mathbf{en_T = \sqrt{[(en_{1/f})^2 + (en_{BB})^2]}}$

$en_T = \sqrt{[(176nV\ rms)^2 + (2490nV\ rms)^2]} = 2496nV\ rms$

Example Current Noise Calculation

PARAMETER	OPA627BM, BP, SM OPA637BM, BP, SM MIN	TYP	MAX	UNITS
NOISE				
Input Voltage Noise				
Noise Density: f = 10Hz		15	40	nV/√Hz
f = 100Hz		8	20	nV/√Hz
f = 1kHz		5.2	8	nV/√Hz
f = 10kHz		4.5	6	nV/√Hz
Voltage Noise, BW = 0.1Hz to 10Hz		0.6	1.6	µVp-p
Input Bias Current Noise				
Noise Density, f = 100Hz		1.6	2.5	fA/√Hz
Current Noise, BW = 0.1Hz to 10Hz		30	60	fAp-p

Note: This example amp doesn't have 1/f component for current noise.



Example Current Noise Calculation

Broadband Current Noise Component:
$BWn \approx (f_H)(K_n)$
$BWn \approx (158kHz)(1.57) = 248kHz$

$R_{eq} = R_f \,||\, R_1 = 100k \,||\, 1k = 0.99k$

$e_{n\text{-}r} = (i_{BB})(R_{eq})$

$e_{n\text{-}r} = (2.5fA/\sqrt{Hz})(0.99k\Omega) = 0.0025nV/\sqrt{Hz}$

$E_{n\text{-}r} = (e_{n\text{-}r})(\sqrt{BW_n})$

$E_{n\text{-}r} = (0.0025nV/\sqrt{Hz})(\sqrt{248kHz}) = 1.23nV\ rms$ neglect

Since the Total Voltage noise is $e_{nvt} = 2496nV\ rms$
the current noise can be neglected.

PARAMETER	OPA627BM, BP, SM OPA637BM, BP, SM			UNITS
	MIN	TYP	MAX	
NOISE				
Input Voltage Noise				
Noise Density: f = 10Hz		15	40	nV/√Hz
f = 100Hz		8	20	nV/√Hz
f = 1kHz		5.2	8	nV/√Hz
f = 10kHz		4.5	6	nV/√Hz
Voltage Noise, BW = 0.1Hz to 10Hz		0.6	1.6	µVp-p
Input Bias Current Noise				
Noise Density, f = 100Hz		1.6	2.5	fA/√Hz
Current Noise, BW = 0.1Hz to 10Hz		30	60	fAp-p

Example Resistor Noise Calculation

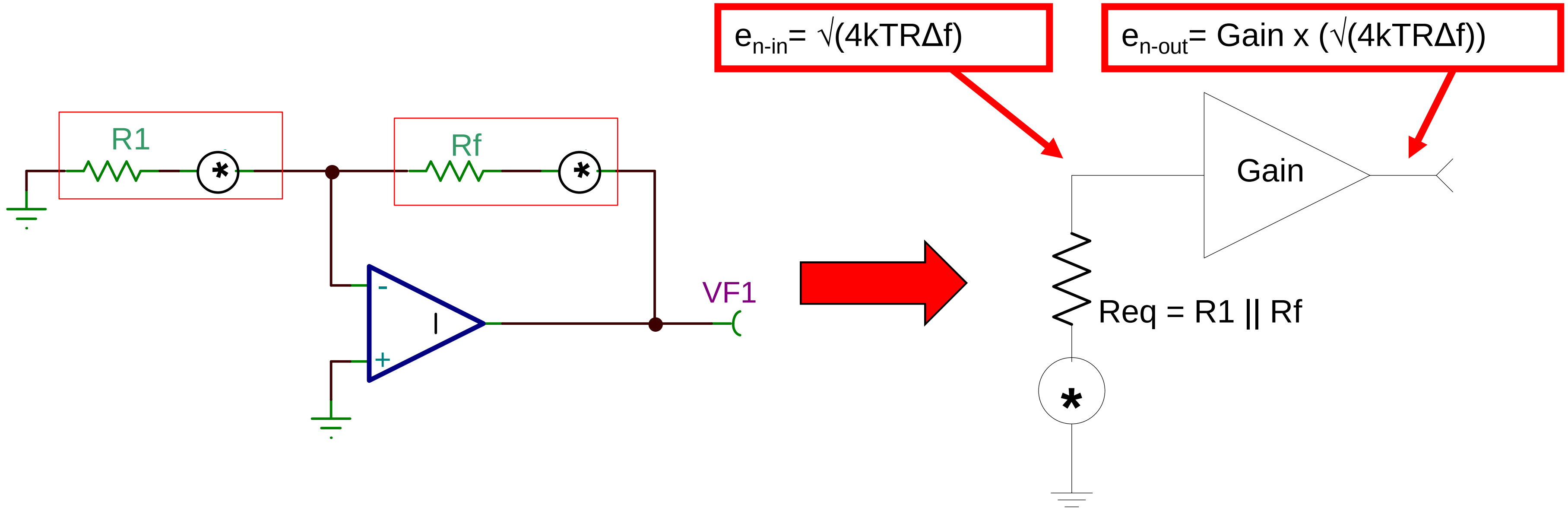
$e_{nr} = \sqrt{(4kT_K R\Delta f)}$

where:

$R = Req = R1||Rf$

$\Delta f = BW_n$

$e_{nr} = \sqrt{(4\ (1.38E\text{-}23)\ (273 + 25)\ (0.99k)(248kHz))} = 2010nV\ rms$



Total Noise Calculation

Voltage Noise From Op-Amp RTI:

e_{nv} = 2510nV rms

Current Noise From Op-Amp RTI (as a voltage):

e_{ni} = 1.24nV rms

Resistor Noise RTI:

e_{nr} = 2020nV rms

Total Noise RTI:

$e_{n\ in} = \sqrt{((2510nV)^2 + ((1.2nV)^2 + ((2010nV)^2)} = 3216nV$ rms

Total Noise RTO:

$e_{n\ out} = e_{n\ in}$ x gain = (3216nV)(101) = 325uV rms

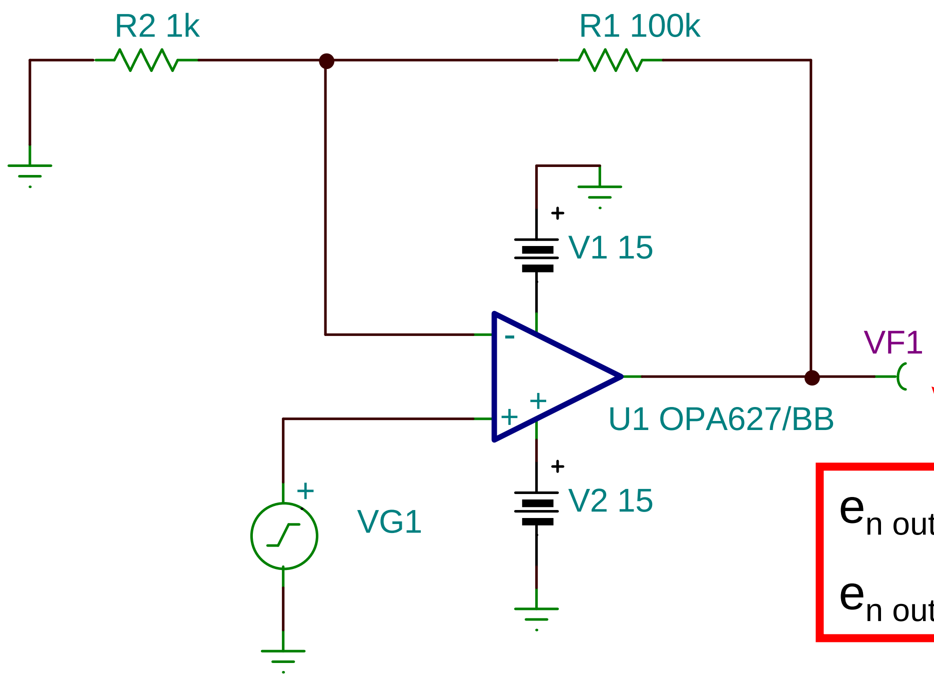
Calculating Noise Vpp from Noise Vrms

Relation of Peak-to-Peak Value of AC Noise Voltage to rms Value

Peak-to-Peak Amplitude	Probability of Having a Larger Amplitude
2 X rms	32%
3 X rms	13%
4 X rms	4.6%
5 X rms	1.2%
6 X rms *	0.3%
6.6 X rms	0.1%

***Common Practice is to use:**
Peak-to-Peak Amplitude = 6 X rms

Peak to Peak Output For our Example



$e_{n\ out}$ = 325uV rms

$e_{n\ out\ p\text{-}p}$ = (325uV rms)x6 = 1.95mVp-p

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Tina Spice Noise Analysis

Tina-TI Spice – Free simulation software

- DC, AC, Transient, and Noise simulation
- Includes all Texas Instruments op-amps
- Unlimited Nodes
- Does not include some options
  (e.g. Monte Carlo analysis)

- Search for "Tina Spice" on www.ti.com
  - Download free
  - Application circuits available

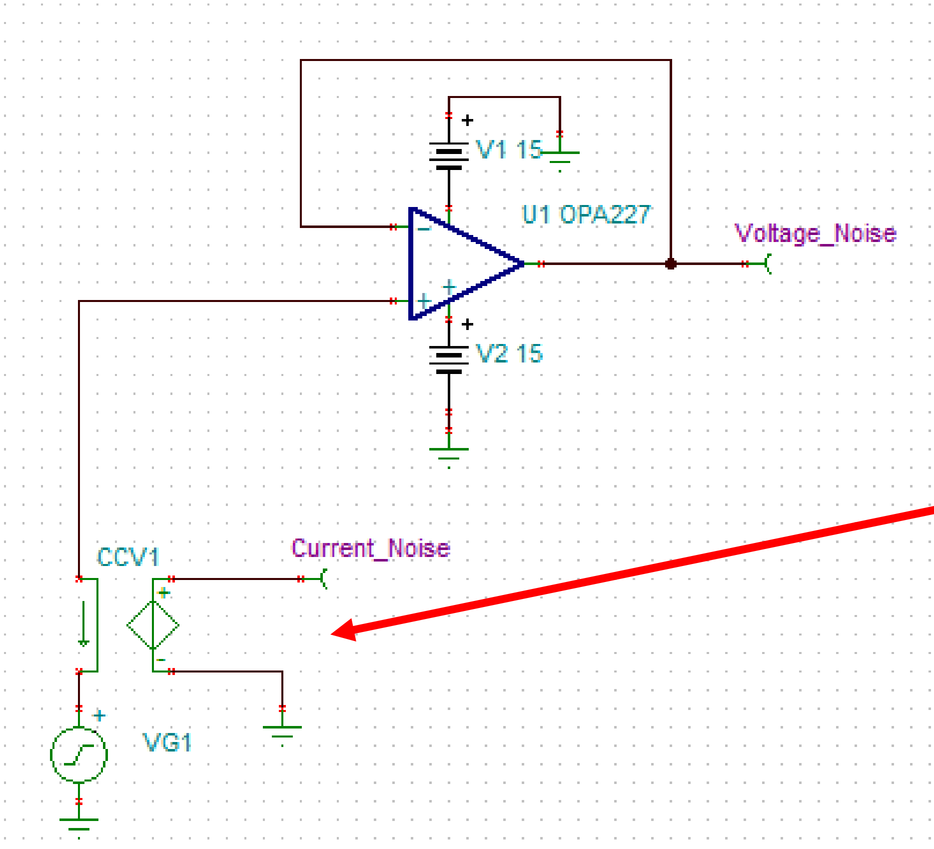
Tina Spice Analysis

1. How to Verify that the Tina Model is Accurate
2. How to Build Your Own Model (see appendix)
3. How to Compute Input and Output Noise

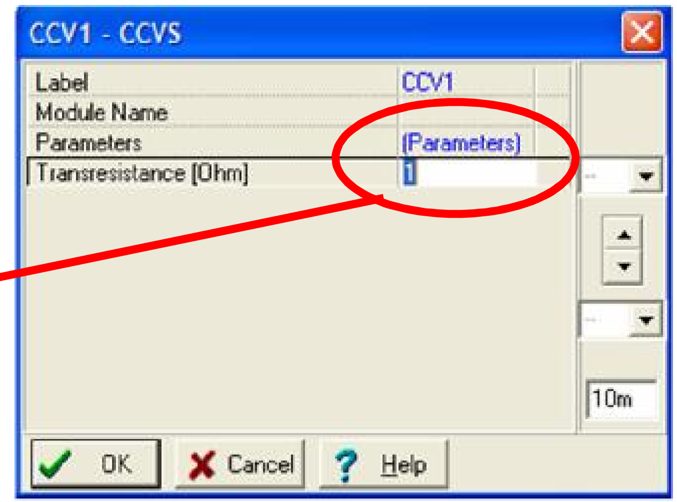
Translate Current Noise to Voltage Nose

Is the Tina Noise Model Accurate?

OPA227 bipolar operational amplifier will be evaluated



Set Gain to 1.



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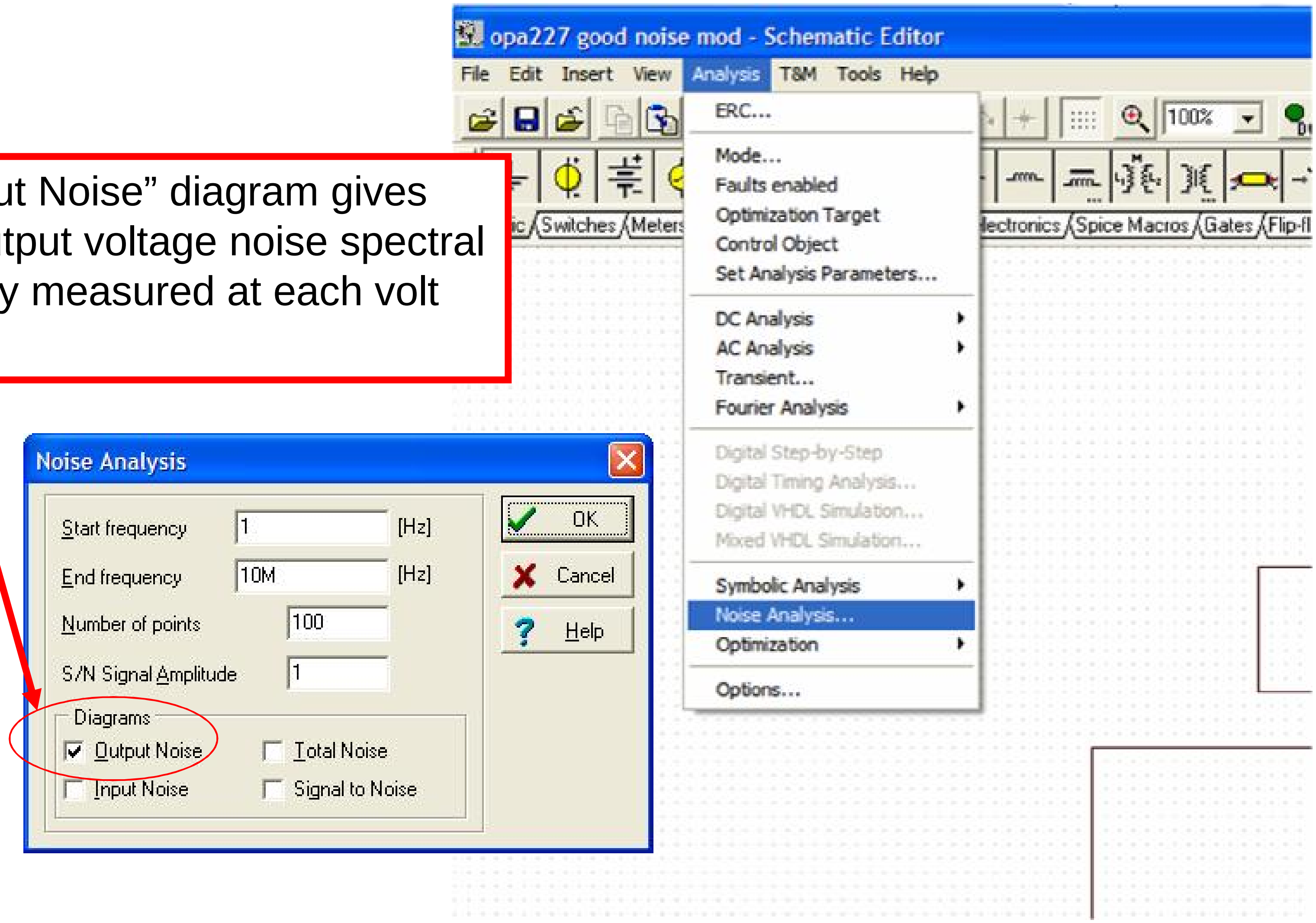
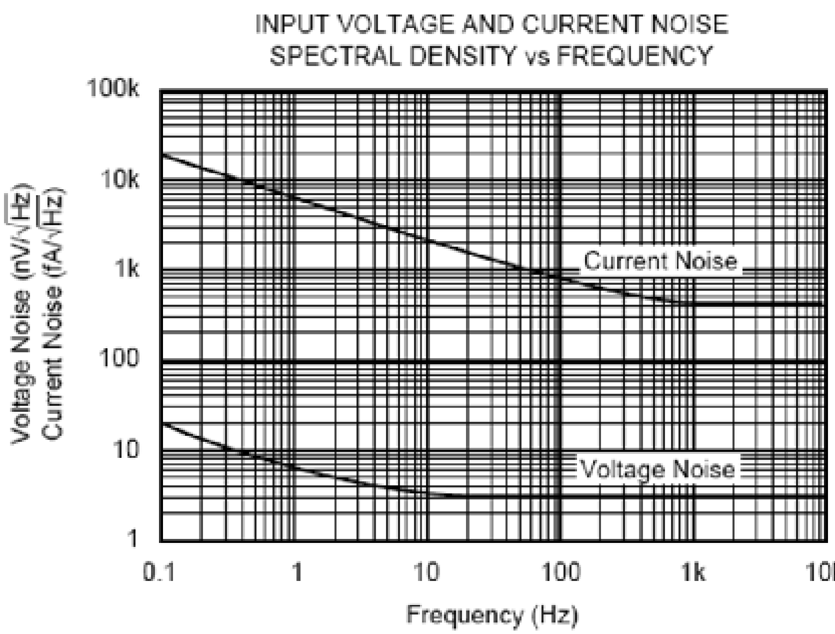
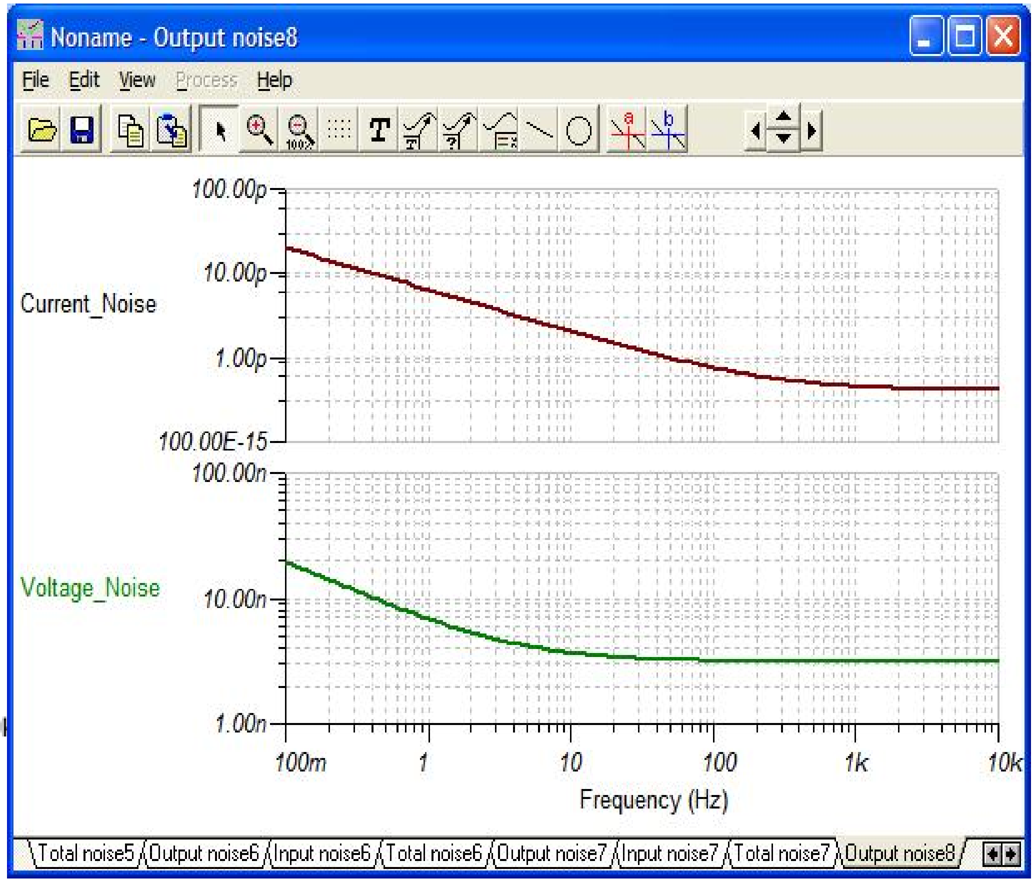
Generate Spectral Noise Plots

"Output Noise" diagram gives the output voltage noise spectral density measured at each volt meter.



OPA227 Matches Tina Model Matches the Data Sheet

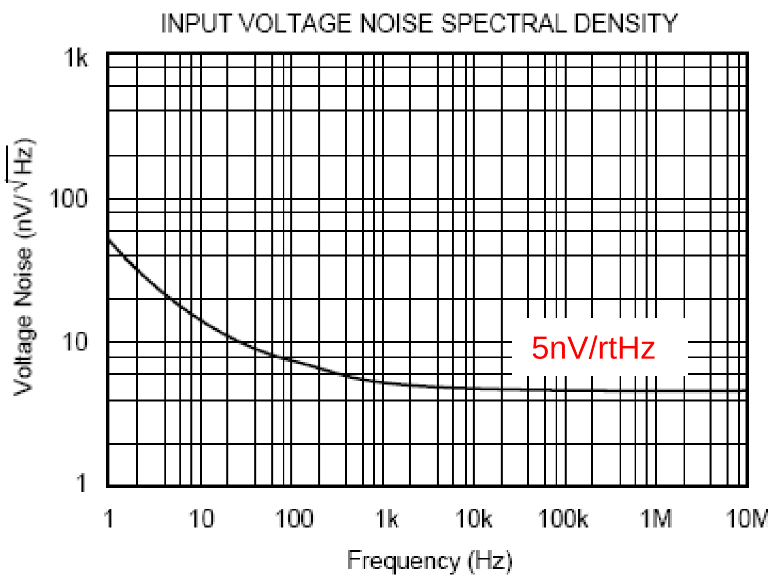


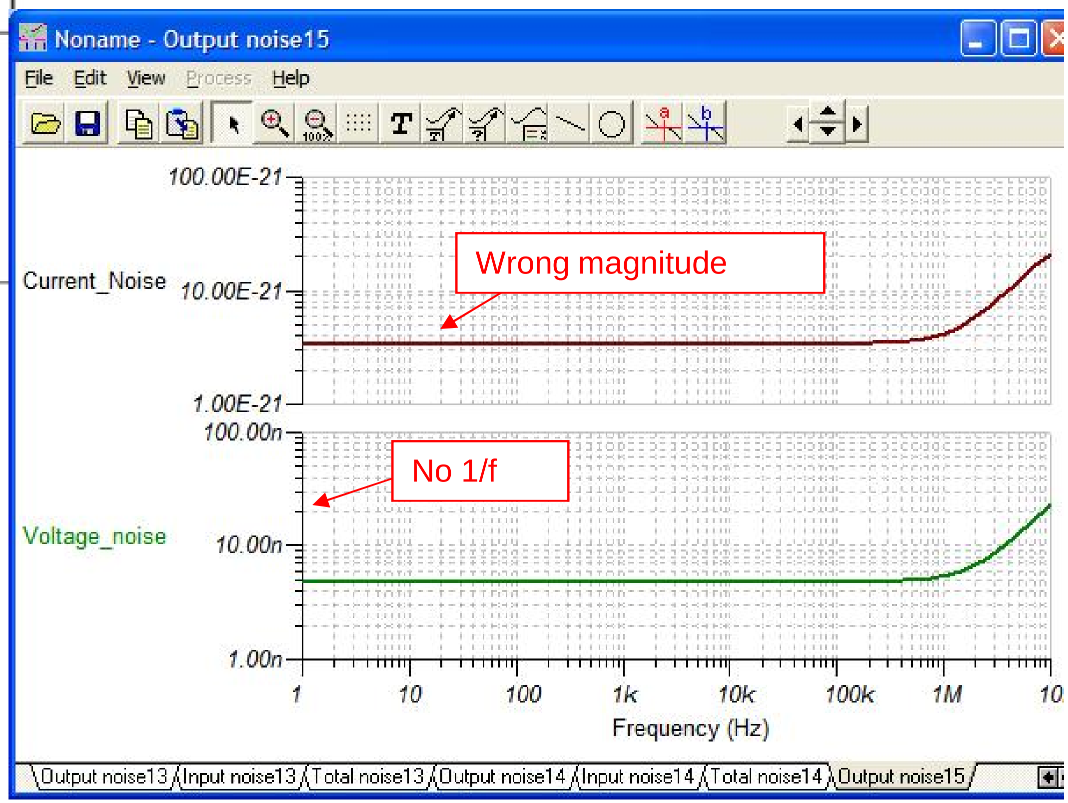


The OPA627 Tina Model Does NOT Match the Data Sheet

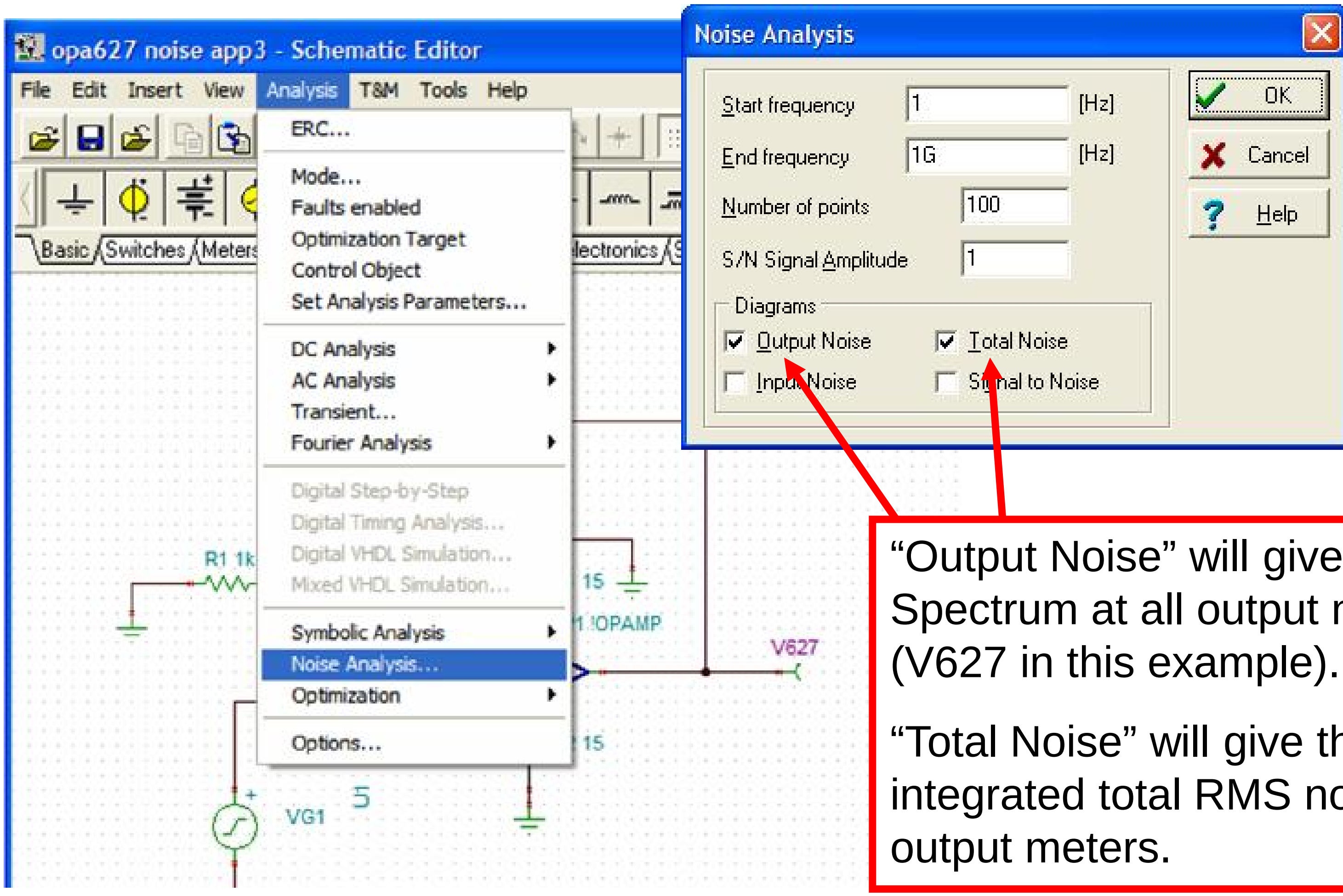
See Appendix for Simple Method to Build your Own Model!

PARAMETER	OPA627BM, BP, SM OPA637BM, BP, SM MIN	TYP	MAX	UNITS
NOISE				
Input Voltage Noise				
Noise Density: f = 10Hz		15	40	nV/√Hz
f = 100Hz		8	20	nV/√Hz
f = 1kHz		5.2	8	nV/√Hz
f = 10kHz		4.5	6	nV/√Hz
Voltage Noise, BW = 0.1Hz to 10Hz		0.6	1.6	µVp-p
Input Bias Current Noise				
Noise Density, f = 100Hz		1.6	2.5	fA/√Hz
Current Noise, BW = 0.1Hz to 10Hz		30	60	fAp-p





Let's Use Tina on the Hand Analysis Circuit



"Output Noise" will give the noise Spectrum at all output meters (V627 in this example).

"Total Noise" will give the integrated total RMS noise at all output meters.

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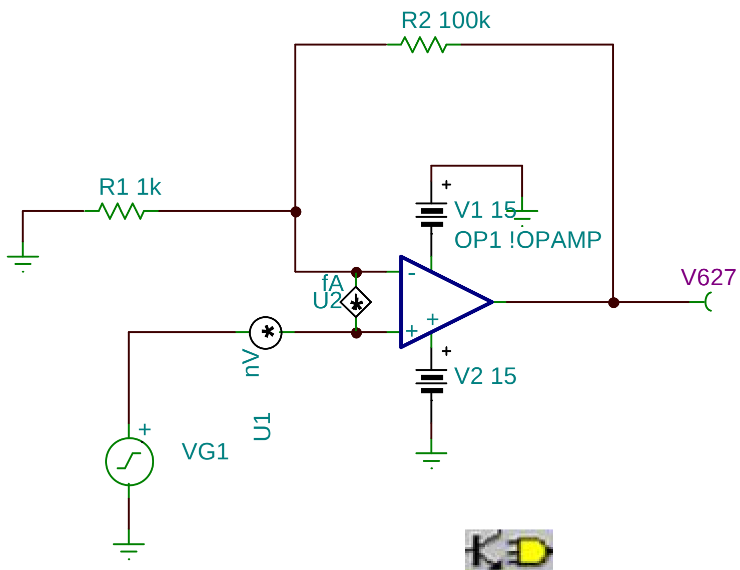
Let's Analyze the Circuit that We did Hand Analysis on

Hand Analysis:

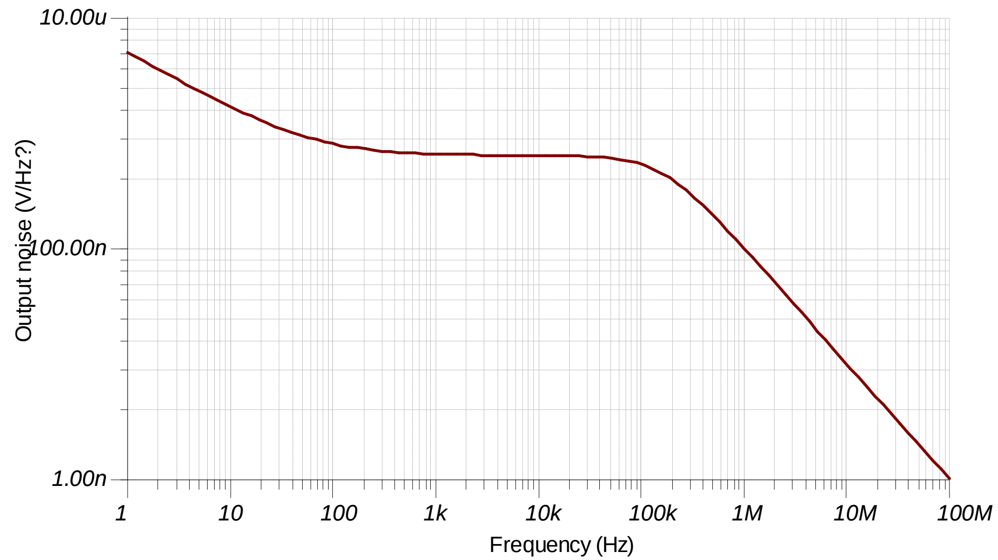
$e_{n\ out}$ = 325uV rms

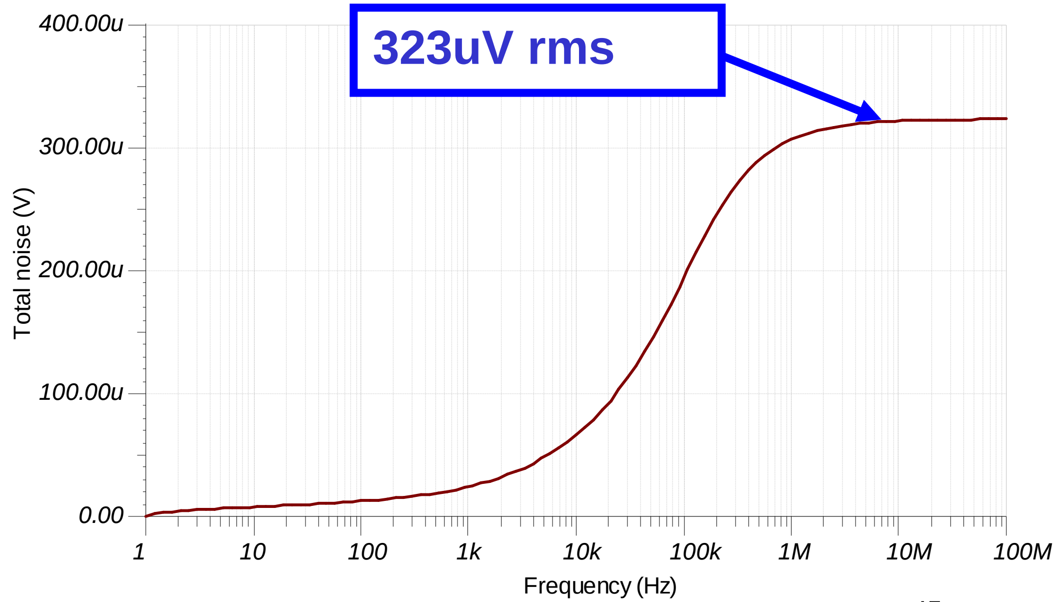
Tina Analysis:

$e_{n\ out}$ = 323uV rms

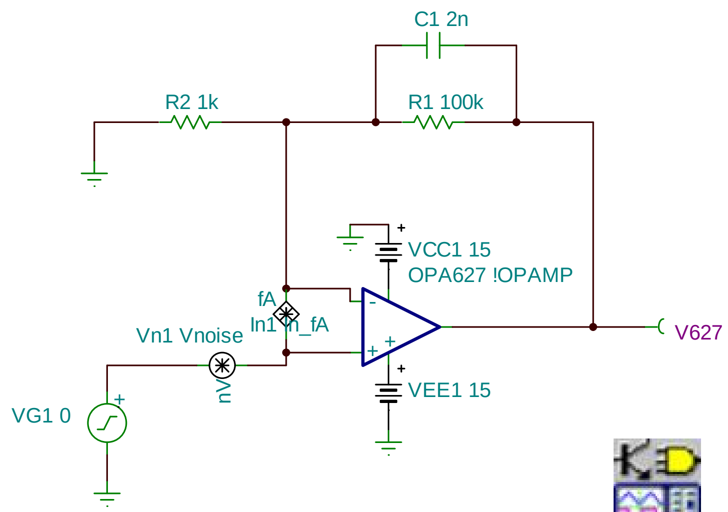


no filter opa627.TSC





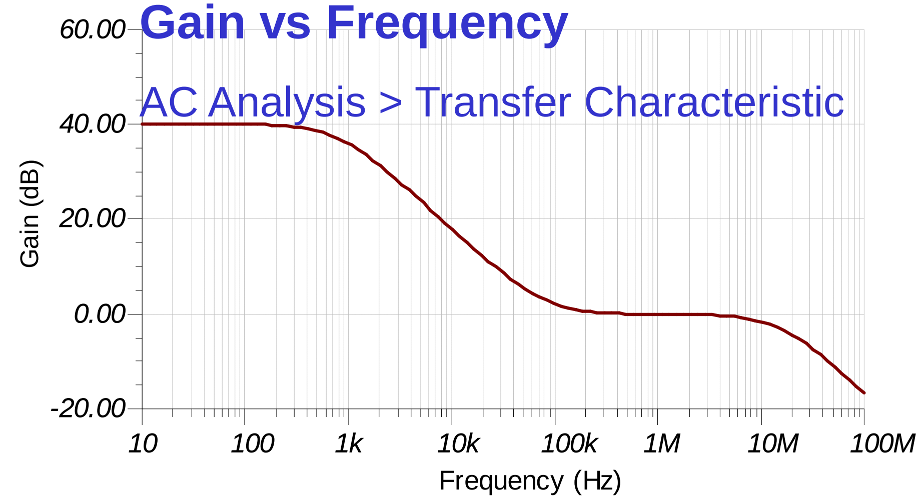
Use Tina to Analyze this Common Topology for Noise Reduction



cf-filter-opa627.TSC

Gain vs Frequency

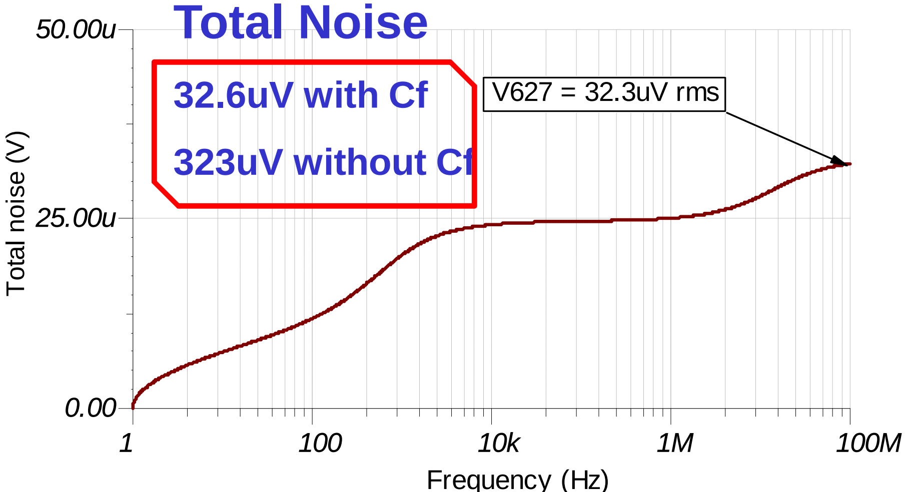
AC Analysis > Transfer Characteristic



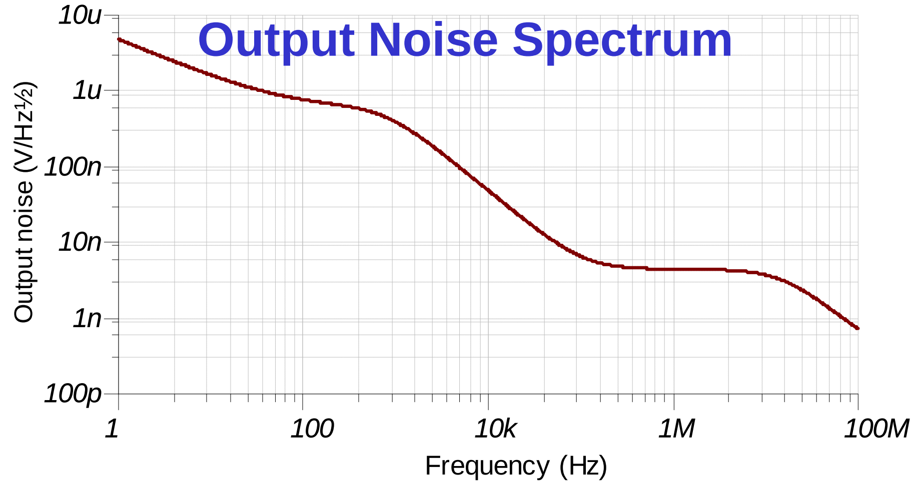
Total Noise

32.6uV with Cf

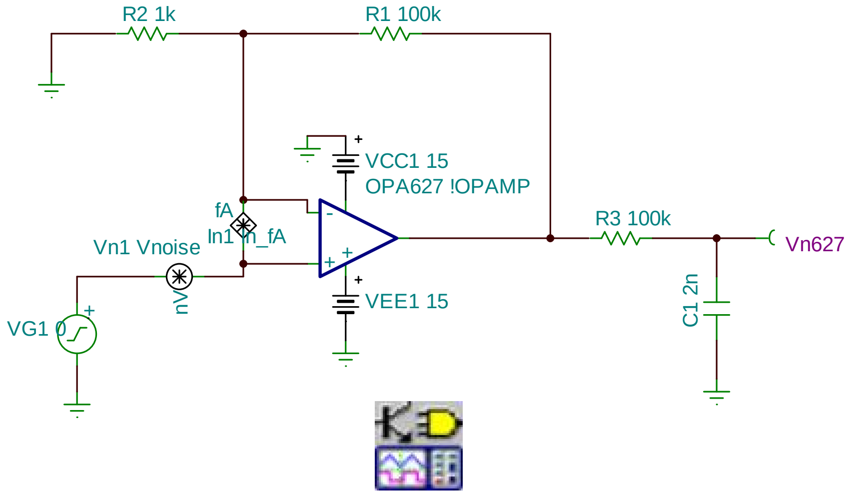
323uV without Cf



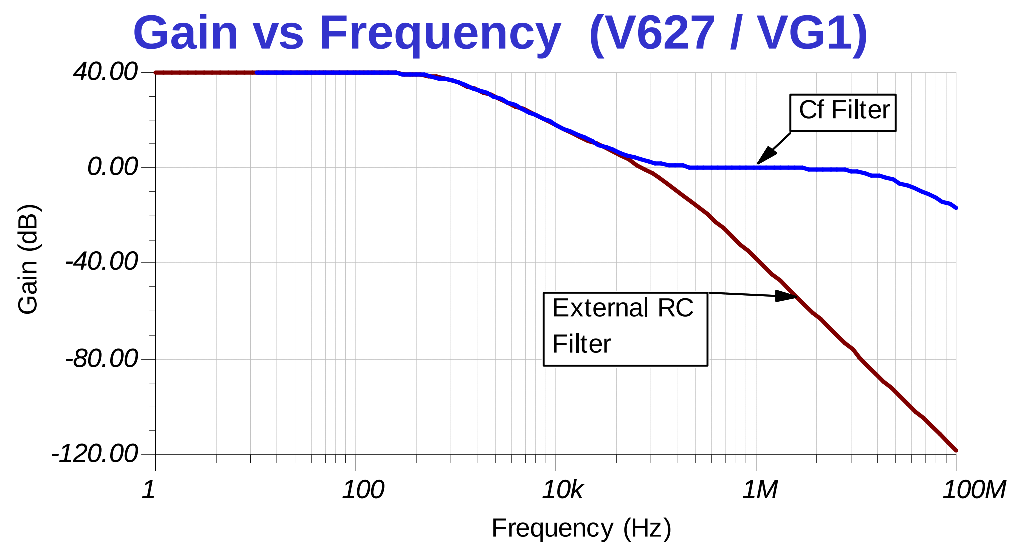
Output Noise Spectrum

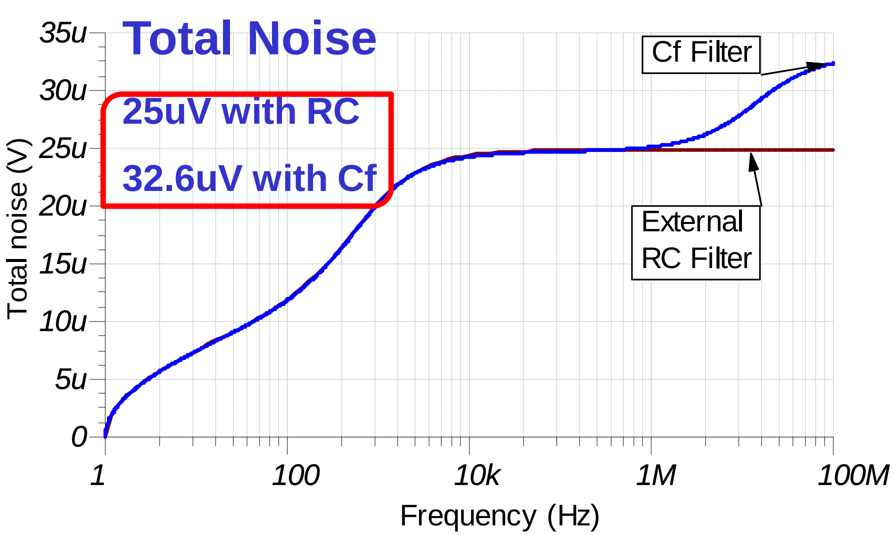


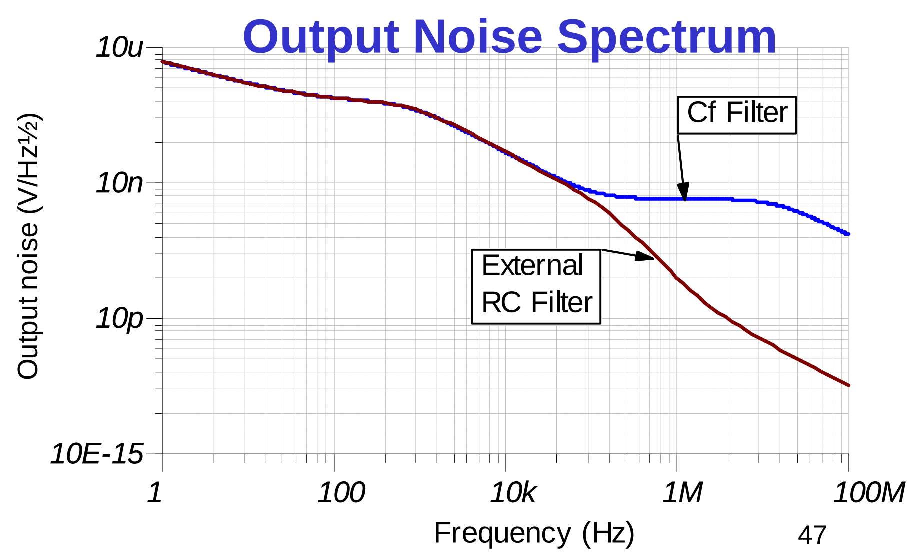
External Filter Improves Noise Performance



external-filter-opa627.TSC







Noise Measurement... The Instruments

Instruments For Noise Measurements

1. Oscilloscope
2. Spectrum Analyzer / Signal Analyzer

Oscilloscope Noise Measurements

- Do NOT use 10x Probes for low noise measurements
- Use direct BNC Connection (10 times better noise floor)
- Use Male BNC Shorting Cap to Measure Noise Floor
- Use BW Limiting if Appropriate
- Use digital scope in dc coupling for 1/f noise measurements (ac coupling has a 60Hz high pass)
- Use AC coupling for broadband measurements if necessary

TDS460A Digitizing Oscilloscope Example

Best Noise Floor = 0.2mV

BW Limit = 20MHz

BNC Shorting Cap

Noise measurements use BNC cables

Noise Floor = 0.8mV

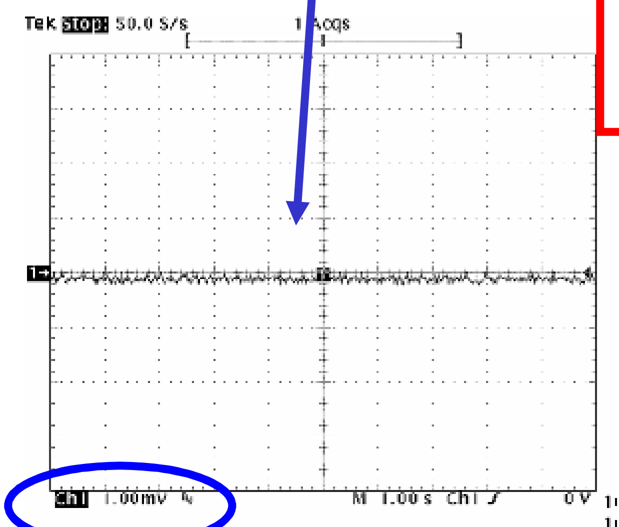
BW FULL = 400MHz

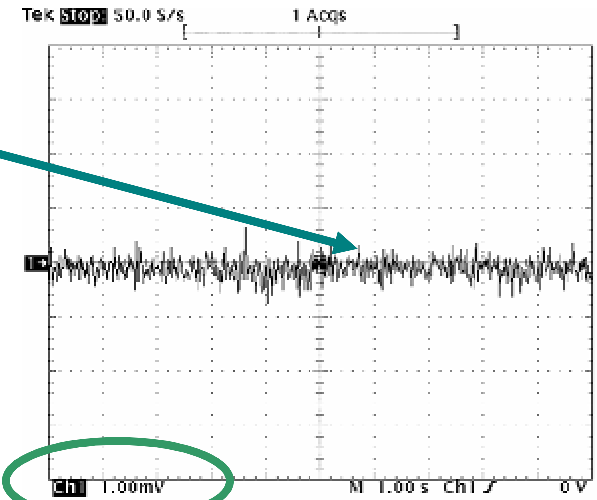
BNC Shorting Cap

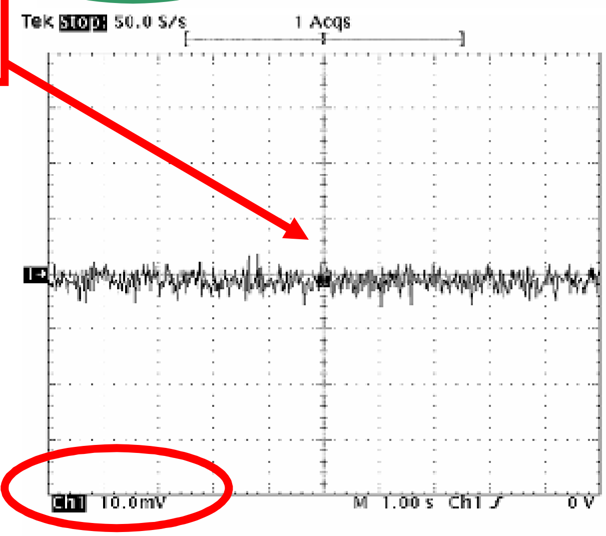
Worst Noise Floor = 8mV

BW FULL = 400MHz

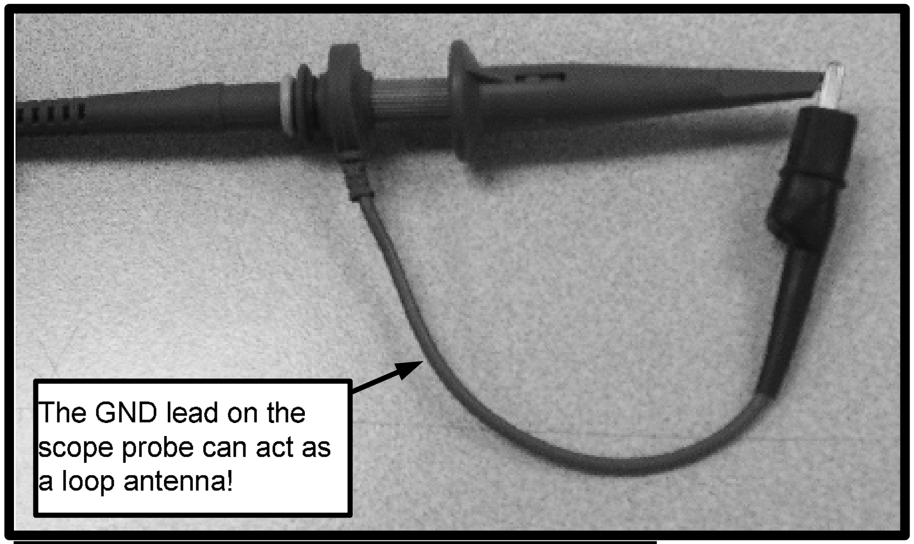
10x Scope Probe

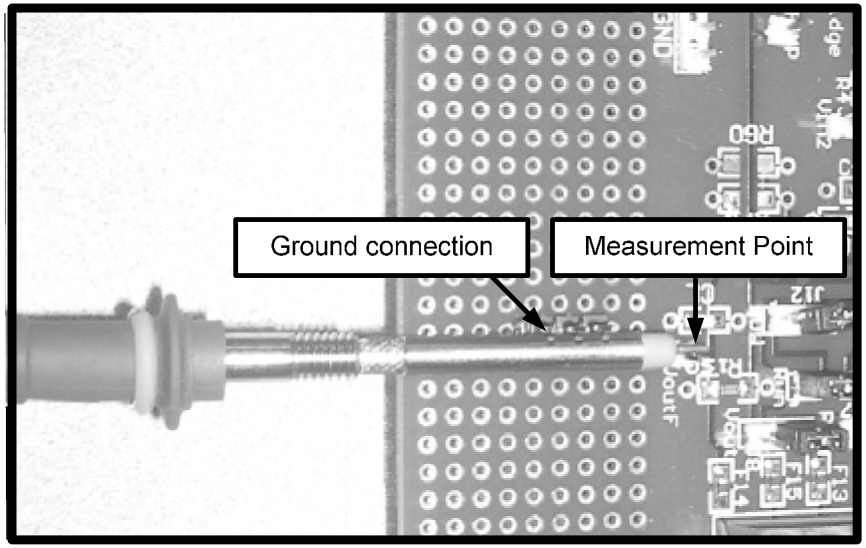


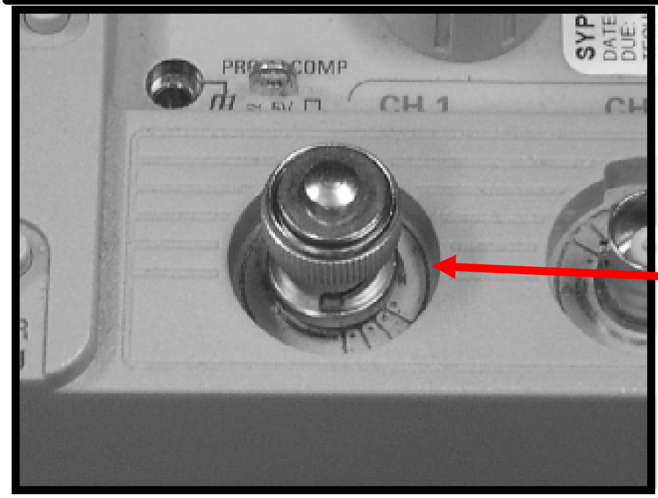




Oscilloscope Noise Measurement



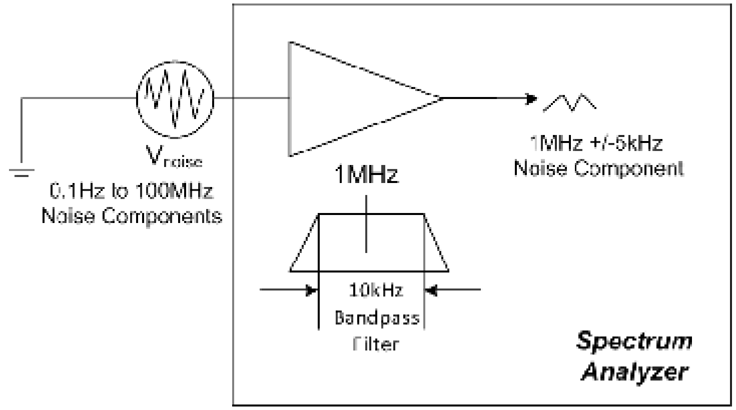


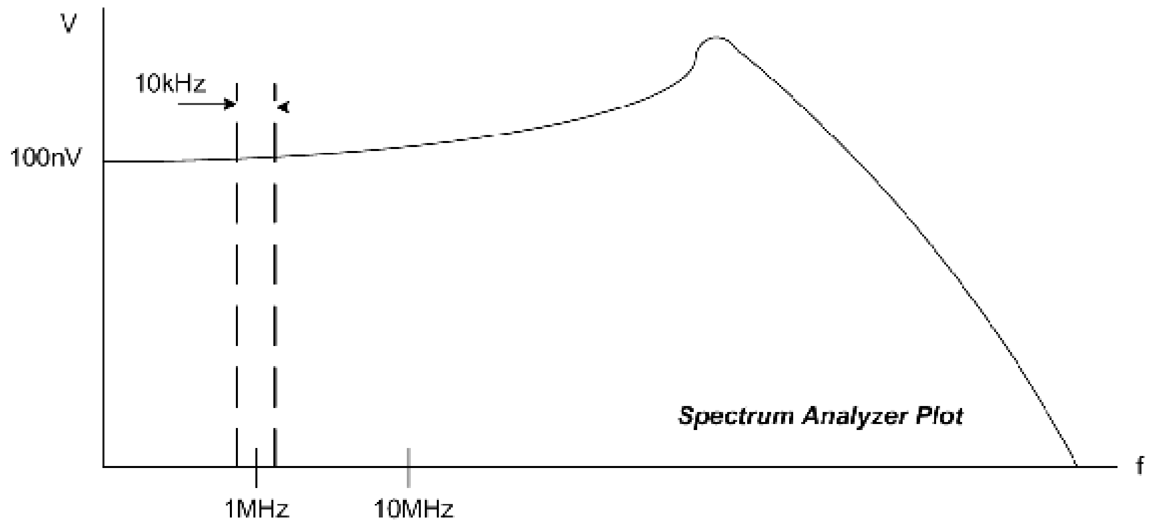


Always measure the noise floor.

BNC Shorting Cap

Spectrum Analyzer -- Convert Result to nV/rt-Hz



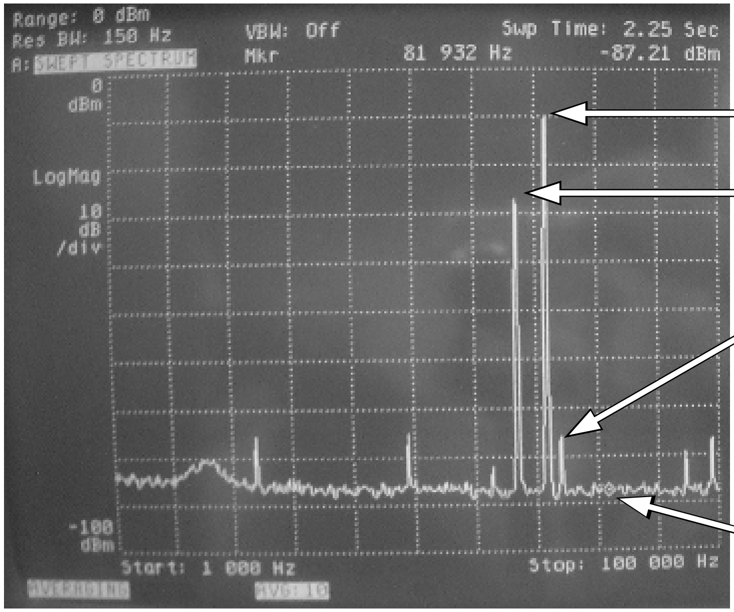


To convert to nV/$\sqrt{Hz}$ plot at 1MHz data point we divide 100nV by $\sqrt{10kHz}$ which gives the Spectral Noise Density (nV/$\sqrt{Hz}$) at 1MHz

TEXAS INSTRUMENTS

Effect of Changing the measurement Bandwidth

Measuring 67kHz and 72kHz

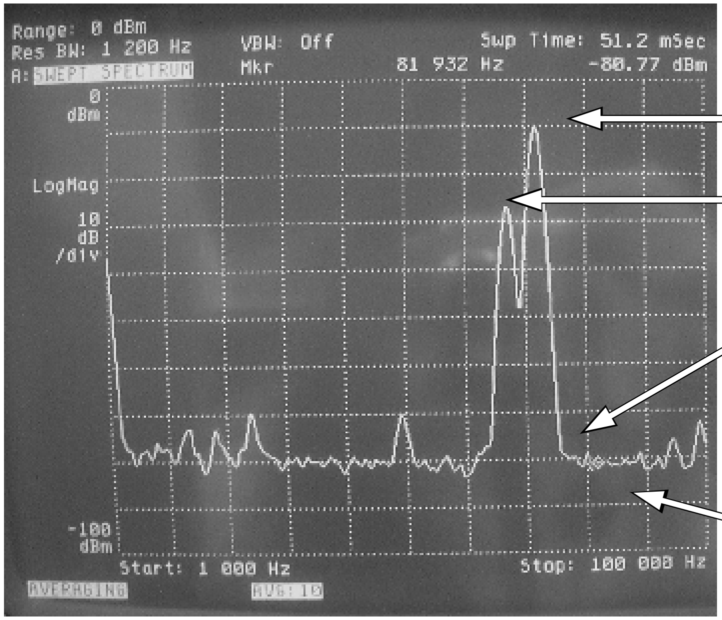


-10dBm @ 72kHz

-26dBm @ 67kHz

-77dBm @ 74kHz

Noise floor -87dBm



-10dBm @ 72kHz

-26dBm @ 67kHz

Lower harmonics obscured

Noise floor -80dBm

BW set to 150Hz

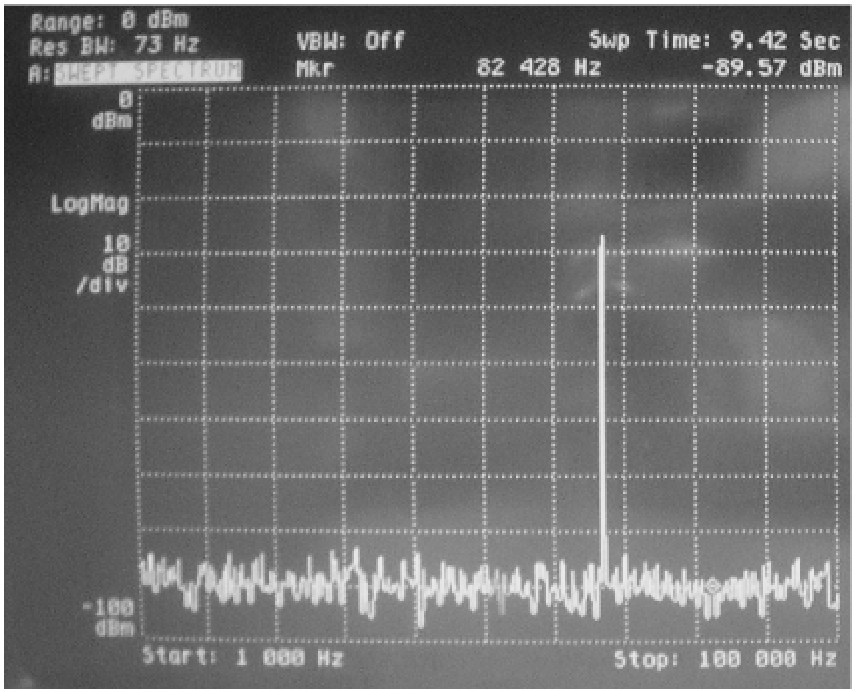
Narrow BW increases resolution & lowers noise floor

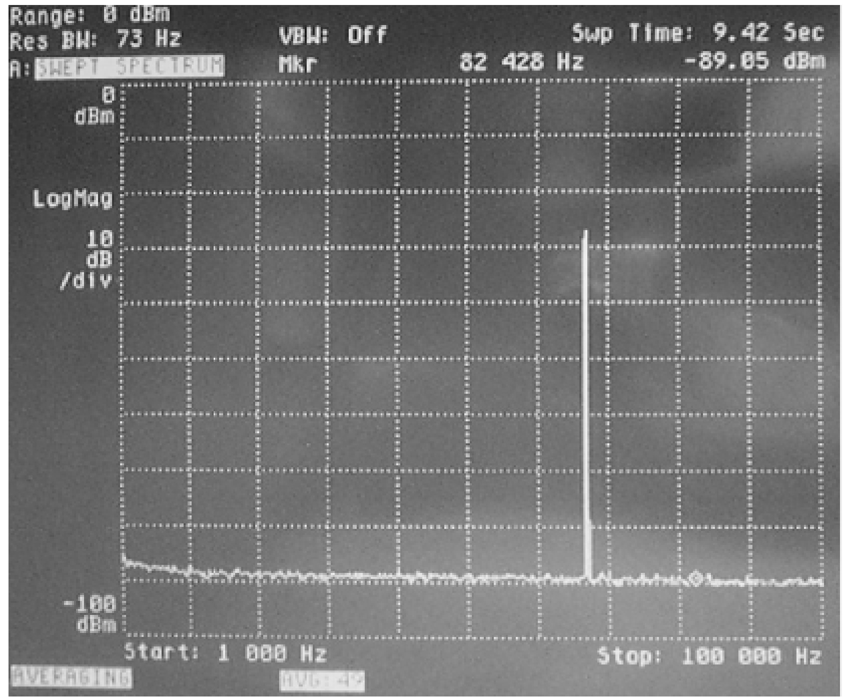
BW set to 1200Hz

Wider BW reduces resolution & raises noise floor

Narrow BW à Longer Sweep Time

Effect of Changing Averaging





No Averaging.

Averaging = 49.

Increase Averaging to Reduce Noise Floor à Increase Measurement Time

dBm to Spectral Density

Convert dBm to nV/rt-Hz

$$V_{spect\_anal} = \sqrt{\left(10^{\frac{NdBm}{10}}\right)\cdot(1mW)\cdot R} \qquad (5.4)$$

$$V_{spect\_den} = \frac{V_{spect\_anal}}{\sqrt{K_n \cdot RBW}} \qquad (5.5)$$

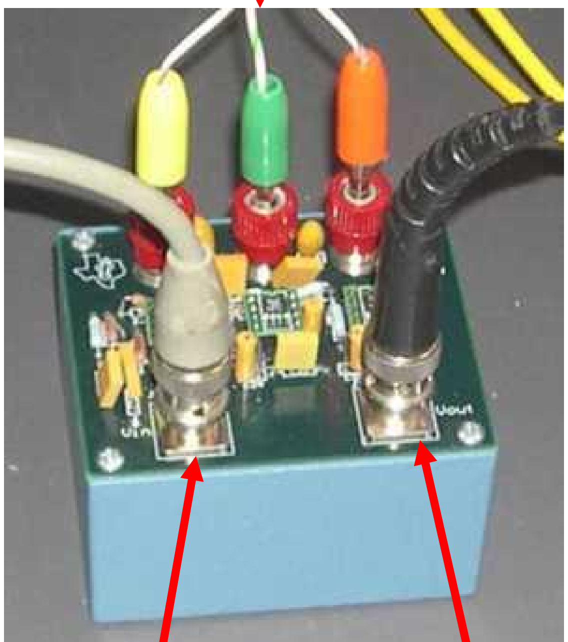
Where
NdBm -- the noise magnitude in dBm from the spectrum analyzer
R -- the reference impedance used for the dBm calculation
V_{spect_anal} -- noise voltage measured by spectrum analyzer per resolution bandwidth
RBW -- resolution bandwidth setting on spectrum analyzer
V_{spect_den} -- spectral density in (nV/rt-Hz)
K_n -- conversion factor that changes the resolution bandwidth to a noise bandwidth

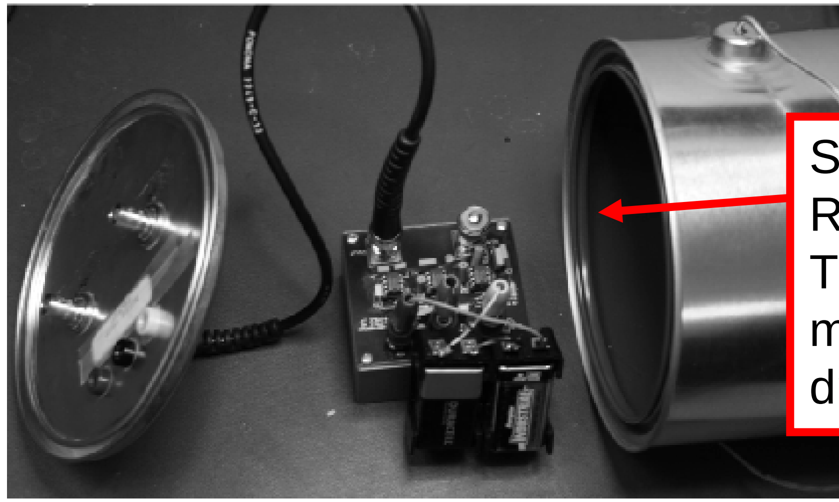
Filter Type	Application	Kn
4-pole sync	Most Spectrum Analyzers Analog	1.128
5-pole sync	Some Spectrum Analyzers Analog	1.111
Typical FFT	FFT-based Spectrum Analyzers	1.056

Noise Measurement... General Precautions

Some General Measurement Precautions

Linear Supply or Battery



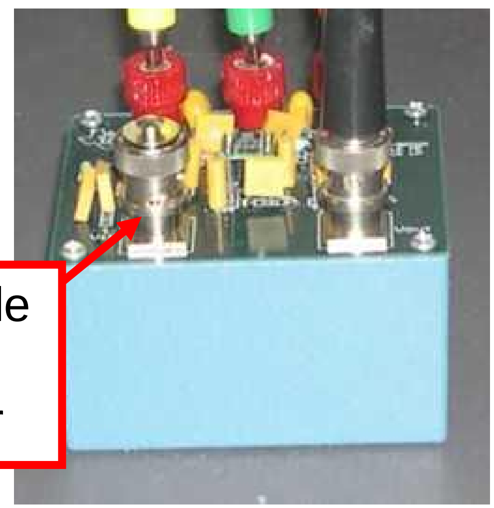


Steel paint can as RFI / EMI Shield. This also minimizes thermal drift

BNC From Circuit Under Test

BNC To Scope

Use BNC Male Shorting cap for noise floor

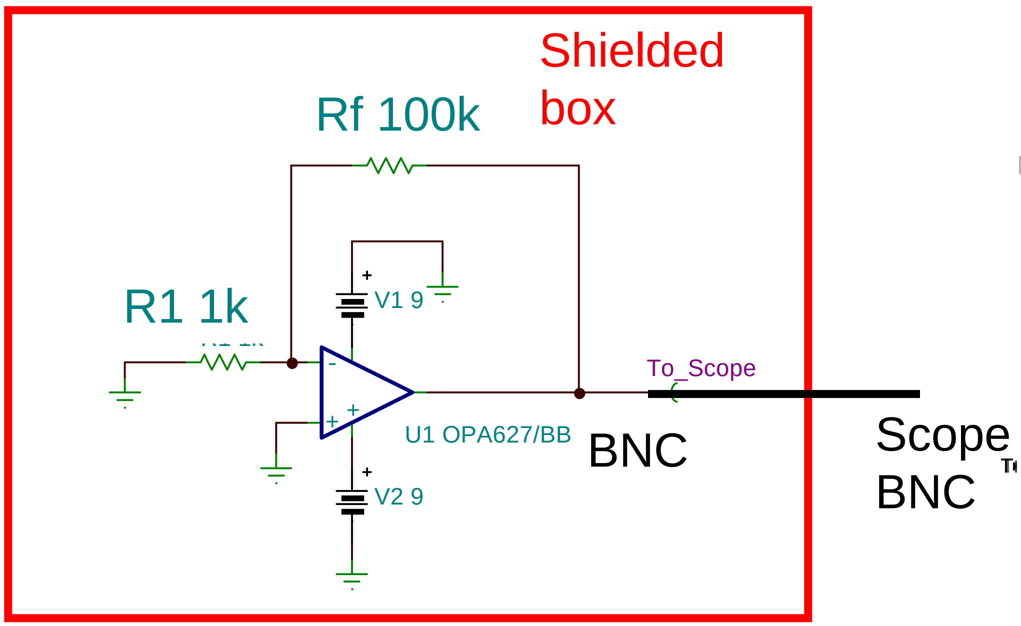


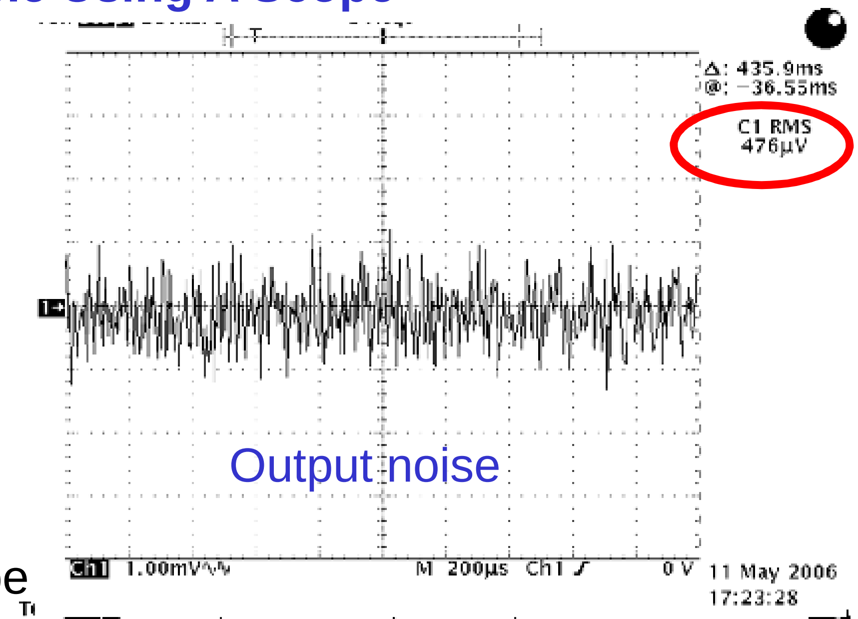
Noise Measurement… Example Measurement

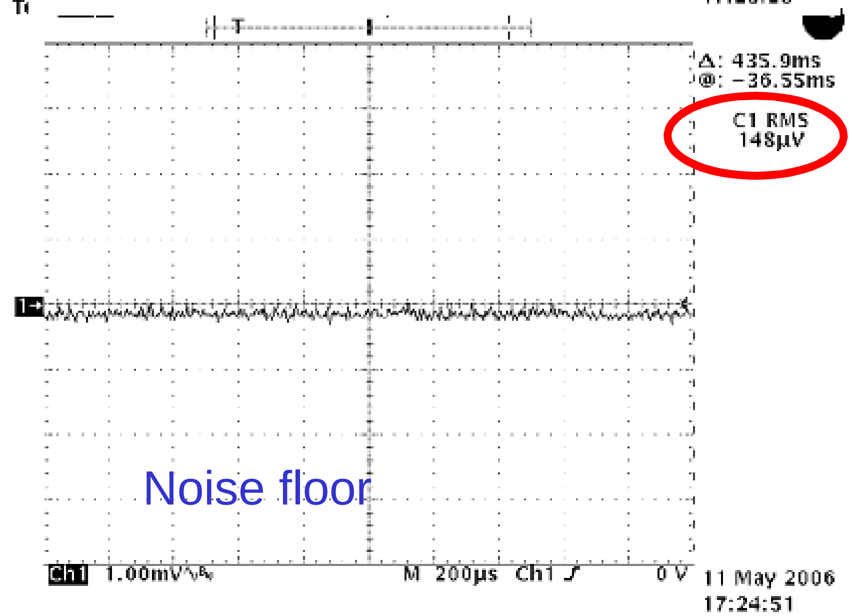
Noise Measurement Circuits

1. Example Circuit Noise Measurement
   - Broadband with scope
2. Voltage Noise Spectral Density Measurement
   - Look at typical spectrum analyzer errors
3. 1/f Noise Measurement (see appendix)

Measure The OPA627 Example Using A Scope







Calculated (Previous Example):

Vn = 325uV rms

Measured: **(noise floor)**

$V_n = \sqrt{(476uV)^2 - (148uV)^2} = 452uV\ rms$

Note: peak to peak reading is roughly 2mVp-p

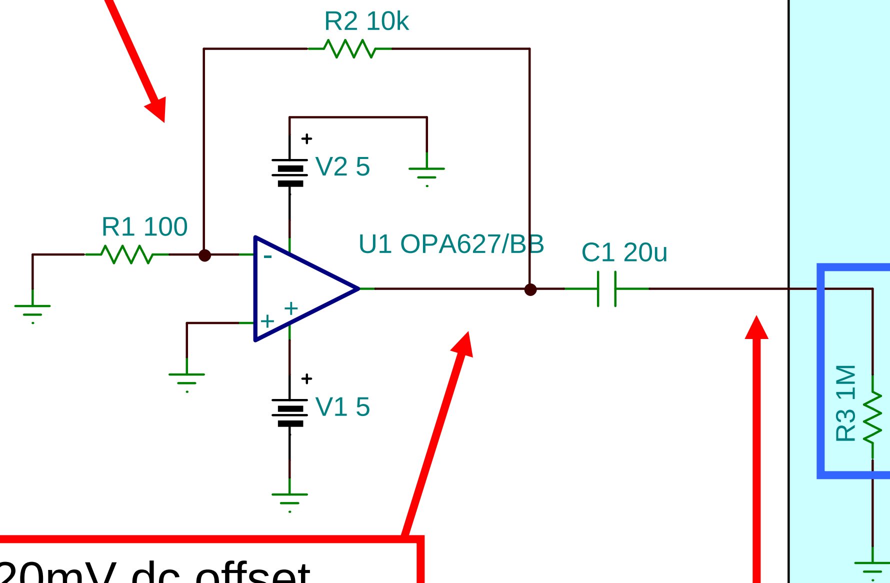
Spectrum Analyzer Measurement of OPA627

(Voltage Noise Spectrum)

Low Req to reduce effect of i_n and thermal noise

Agilent 35770a Dynamic Signal Analyzer

- Has nV/rt-Hz Mode
- Bandwidth 0Hz to 100kHz



1M Input

Impedance

20mV dc offset

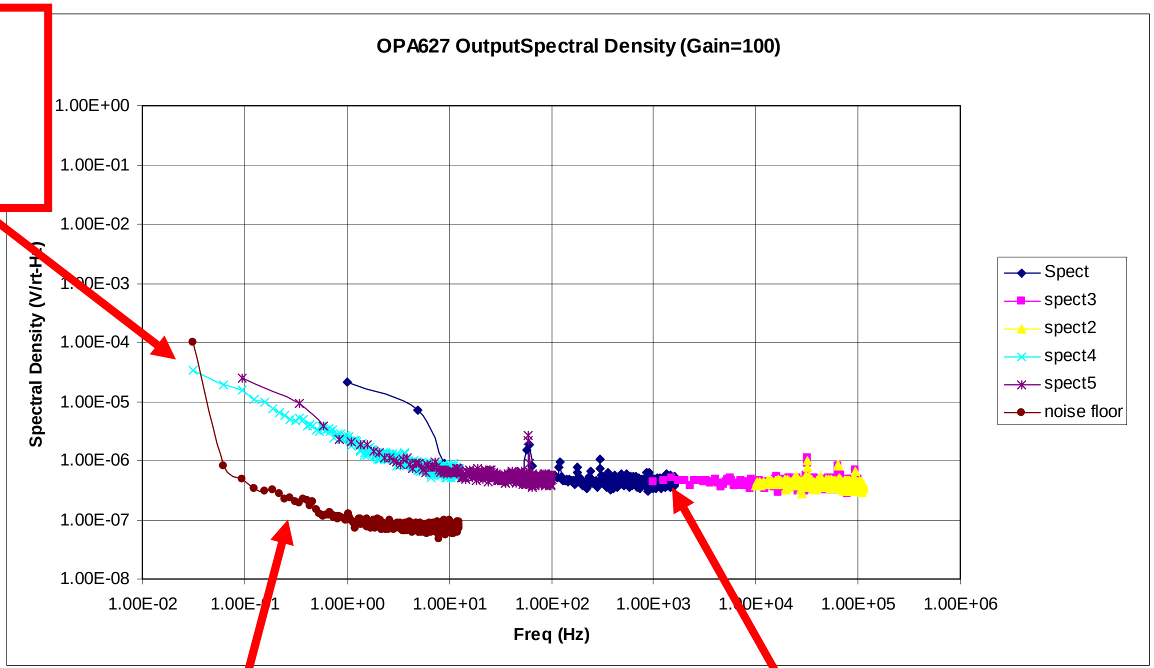
0.2mVp-p noise

(Bad SNR)

0.008Hz HPF removes dc component, but still allows 1/f measurement

Spectrum Analyzer Measurement of OPA627

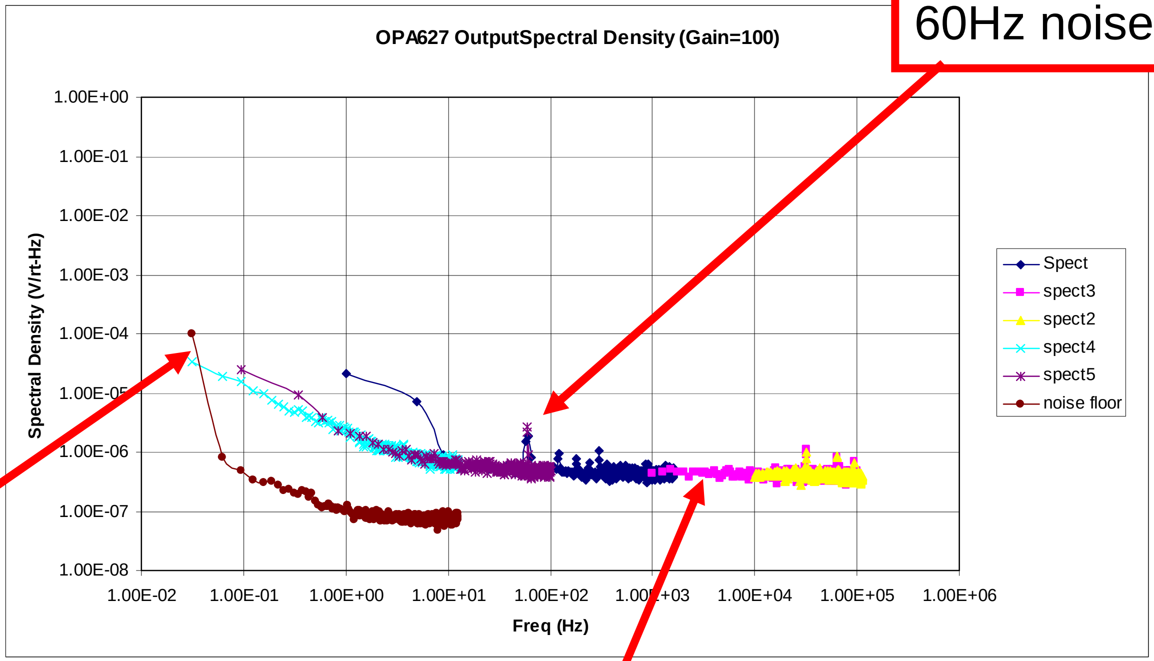
The low frequency run is time consuming. Approximately 12 min.



Noise floor verification

Data was collected over five different frequency ranges

Spectrum Analyzer Measurement of OPA627

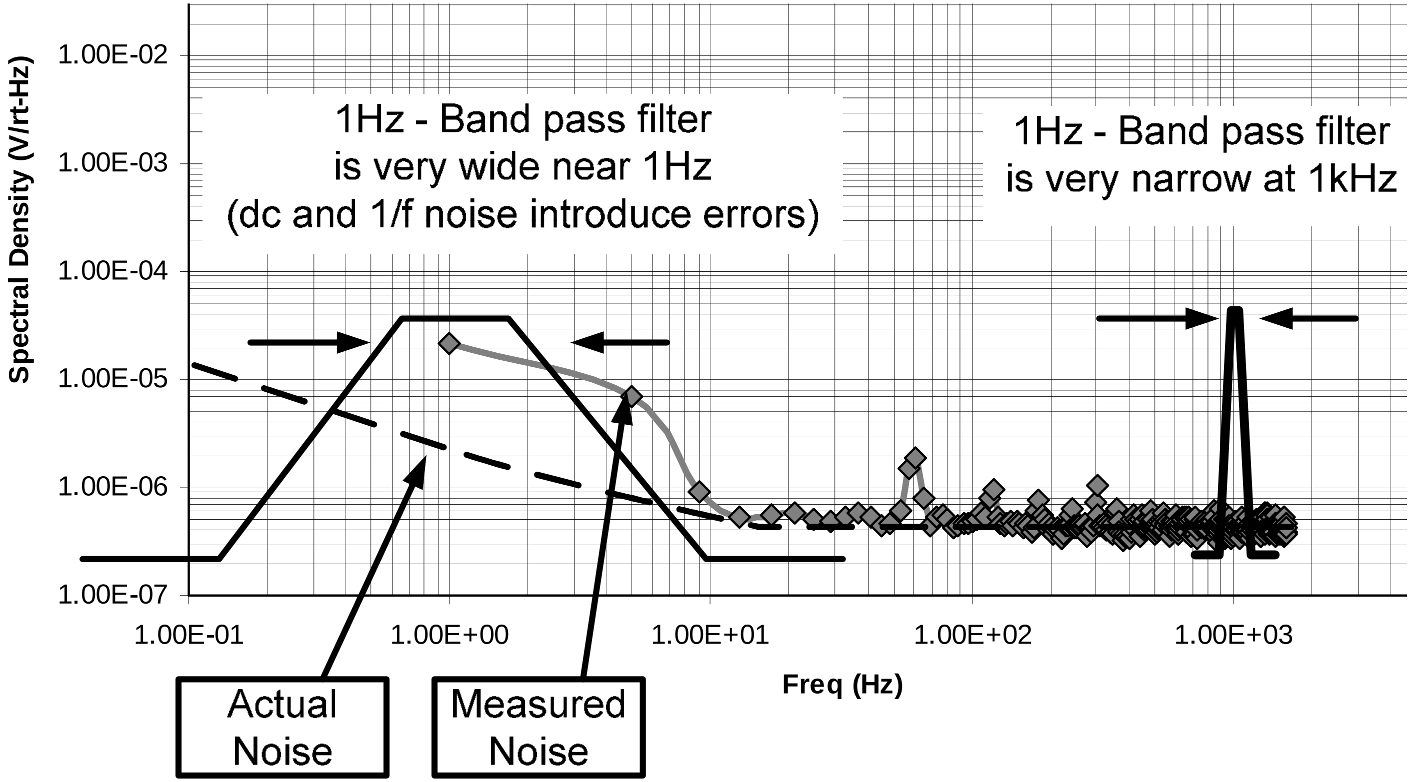


60Hz noise pickup

Low frequency tail on each run must be discarded.

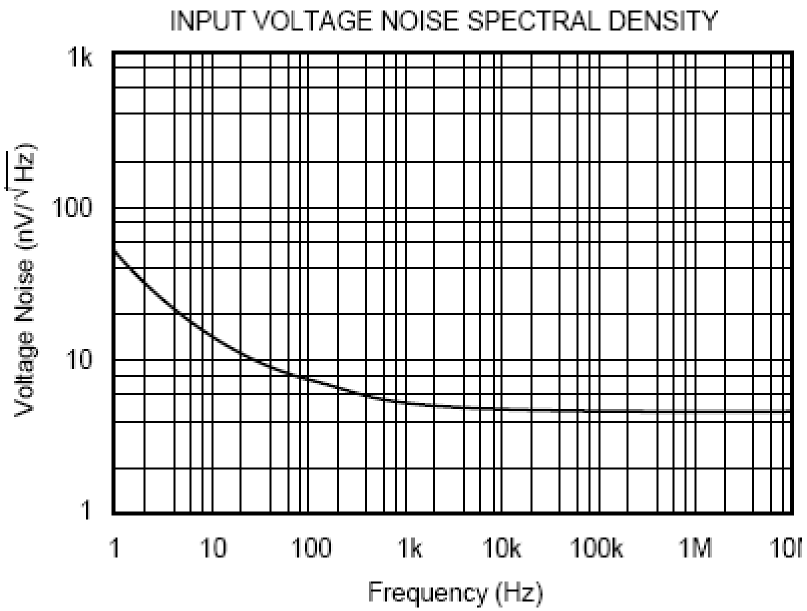
1. Combine to one curve
2. Discard bad information
3. Divide by gain of 100

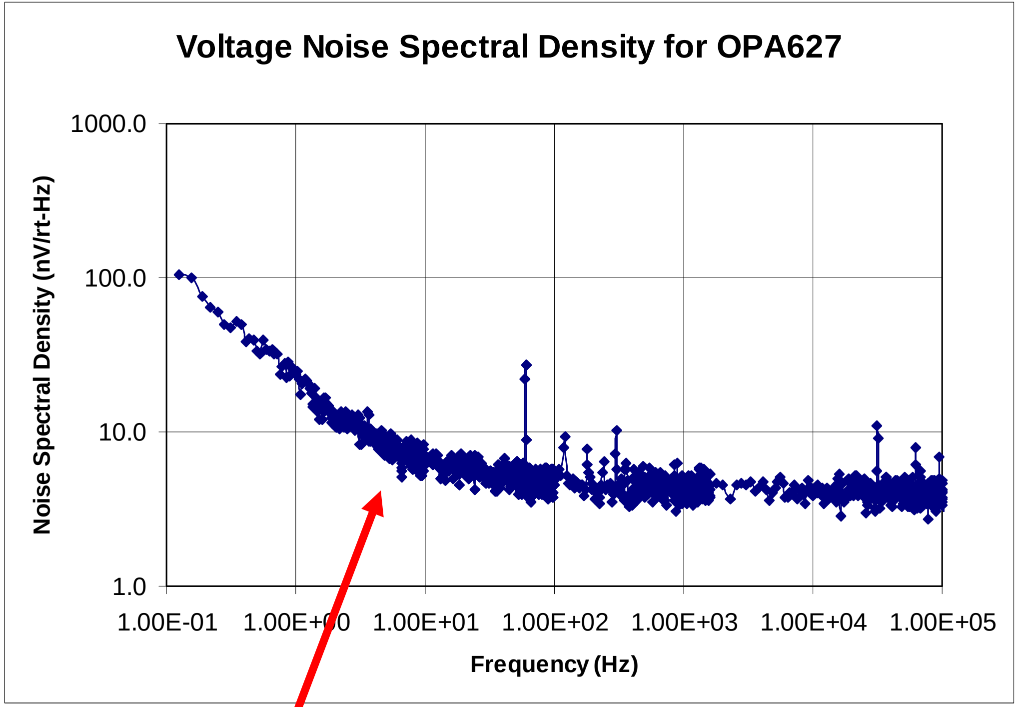
The "Tail Error" is from relatively wide measurement bandwidth at low frequency.



Spectrum Analyzer Measurement of OPA627

OPA627 PDS





Measured 1/f noise corner is better then data sheet.

References

1. Robert V. Hogg, and Elliot A Tanis, Probability and Statistical Inference, 3rd Edition, Macmillan Publishing Co
2. C. D. Motchenbacher, and J. A. Connelly, Low-Noise Electronic System Design, A Wiley-Interscience Publication
3. Henry W. Ott, Noise Reduction Techniques in Electronics Systems, John Wiley and Sons

Acknowledgments:

1. R. Burt, Technique for Computing Noise based on Data Sheet Curves, General Noise Information
2. N. Albaugh, General Noise Information, AFA Deep-Dive Seminar
3. T. Green, General Information
4. B. Trump, General Information
8. B. Sands, Noise Models

Noise Article Series (www.en-genius.net)

http://www.en-genius.net/site/zones/audiovideoZONE/technical_notes/avt_022508

Noise Book: Operational Amplifier Noise by Art Kay

http://www.amazon.com/Operational-Amplifier-Noise-Techniques-Analyzing/dp/0750685255

Thank You

for

Your Interest

in

Noise – Calculation and Measurement

Comments, Questions, Technical Discussions Welcome:

Art Kay 520-746-6072 kay_art@ti.com

Appendix 2

Statistics Summary (Formulas)

Mean defined for a Probability Distribution Function

$$\mu = \int_{-\infty}^{\infty} (x) f(x)\, dx$$ (1) Continuous form

$$\mu = \sum_{x=-\infty}^{\infty} (x) \cdot f(x)$$ (2) Discrete form

Variance defined for a Probability Distribution Function

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$ (3) Continuous form

$$\sigma^2 = \sum_{x=-\infty}^{\infty} (x-\mu)^2 \cdot f(x)$$ (4) Discrete form

Standard deviation defined for a Probability Distribution Function

$$\sigma = \sqrt{\sigma^2} = \sqrt{\int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx}$$ (5) Continuous form

$$\sigma = \sqrt{\sigma^2} = \sqrt{\sum_{x=-\infty}^{\infty} (x-\mu)^2 \cdot f(x)}$$ (6) Discrete form

Root Mean Squared (RMS) defined for a Probability Distribution Function
This is the same as σ if $\mu = 0$

$$RMS = \sqrt{\int_{-\infty}^{\infty} (x)^2 f(x)\, dx}$$ (7) Continuous form

$$RMS = \sqrt{\sum_{x=-\infty}^{\infty} (x)^2 \cdot f(x)}$$ (8) Discrete form

Mean defined for a Discrete Statistical Population

$$\mu = \frac{1}{b-a} \int_a^b g(t)\, dt$$ (9) Continuous form

$$\mu = \frac{1}{n} \sum_{i=1}^{n} x_i$$ (10) Discrete form

Variance defined for a Probability Distribution Function

$$\sigma^2 = \frac{1}{b-a} \int_a^b (g(t)-\mu)^2\, dt$$ (11) Continuous form

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i-\mu)^2$$ (12) Discrete form

Standard deviation defined for a Probability Distribution Function

$$\sigma = \sqrt{\frac{1}{b-a} \int_a^b (g(t)-\mu)^2\, dt}$$ (13) Continuous form

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i-\mu)^2}$$ (14) Discrete form

Root Mean Squared (RMS) defined for a Probability Distribution Function
This is the same as σ if $\mu = 0$

$$RMS = \sqrt{\frac{1}{b-a} \left(\int_a^b g(t)^2\, dt \right)}$$ (15) Continuous form

$$RMS = \sqrt{\frac{1}{n} \sum_{i=1}^{n} x_i^2}$$ (16) Discrete form

Statistics Summary

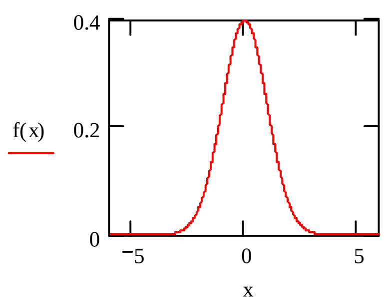
(PDF vs Discrete Population)

Example: Statistics on the Probability Distribution Function

$\sigma := 1 \qquad \mu := 0$

$$f(x) := \frac{1}{\sigma\cdot(2\pi)^{.5}} \cdot e^{\left[\frac{-(x-\mu)^2}{2\sigma^2}\right]}$$

Probability Distribution Function for Normal Curve



$\mu := \int_{-\infty}^{\infty} (x) f(x)\, dx \qquad \mu = 0$ Mean

$\text{var} := \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx \qquad \text{var} = 1$ Variance

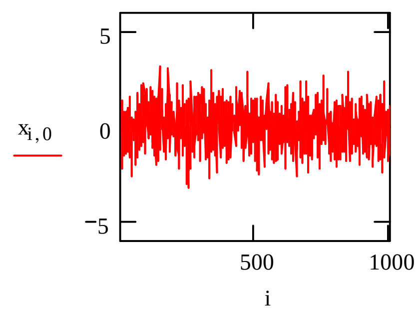
$\sigma := \sqrt{\text{var}} \qquad \sigma = 1$ Standard Deviation

$\text{RMS} := \sqrt{\int_{-\infty}^{\infty} (x)^2 f(x)\, dx} \qquad \text{RMS} = 1$ Root Mean Squared

Example: Statistics on the Discrete Statistical Population

$x := \text{rnorm}(1001, 0, 1)$

$i := 1 .. 1000$



$\mu := \frac{1}{1000} \cdot \sum_{i=1}^{1000} x_{i,0} \qquad \mu = -4.181 \times 10^{-3}$ Mean

$\text{var} := \frac{1}{1000} \sum_{i=1}^{1000} \left(x_{i,0} - \mu\right)^2 \qquad \text{var} = 1.021$ Variance

$\sigma := \sqrt{\text{var}} \qquad \sigma = 1.01$ Standard Deviation

$\text{RMS} := \sqrt{\frac{1}{1000} \sum_{i=1}^{1000} \left(x_{i,0}\right)^2} \qquad \text{RMS} = 1.01$ Root Mean Squared

Statistics Summary
(RMS and STDEV)

Example where $RMS \neq STDEV$

$g(t) := \sin(t) + 0.3$

$\mu := \frac{1}{2\pi - 0} \int_0^{2\pi} g(t)\,dt \qquad \mu = 0.3$

Variance for a Discrete Statistical Population

$var := \frac{1}{2\pi - 0} \int_0^{2\pi} (g(t) - \mu)^2\,dt \qquad var = 0.5$

Standard deviation for a Discrete Statistical Population

$\sigma := \sqrt{var} \qquad \sigma = 0.707$

Root Mean Squared (RMS) for a Discrete Statistical Population
This is the same as σ if $\mu = 0$

$RMS := \sqrt{\frac{1}{2\pi - 0}\left(\int_0^{2\pi} g(t)^2\,dt\right)} \qquad RMS = 0.768$

$RMS = \sqrt{\sigma^2 + \mu^2}$ So $\sigma = \sqrt{RMS^2 - \mu^2}$

$\sigma := \sqrt{RMS^2 - \mu^2} \qquad \sigma = 0.707$

Example where RMS = STDEV

$g(t) := \sin(t)$

$\mu := \frac{1}{2\pi - 0} \int_0^{2\pi} g(t)\,dt \qquad \mu = 0$

Variance defined for a Probability Distribution Function

$var := \frac{1}{2\pi - 0} \int_0^{2\pi} (g(t) - \mu)^2\,dt \qquad var = 0.5$

Standard deviation defined for a Probability Distribution Function

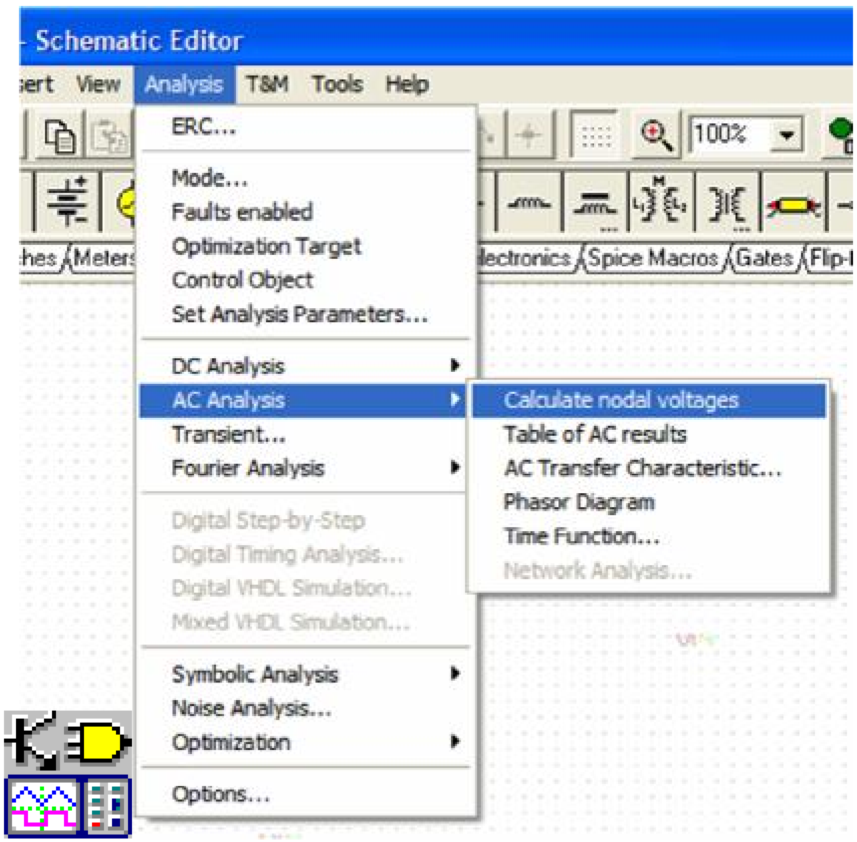
$\sigma := \sqrt{var} \qquad \sigma = 0.707$

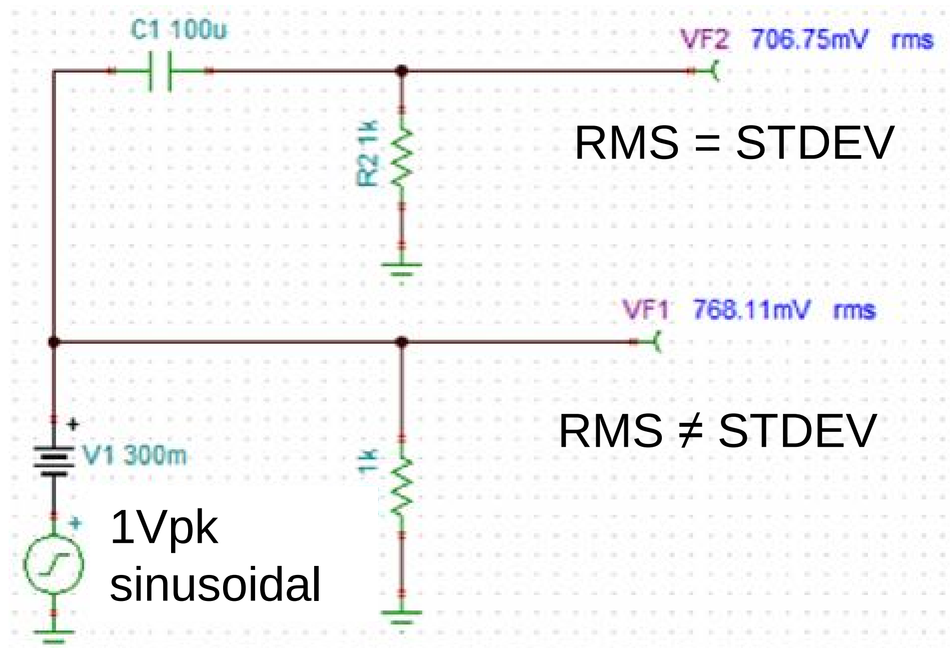
Root Mean Squared (RMS) defined for a Probability Distribution Function
This is the same as σ if $\mu = 0$

$RMS := \sqrt{\frac{1}{2\pi - 0}\left(\int_0^{2\pi} g(t)^2\,dt\right)} \qquad RMS = 0.707$

RMS vs STDEV

Stdev = RMS when the Mean is zero (No DC component).   Tina gives the true RMS result in AC calculations. See mathematical proof in appendix.





Useful Numerical Methods

Method for generating a time domain approximation of 1/f noise

```
For i = 1 To 32768
        white = Rnd(1) - 0.5
        buf0 = 0.997 * old_buf0 + 0.029591 * white
        buf1 = 0.985 * old_buf1 + 0.032534 * white
        buf2 = 0.95 * old_buf2 + 0.048056 * white
        buf3 = 0.85 * old_buf3 + 0.090579 * white
        buf4 = 0.62 * old_buf4 + 0.10899 * white
        buf5 = 0.25 * old_buf5 + 0.255784 * white
        pink = buf0 + buf1 + buf2 + buf3 + buf4 + buf5
Next i
```

Useful Numerical Methods

Method for generating average, standard deviation, and RMS for a discrete population (sampled data)

```
'N is the Number of samples, and f(i) is an array of measured data
Average=0 'Initialize the variables
Stdev=0
RMS=0

For i = 1 To N
   Average=Average + f(i)
Next I
Average = Average/N

For i = 1 To N
   stdev=(f(i) – Average)^2
Next I
Stdev = Stdev/N

For i = 1 To N
   RMS=(f(i))^2
Next I
RMS = RMS/N
```

Brick Wall Factor Calculation for first order filter.

$$e_{rms}^{2} = \int_{f_1}^{f_2} e_n^{2} \cdot (|G|)^2 \, df$$

where
erms -- total rms noise from f1 to f2 in Vrms
e_n -- magnatitude of noise spectral density at f1 in V/ $\sqrt{Hz}$
G -- gain function for a single pole filter

$$G = \frac{1}{1 + \frac{j\omega}{\omega_p}} \qquad |G| = \frac{1}{\sqrt{1^2 + \frac{f^2}{f_p^{\,2}}}} \qquad (|G|)^2 = \frac{1}{1 + \frac{f^2}{f_p^{\,2}}}$$

$$e_{rms}^{2} = \int_{f_1}^{f_2} e_n^{2} \cdot \frac{1}{1 + \frac{f^2}{f_p^{\,2}}} \, df = \int_{f_1}^{f_2} e_n^{2} \cdot \frac{f_p^{\,2}}{f_p^{\,2} + f^2} \, df$$

$$e_{rms}^{2} = e_n^{2} \cdot f_p \cdot \operatorname{atan}\left(\frac{f_2}{f_p}\right) - e_n^{2} \cdot f_p \cdot \operatorname{atan}\left(\frac{f_1}{f_p}\right)$$

Let $f_1 = 0$, $f_2 = \infty$

$$e_{rms}^{2} = e_n^{2} \cdot f_p \cdot \operatorname{atan}(\infty) - e_n^{2} \cdot f_p \cdot \operatorname{atan}\left(\frac{f_1}{f_p}\right) = e_n^{2} \cdot f_p \cdot \frac{\pi}{2}$$

$$e_{rms}^{2} = e_n^{2} \cdot f_p \cdot \frac{\pi}{2}$$

$$e_{rms} = \sqrt{e_n^{2} \cdot f_p \cdot \frac{\pi}{2}}$$

1/f Noise Derivation

$$e_n = \frac{e_{normal}}{f^{0.5}} \qquad e_n^{\,2} = \frac{e_{normal}^{\,2}}{\left(f^{0.5}\right)^2} = \frac{e_{normal}^{\,2}}{f}$$

Units for
$e_n = V/\sqrt{Hz}$
$e_{normal} = V$
$f = Hz$

$$e_{rms}^{\,2} = \int_a^b \frac{e_{normal}^{\,2}}{f}\,df = e_{normal}^{\,2}\cdot\ln(f)\Big|_a^b$$

erms units of V rms

$$e_{rms}^{\,2} = e_{normal}^{\,2}\cdot\ln(b) - e_{normal}^{\,2}\cdot\ln(a) = e_{normal}^{\,2}\cdot\ln\left(\frac{b}{a}\right)$$

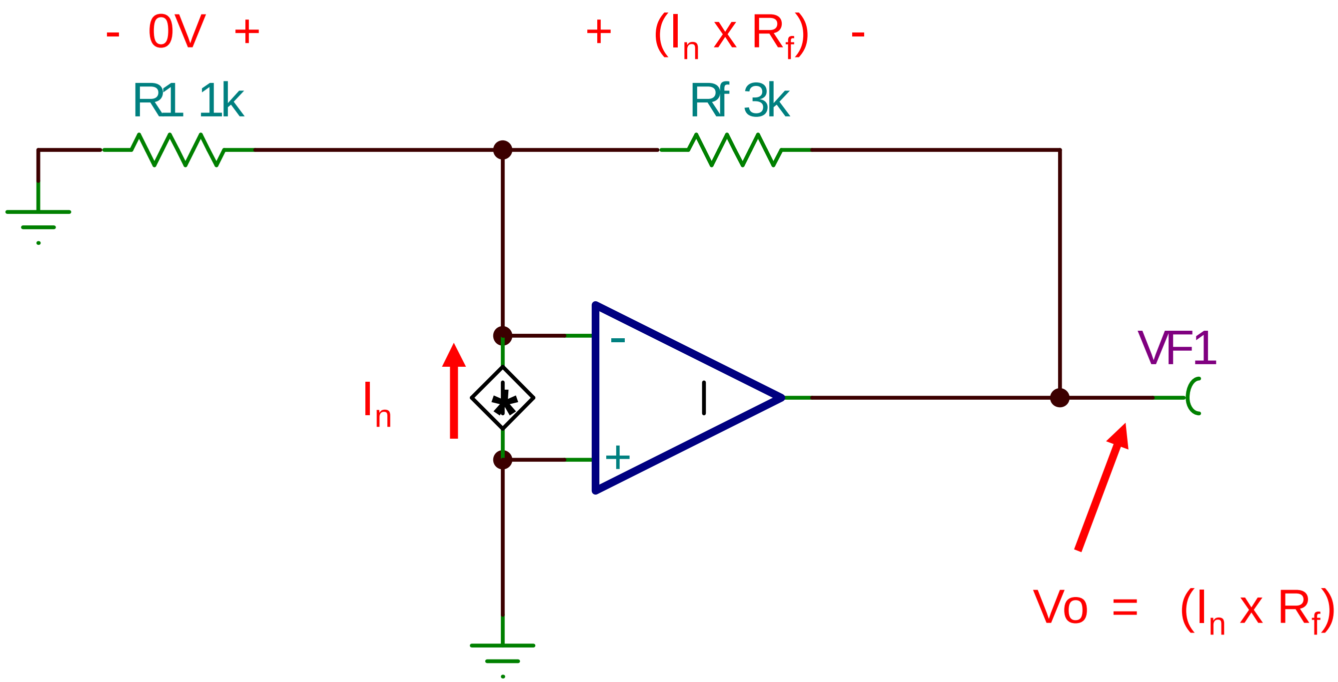
$$e_{rms}^{\,2} = e_{normal}^{\,2}\cdot\ln\left(\frac{b}{a}\right)$$

$$e_{rms} = e_{normal}\cdot\sqrt{\ln\left(\frac{b}{a}\right)}$$

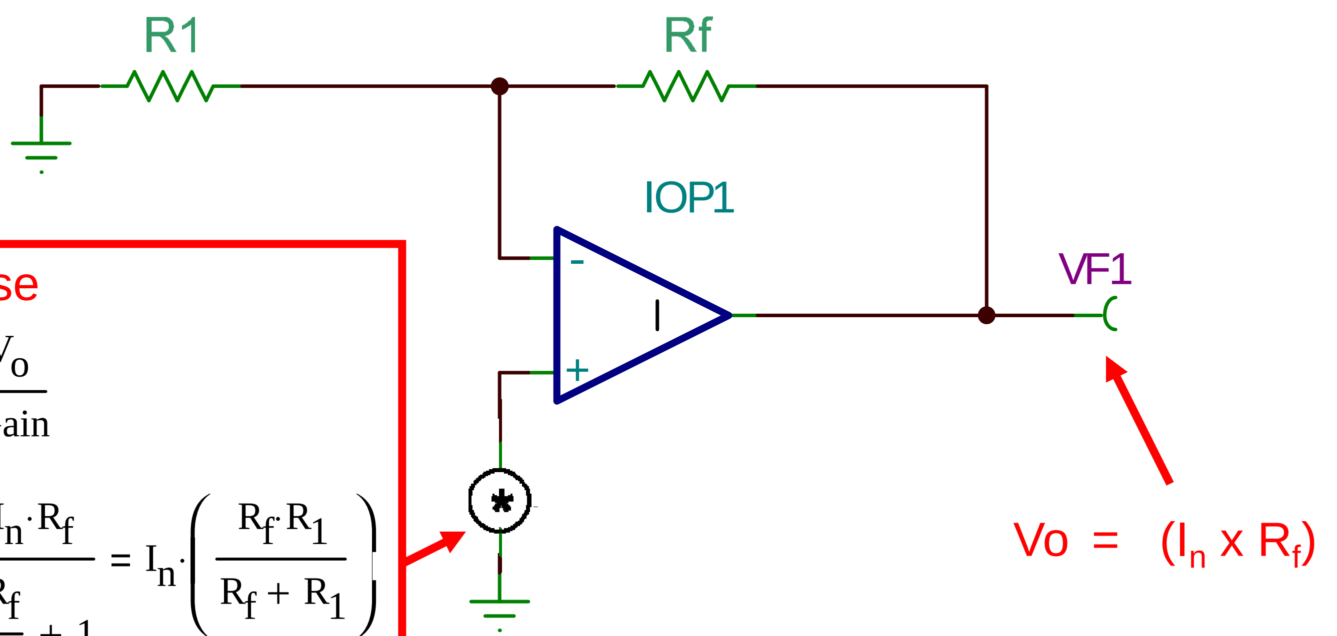
$\sqrt{\ln\left(\frac{b}{a}\right)}$ has no units

e_{normal} has units of V

Current Noise



Current Noise



Equ. Input Noise

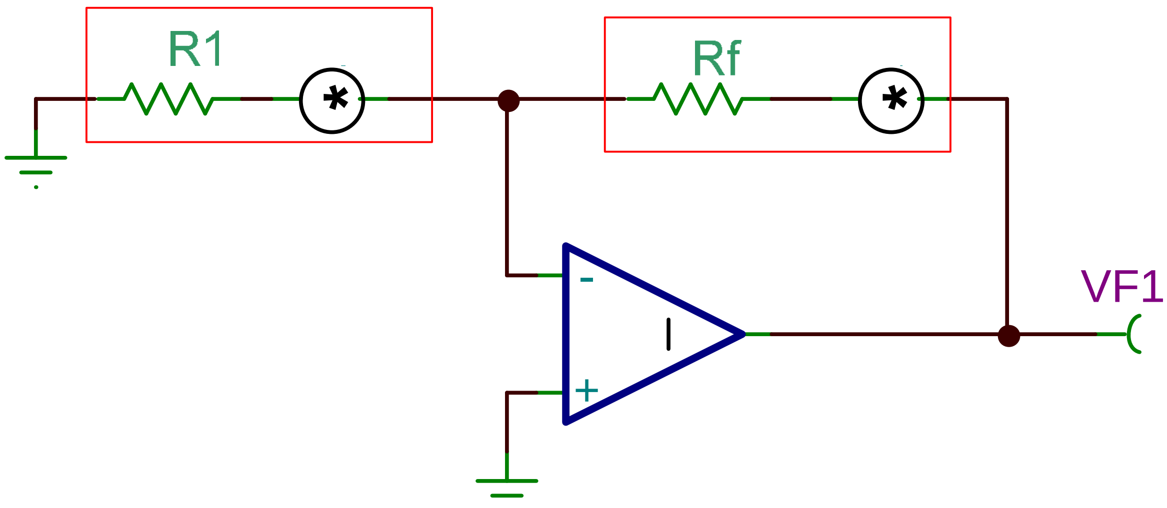
$$Equ\_Input\_Noise = \frac{V_o}{Gain}$$

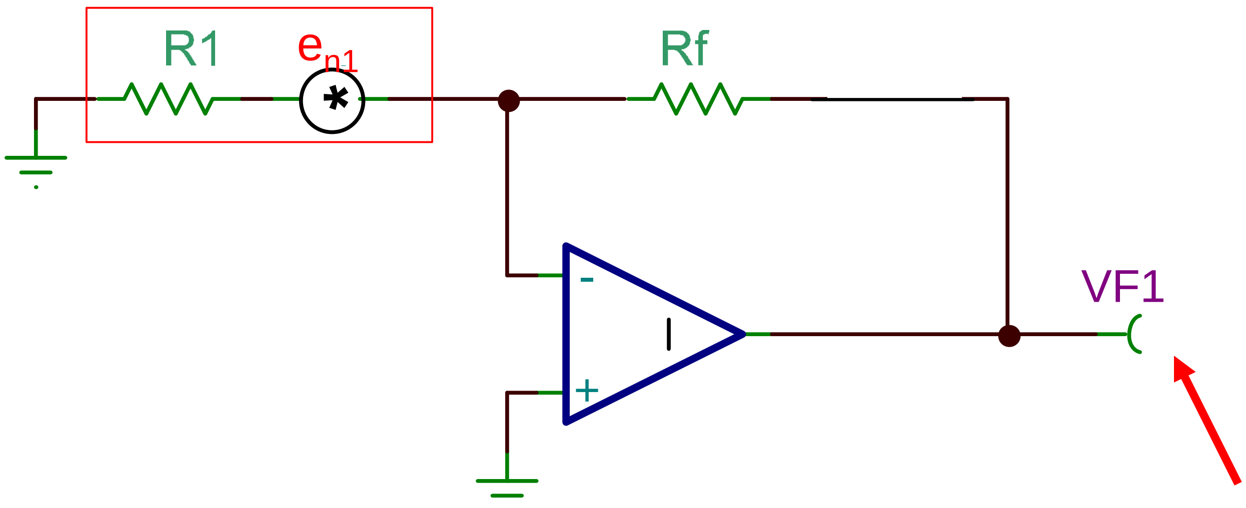
$$Equ\_Input\_Noise = \frac{I_n \cdot R_f}{\frac{R_f}{R_1} + 1} = I_n \cdot \left(\frac{R_f \cdot R_1}{R_f + R_1} \right)$$

$$Equ\_Input\_Noise = I_n \, (R_f \, || \, R_1)$$

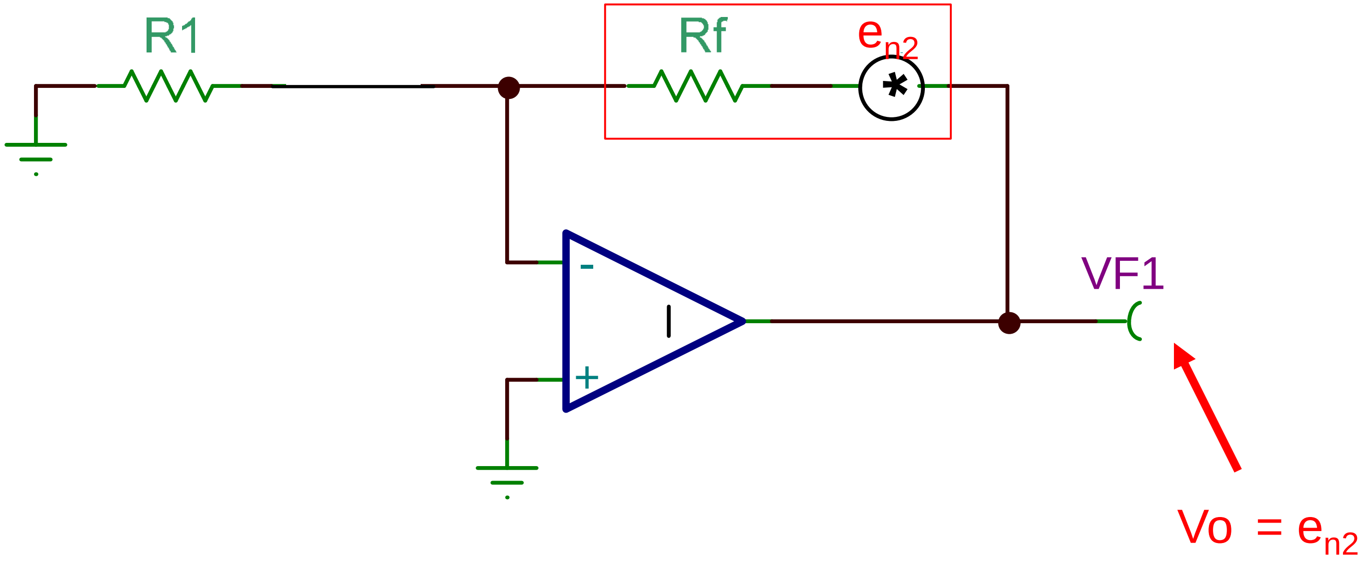
Solve for Resistor Noise Components

Noise Source with Each Resistor



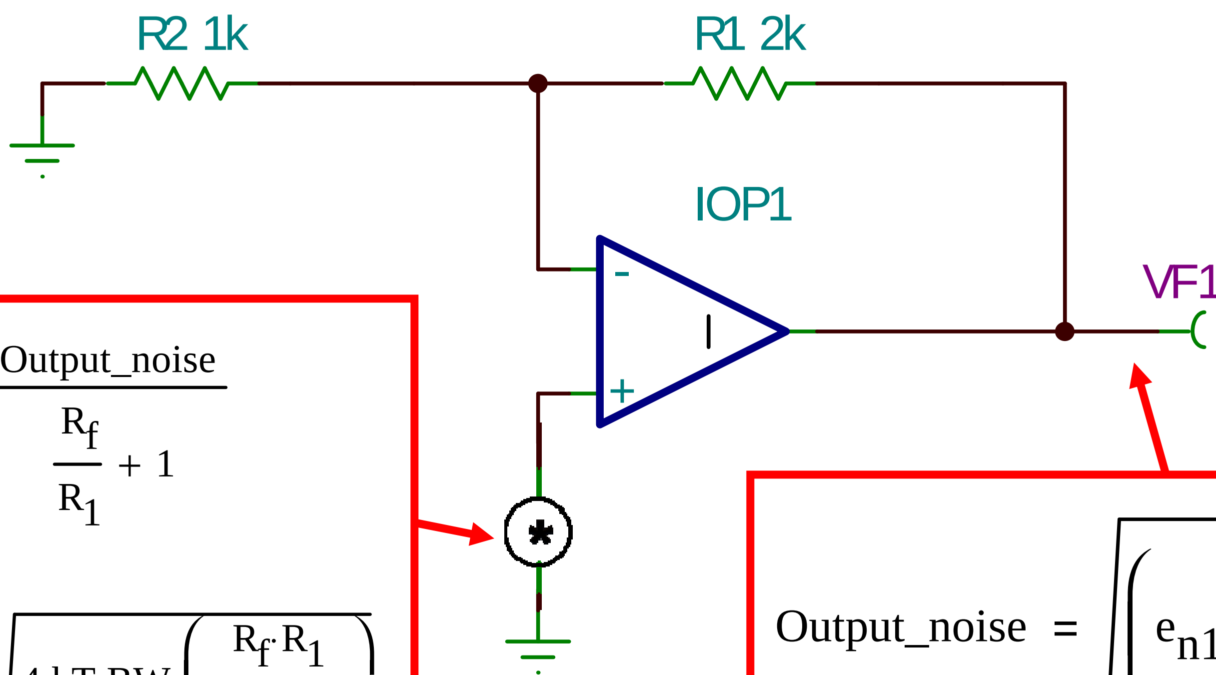


$V_o = e_{n1} \times R_f/R_1$


R1
Rf
e_{n2}
VF1
$Vo = e_{n2}$

Add noise components and refer to the input.

Note: the input noise is equivalent to Rf || R1 (proof on next page)



$$\text{Input\_noise} = \frac{\text{Output\_noise}}{\frac{R_f}{R_1} + 1}$$

$$\text{Input\_noise} = \sqrt{4 \cdot kT \cdot BW \cdot \left(\frac{R_f \cdot R_1}{R_f + R_1} \right)}$$

Equivalent to noise of Rf || R1

$$\text{Output\_noise} = \sqrt{\left(e_{n1} \cdot \frac{R_f}{R_1} \right)^2 + e_{n2}^{\ 2}}$$

$$Output\_Noise^2 = \sqrt{e_{n1}^2 + e_{n2}^2} = \sqrt{\left(e_{n1}\cdot\frac{R_f}{R_1}\right)^2 + e_{n2}^2}$$

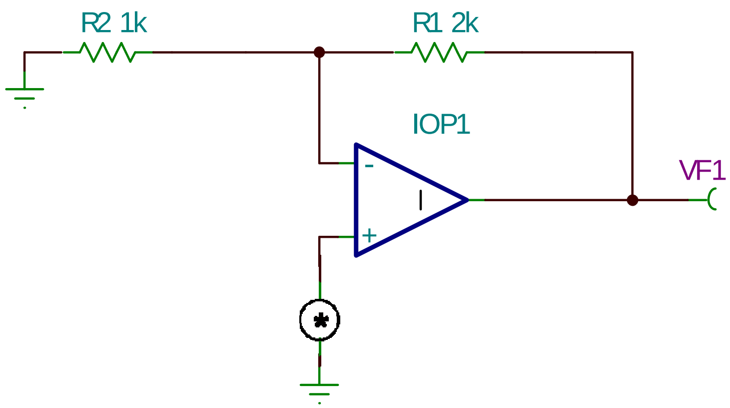
Let $\beta = 4\cdot kT\cdot BW$

$$Output\_noise = \sqrt{\left(\sqrt{\beta\cdot R_1}\cdot\frac{R_f}{R_1}\right)^2 + \left(\sqrt{\beta\cdot R_f}\right)^2}$$

Proof: Simple Amp Resistor Noise

Input Noise is equivalent to noise from parallel combination of Rf and R1.

$$Input\_noise = \frac{\sqrt{\left(\sqrt{\beta\cdot R_1}\cdot\frac{R_f}{R_1}\right)^2 + \left(\sqrt{\beta\cdot R_f}\right)^2}}{\frac{R_f}{R_1} + 1}$$



$$Input\_noise^2 = \frac{\beta\cdot\frac{R_f^2}{R_1} + \beta\cdot R_f}{\frac{(R_f+R_1)^2}{R_1^2}} = \frac{\beta\cdot R_f^2\cdot R_1 + \beta\cdot R_f\cdot R_1^2}{(R_f + R_1)^2}$$

$$Input\_noise = \sqrt{\frac{\beta\cdot R_f^2\cdot R_1 + \beta\cdot R_f\cdot R_1^2}{(R_f + R_1)^2}} = \sqrt{\beta\cdot\frac{R_f\cdot R_1}{R_f + R_1}}$$

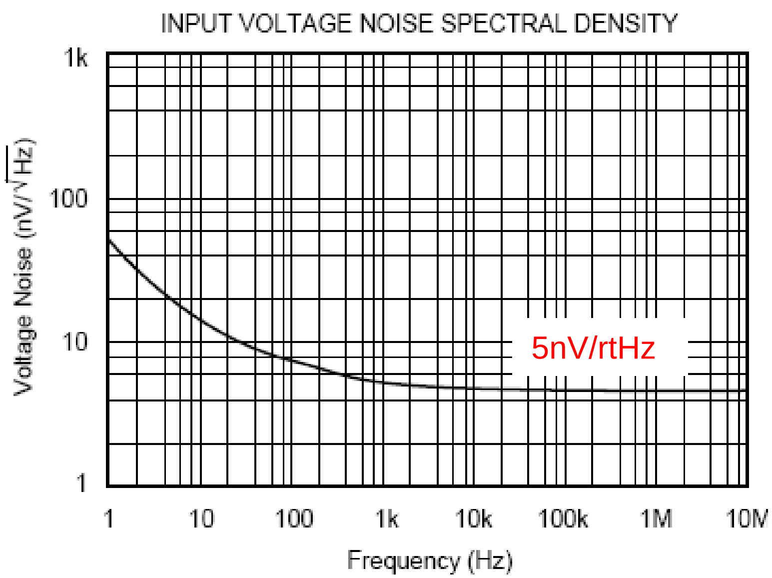
$$Input\_noise = \sqrt{4\cdot kT\cdot BW\cdot\left(\frac{R_f\cdot R_1}{R_f + R_1}\right)}$$

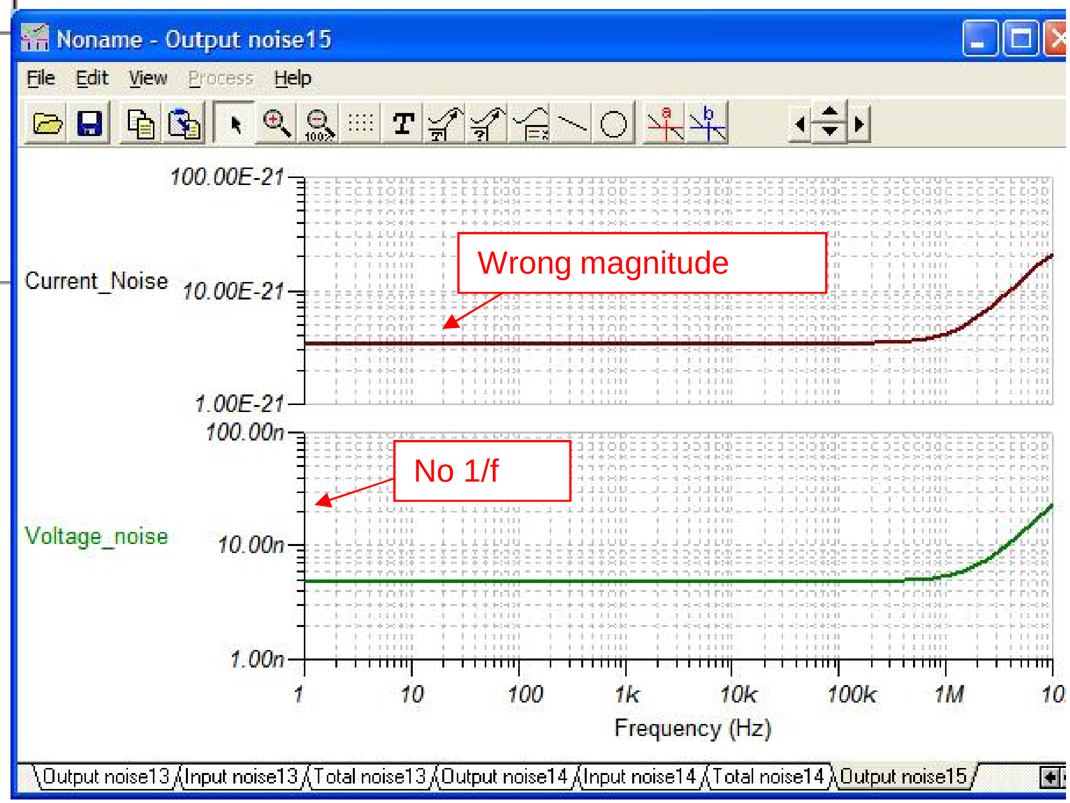
Equ to noise of Rf || R1

Appendix 2

The OPA627 Tina Model Does **NOT** Match the Data Sheet

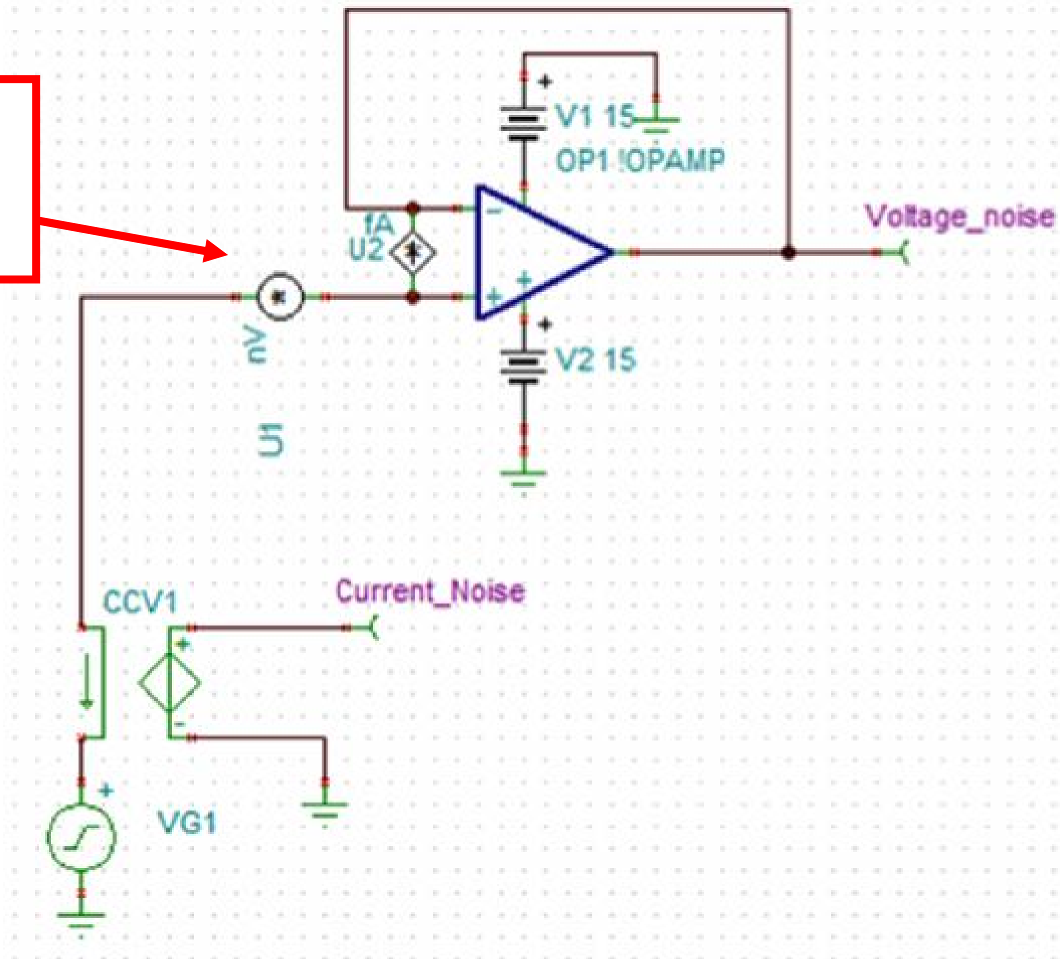
PARAMETER	OPA627BM, BP, SM OPA637BM, BP, SM MIN	TYP	MAX	UNITS
NOISE				
Input Voltage Noise				
Noise Density: f = 10Hz		15	40	nV/√Hz
f = 100Hz		8	20	nV/√Hz
f = 1kHz		5.2	8	nV/√Hz
f = 10kHz		4.5	6	nV/√Hz
Voltage Noise, BW = 0.1Hz to 10Hz		0.6	1.6	µVp-p
Input Bias Current Noise				
Noise Density, f = 100Hz		1.6	2.5	fA/√Hz
Current Noise, BW = 0.1Hz to 10Hz		30	60	fAp-p





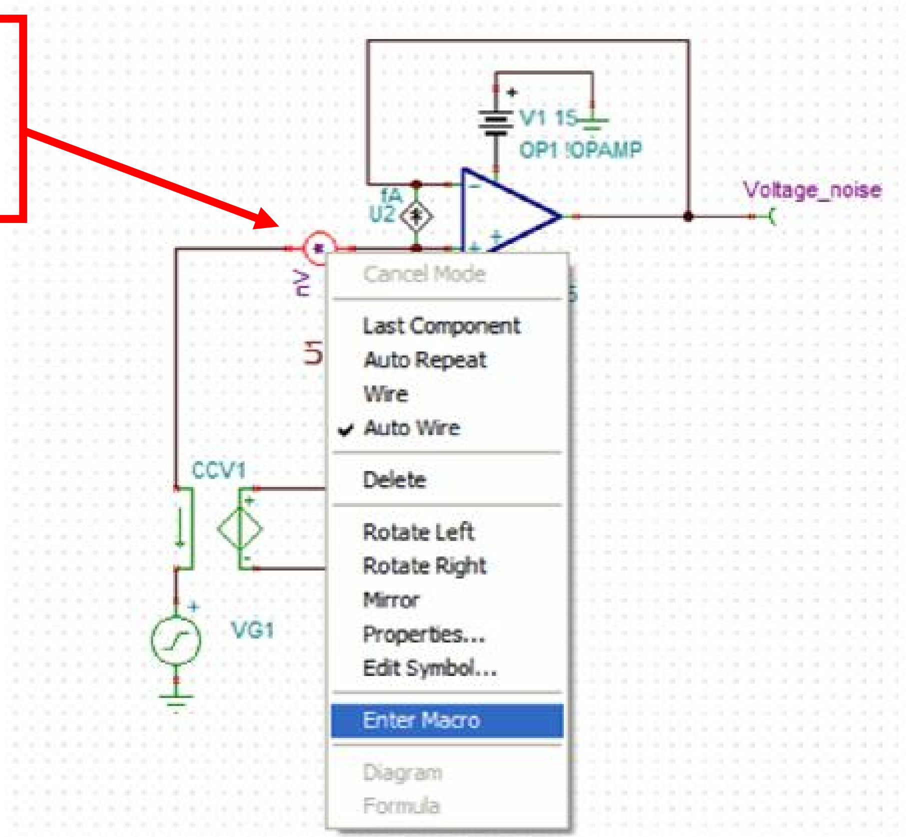
Build Your Own Noise Model Using "Burr-Brown" Macro Model Noise Sources and Generic Op-Amp

The voltage and current noise source is available at www.ti.com (search for "noise sources").



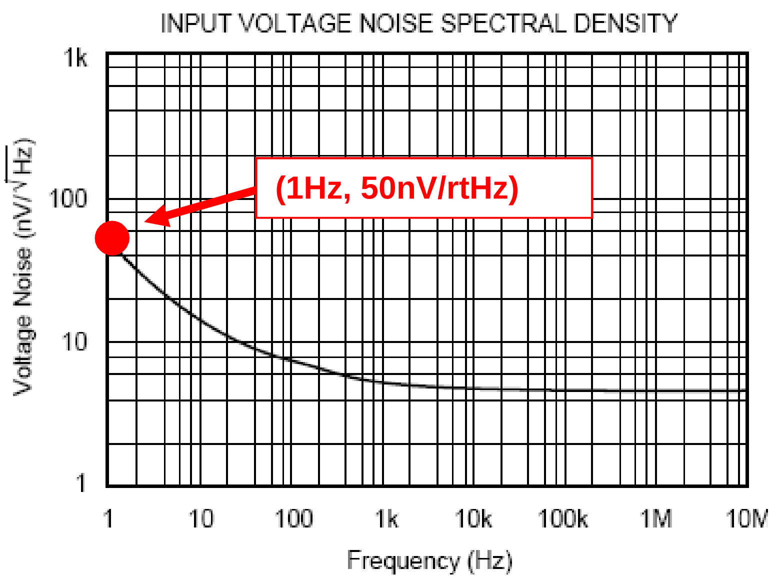
Right Click on Noise Source to Edit the Macro

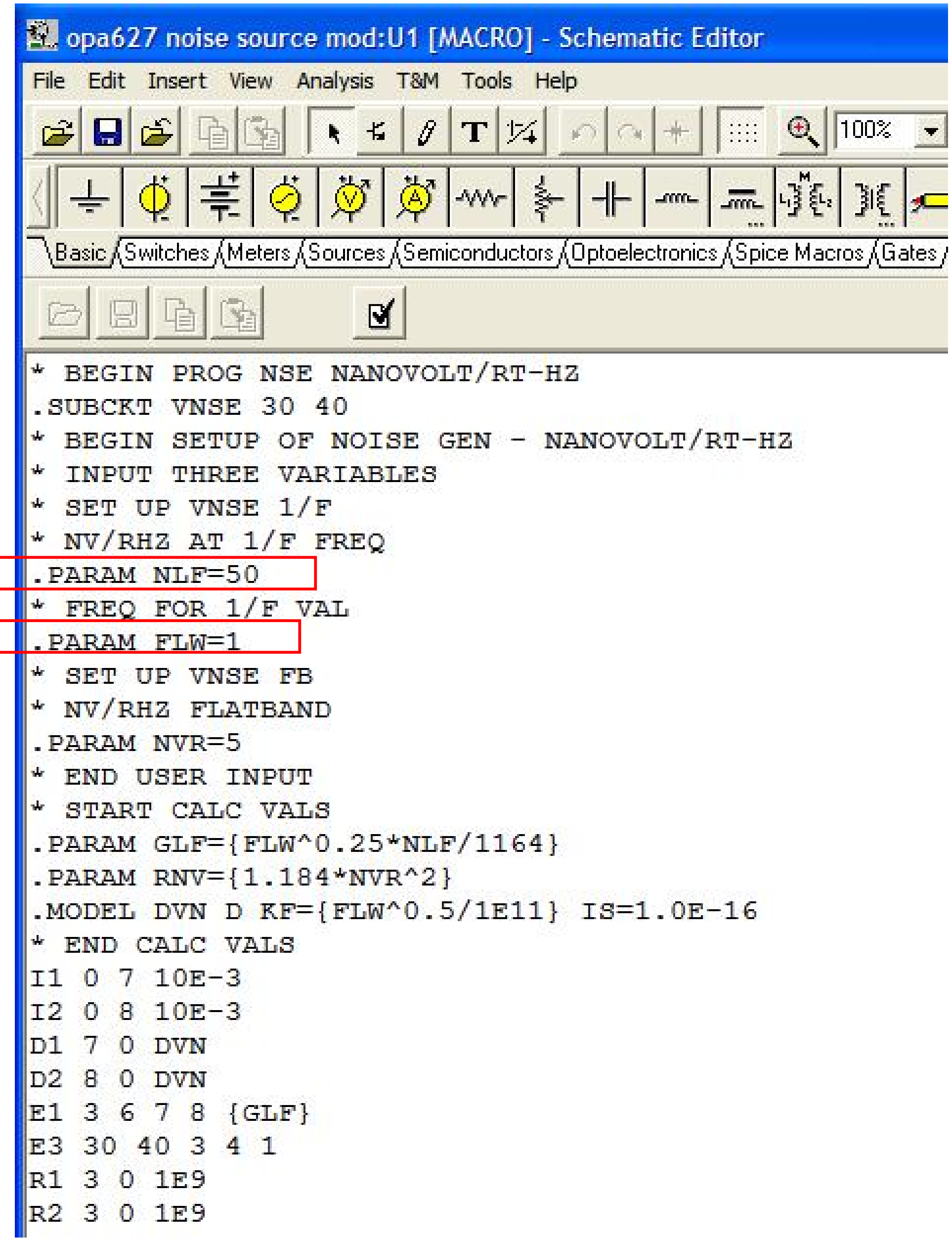
Enter magnitude of 1/f and broadband noise into the macro.



1/f Region

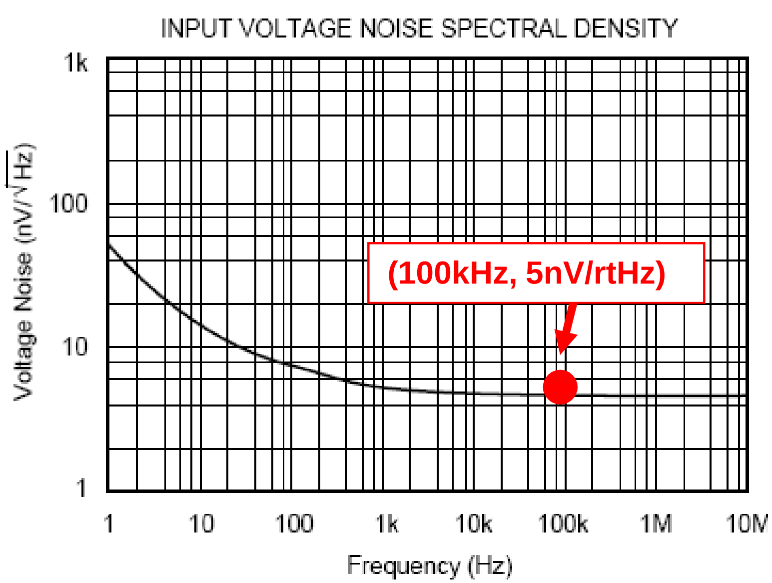
Look for a point in the 1/f region. Enter the frequency and magnitude at this point

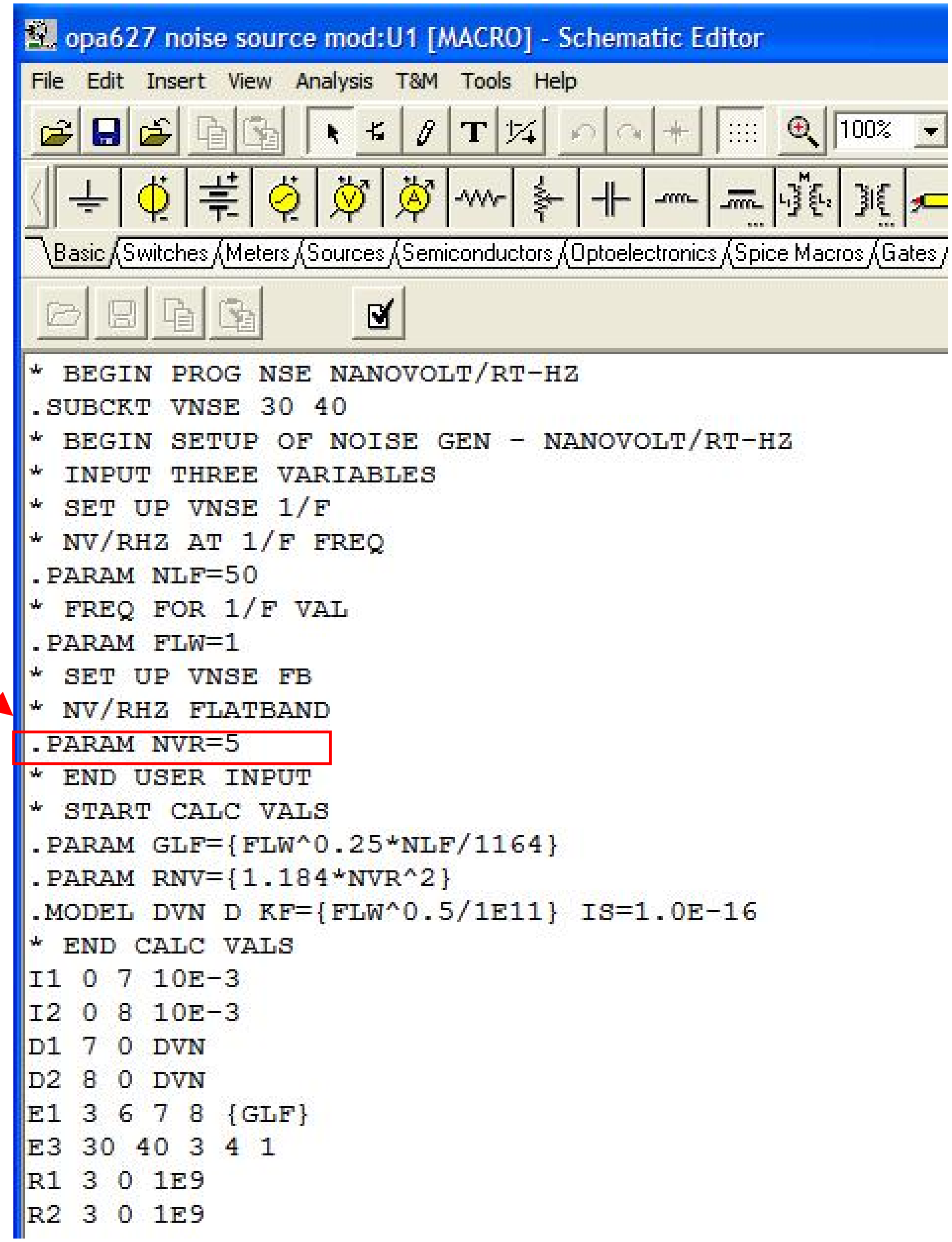




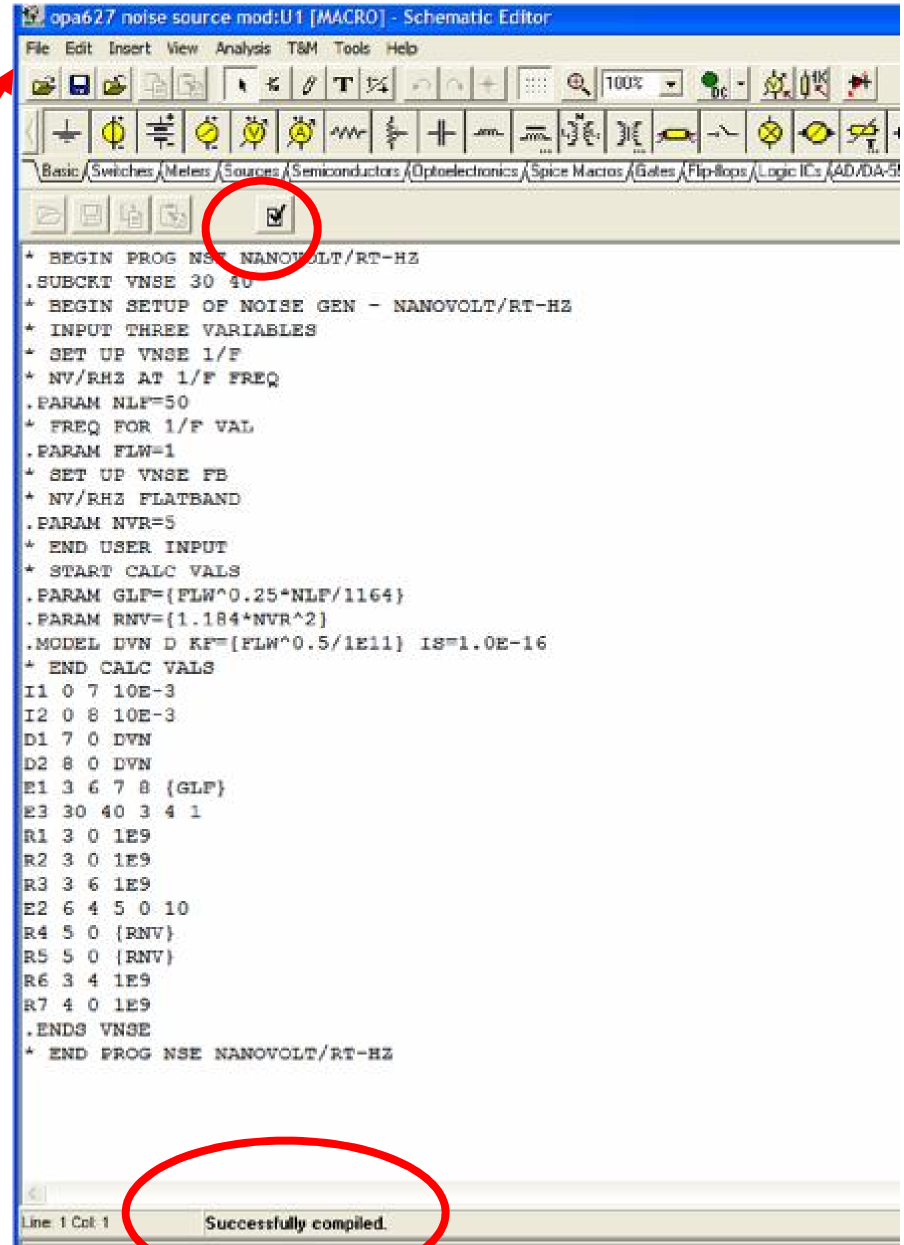
Broadband Region

Look for a point in the broad band region. Enter the magnitude at this point





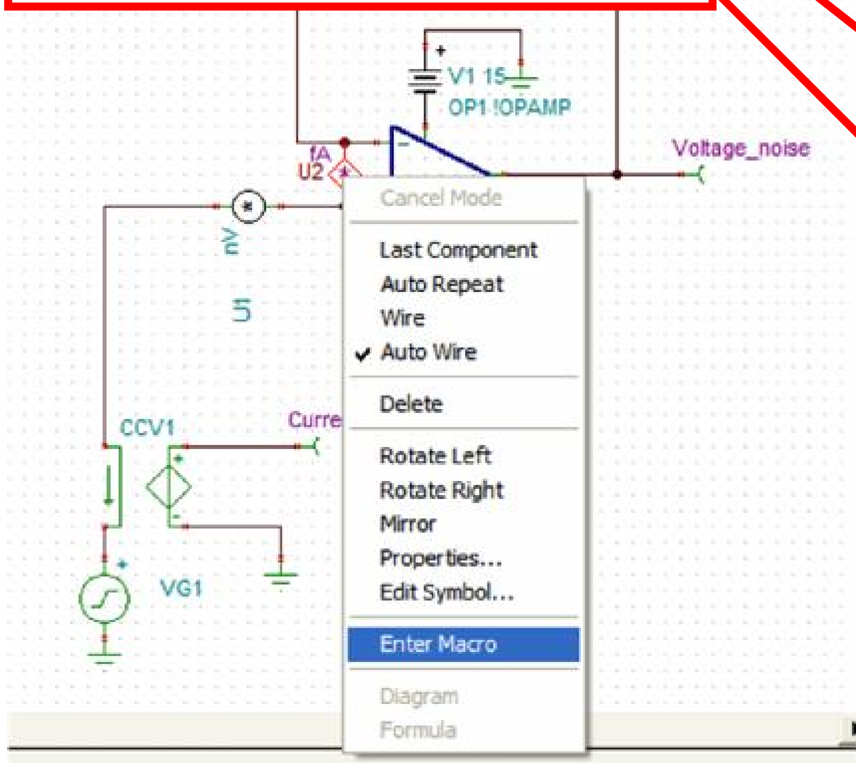
Compile the Macro



After macro is compiled press "file > close" and return to schematic editor.

Same Procedure for Current Noise Source

Follow the same procedure for current noise. This example has no 1/f component (set FLWF = 0.001).




```
* BEGIN PROG NSE FEMTO AMP/RT-HZ
.SUBCKT FEMTO 1 2
* BEGIN SETUP OF NOISE GEN - FEMPTOAMPS/RT-HZ
* INPUT THREE VARIABLES
* SET UP INSE 1/F
* FA/RHZ AT 1/F FREQ
.PARAM NLFF=2.5
* FREQ FOR 1/F VAL
.PARAM FLWF=0.001
* SET UP INSE FB
* FA/RHZ FLATBAND
.PARAM NVRF=2.5
* END USER INPUT
* START CALC VALS
```

PARAMETER	OPA627BM, BP, SM OPA637BM, BP, SM MIN	TYP	MAX	UNITS
NOISE				
Input Voltage Noise				
Noise Density: f = 10Hz		15	40	nV/√Hz
f = 100Hz		8	20	nV/√Hz
f = 1kHz		5.2	8	nV/√Hz
f = 10kHz		4.5	6	nV/√Hz
Voltage Noise, BW = 0.1Hz to 10Hz		0.6	1.6	μVp-p
Input Bias Current Noise				
Noise Density, f = 100Hz		1.6	2.5	fA/√Hz
Current Noise, BW = 0.1Hz to 10Hz		30	60	fAp-p

Important Op-Amp Characteristics

OPA627 Data Sheet

OPEN-LOOP GAIN					
Open-Loop Voltage Gain	$V_O = \pm10V, R_L = 1k\Omega$	112	120		dB
Over Specified Temperature	$V_O = \pm10V, R_L = 1k\Omega$	106	117		dB
SM Grade	$V_O = \pm10V, R_L = 1k\Omega$	100	114		dB
FREQUENCY RESPONSE					
Slew Rate: OPA627	G = –1, 10V Step	40	55		V/µs
OPA637	G = –4, 10V Step	100	135		V/µs
Settling Time: OPA627 0.01%	G = –1, 10V Step		550		ns
0.1%	G = –1, 10V Step		450		ns
OPA637 0.01%	G = –4, 10V Step		450		ns
0.1%	G = –4, 10V Step		300		ns
Gain-Bandwidth Product: OPA627	G = 1		16		MHz
OPA637	G = 10		80		MHz
Total Harmonic Distortion + Noise	G = +1, f = 1kHz		0.00003		%
POWER SUPPLY					

$OLG = 10^{(Ndb/20)}$ = 1E6 (From Data Sheet)

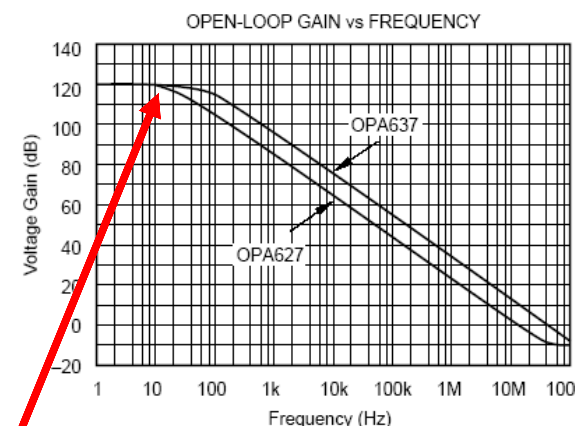
GBW = 16MHz (From Data Sheet)

Dominant Pole = GBW / OLG = (16MHz) / (1E6) = 16Hz

where:

GBW – Unity Gain-Bandwidth Product

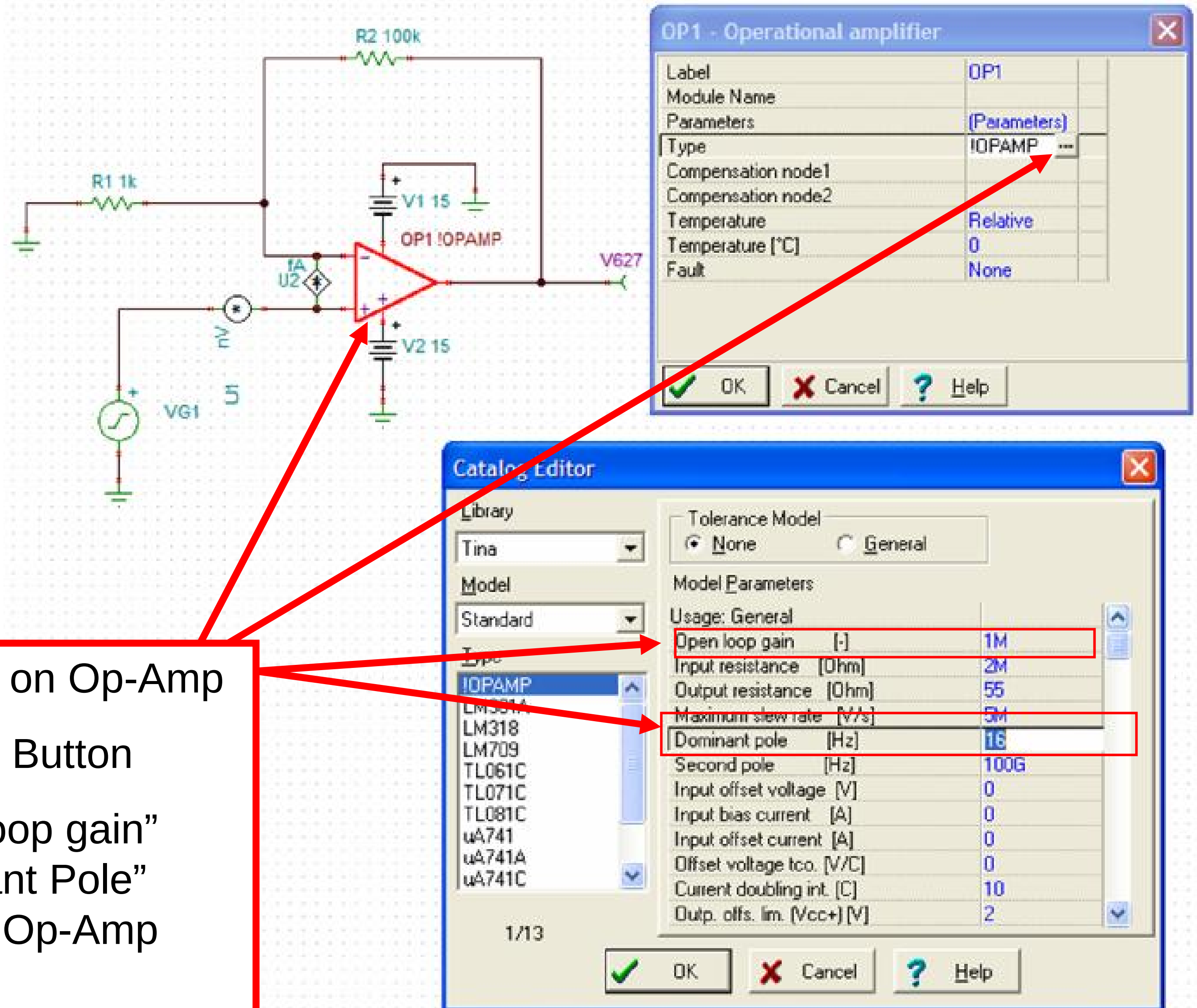
OLG – Open Loop Gain



Dominant Pole

Texas Instruments

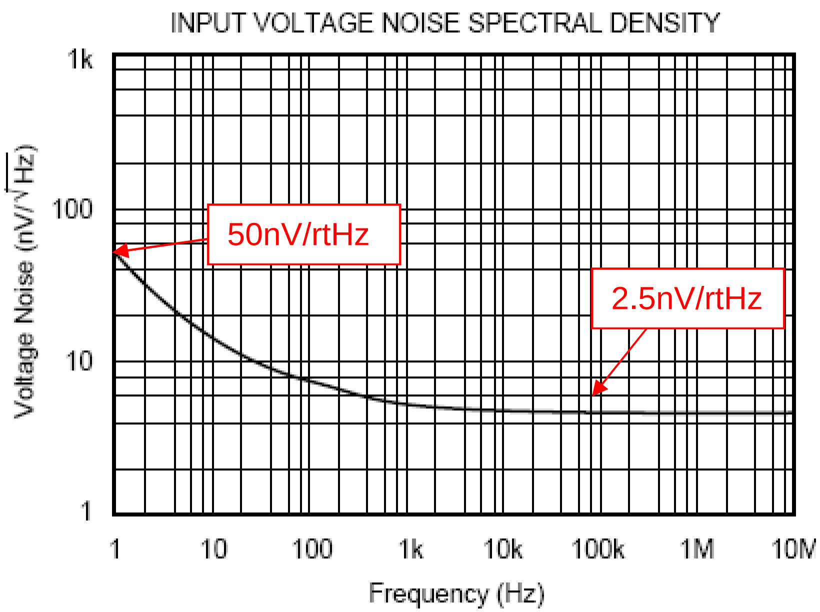
Edit Generic Op-Amp Macro-model

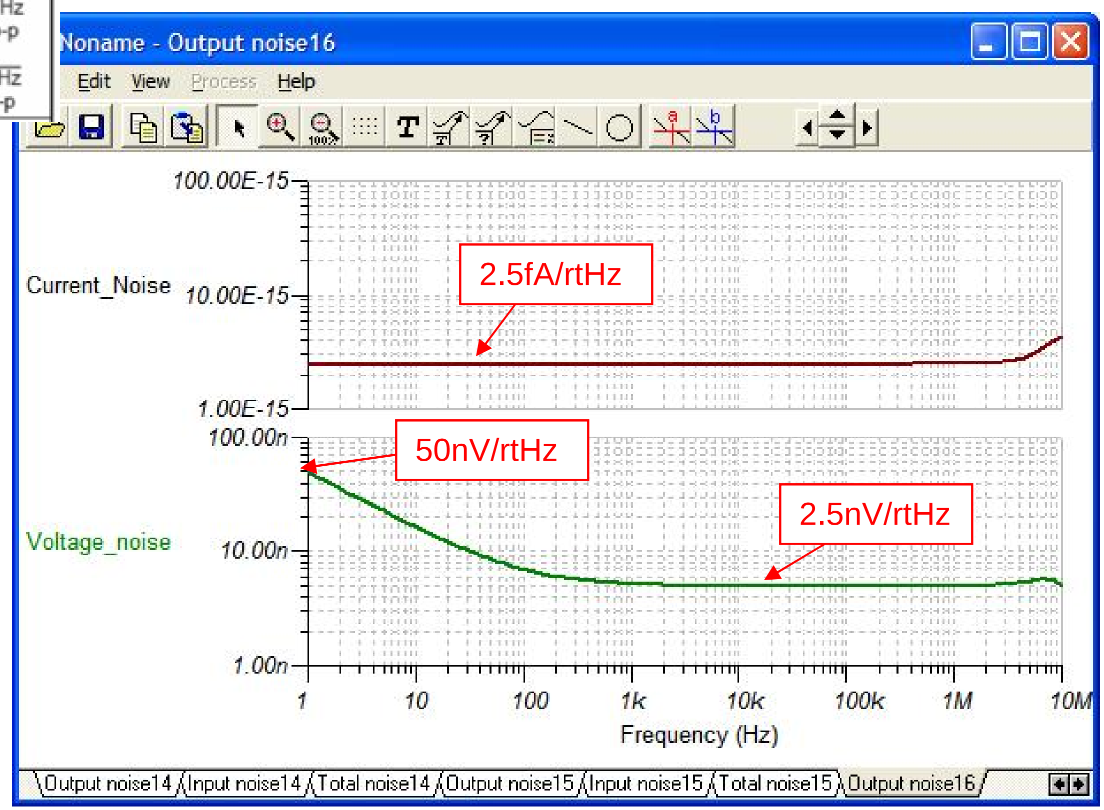


1. Double Click on Op-Amp
2. Press “Type” Button
3. Edit “Open loop gain” and “Dominant Pole” according to Op-Amp data sheet

Verify the Noise Model is Correct Using the Test Procedure

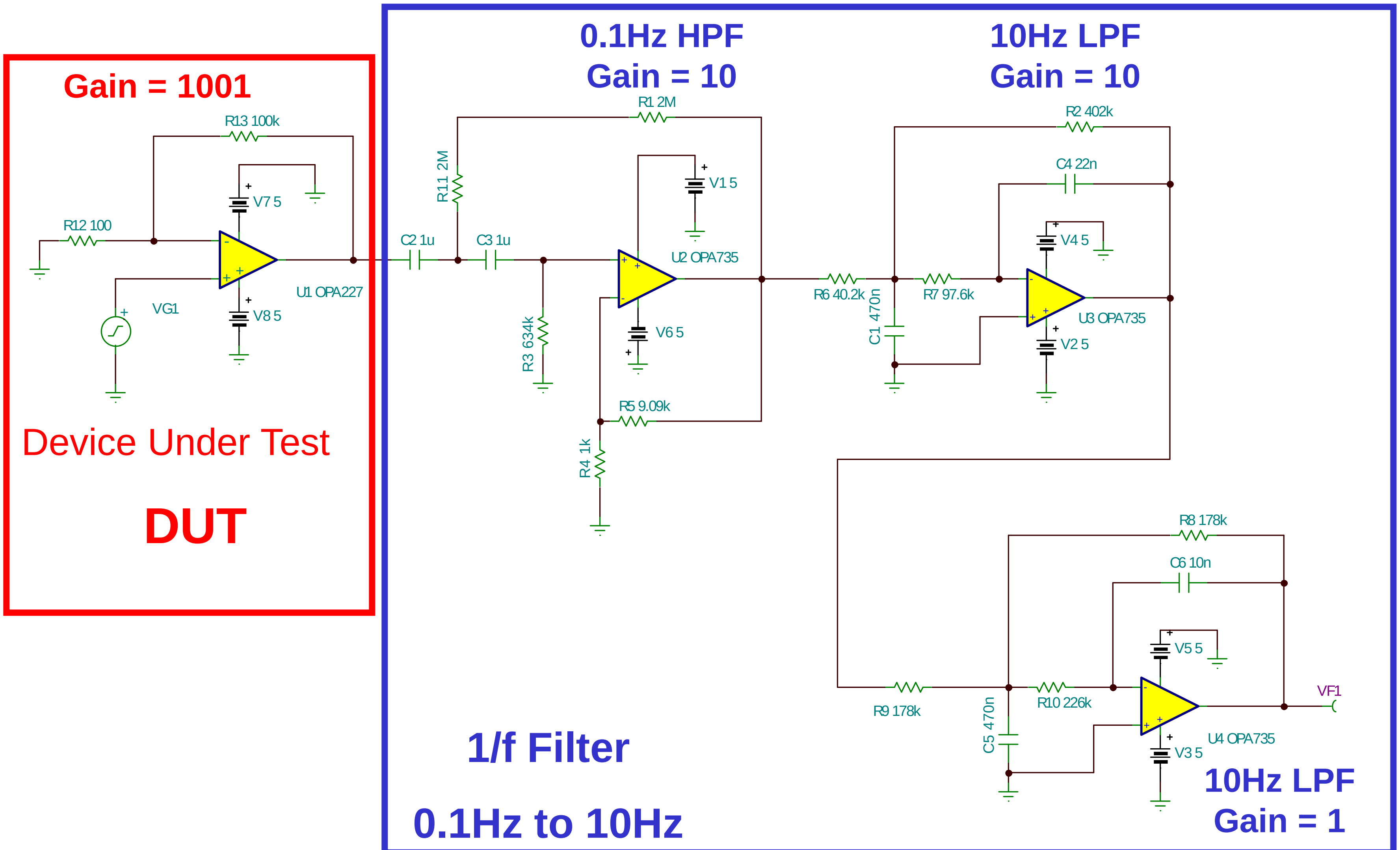
PARAMETER	OPA627BM, BP, SM OPA637BM, BP, SM MIN	TYP	MAX	UNITS
NOISE				
Input Voltage Noise				
Noise Density: f = 10Hz		15	40	nV/√Hz
f = 100Hz		8	20	nV/√Hz
f = 1kHz		5.2	8	nV/√Hz
f = 10kHz		4.5	6	nV/√Hz
Voltage Noise, BW = 0.1Hz to 10Hz		0.6	1.6	µVp-p
Input Bias Current Noise				
Noise Density, f = 100Hz		1.6	2.5	fA/√Hz
Current Noise, BW = 0.1Hz to 10Hz		30	60	fAp-p





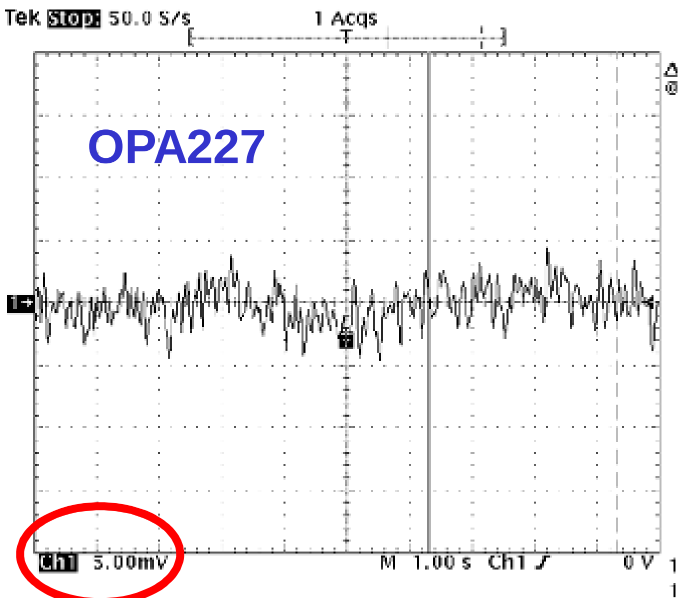
Appendix 2

0.1Hz Second Order 10.0Hz Fourth Order 1/f Filter

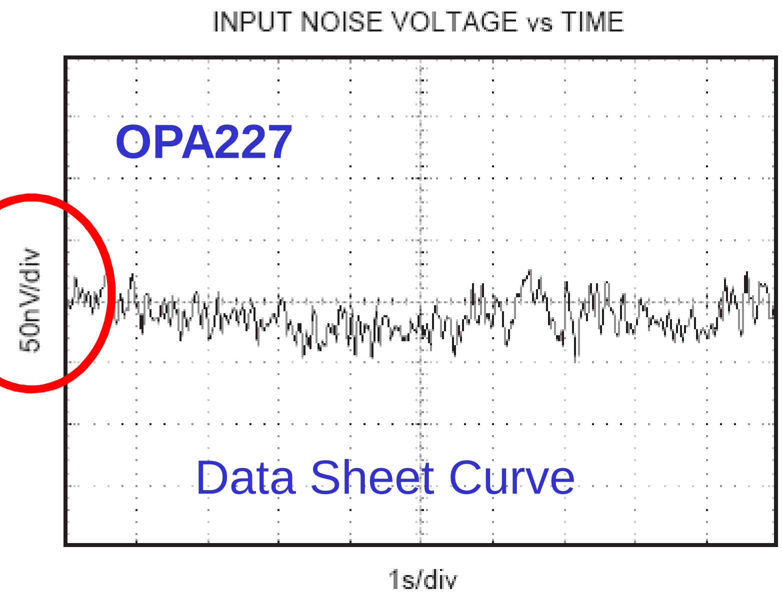


The Circuit Generates the Data Sheet 1/f Plots
(Example OPA227)

Tektronix TDS460A Measurement



PDS 0.1Hz to 10Hz Noise Curve

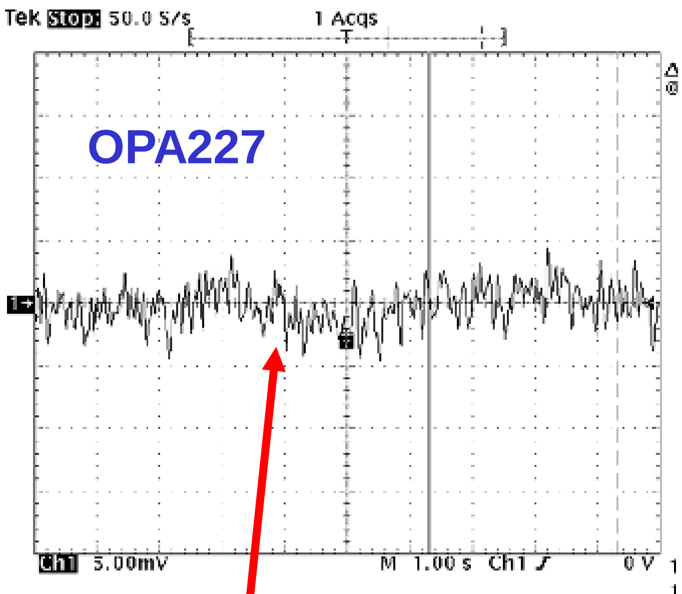


Y-Axis scale must be adjusted to account for the gain to match data sheet.

5mV/(10x10x1000) = 50nV/div

1/f Filter Measured vs Tina
(Example OPA227)

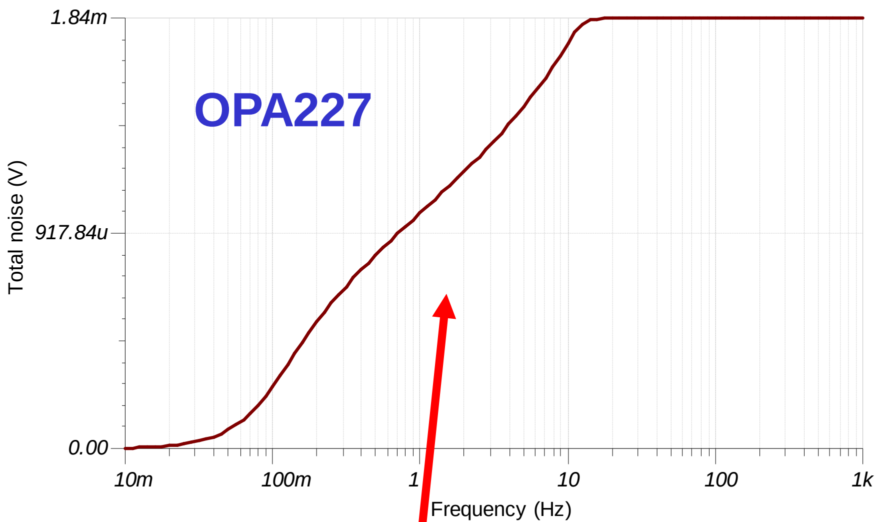
Tektronix TDS460A Measurement



Measured Output is approximately

10mVp-p

Tina Simulation



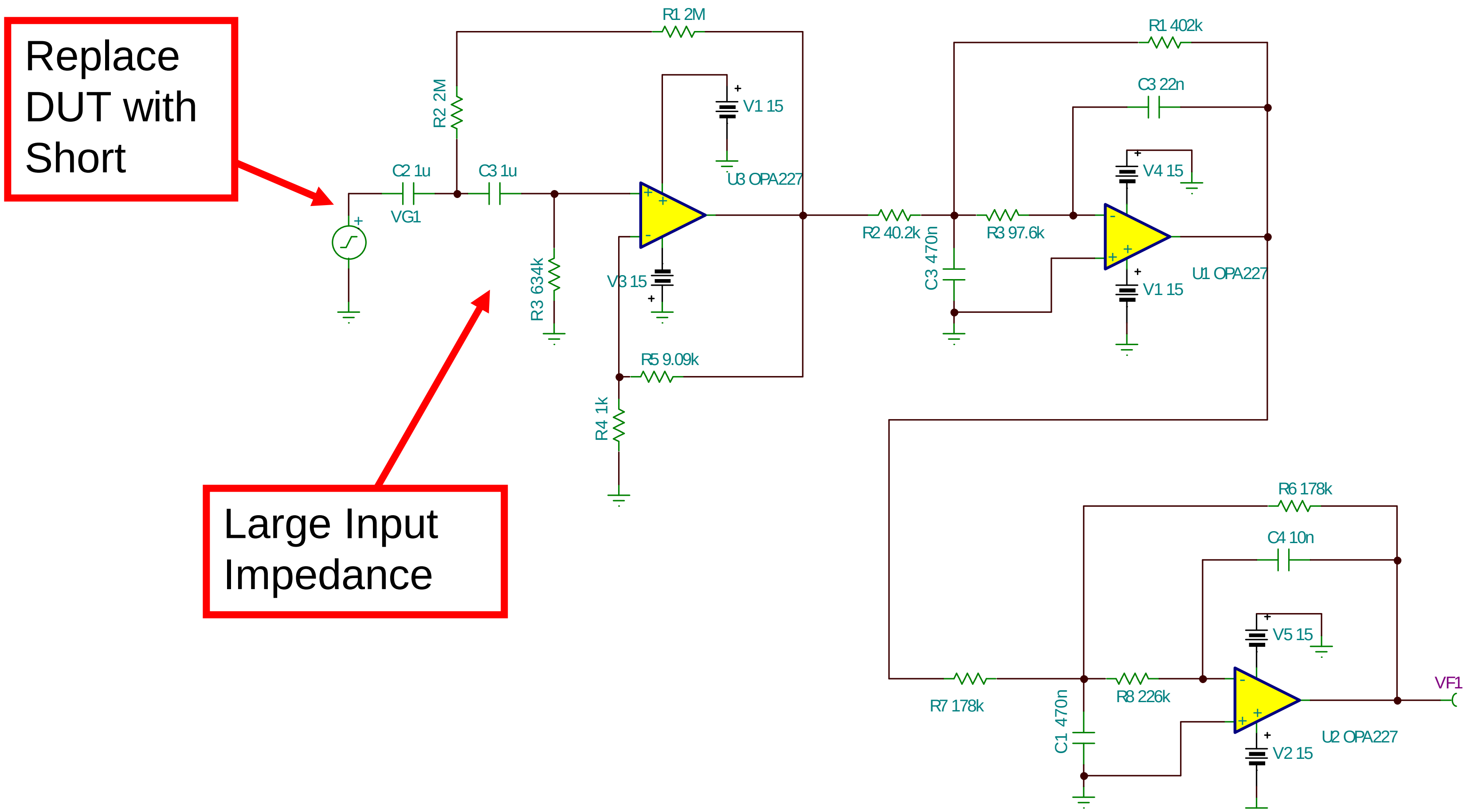
Tina Simulation

1.84mV rms

(1.84mV)(6) = 11mVp-p

Measure Noise Floor of 1/f Filter

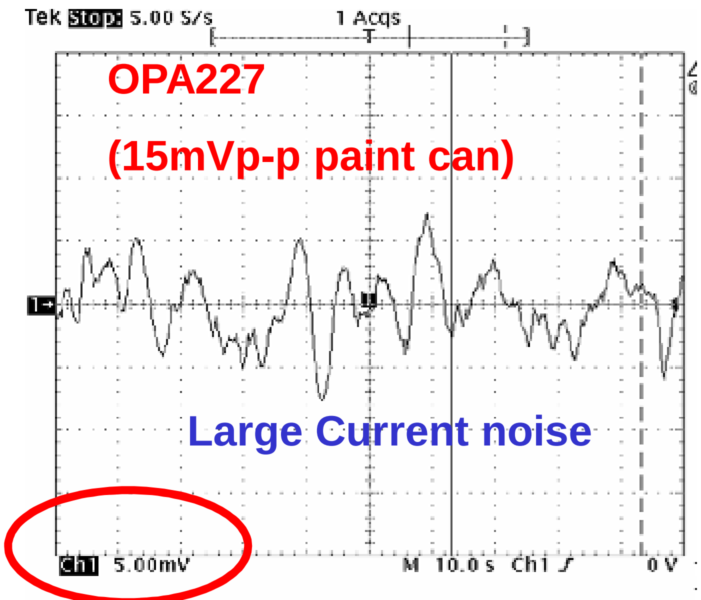
What Op-Amp will give us the lowest noise Floor?

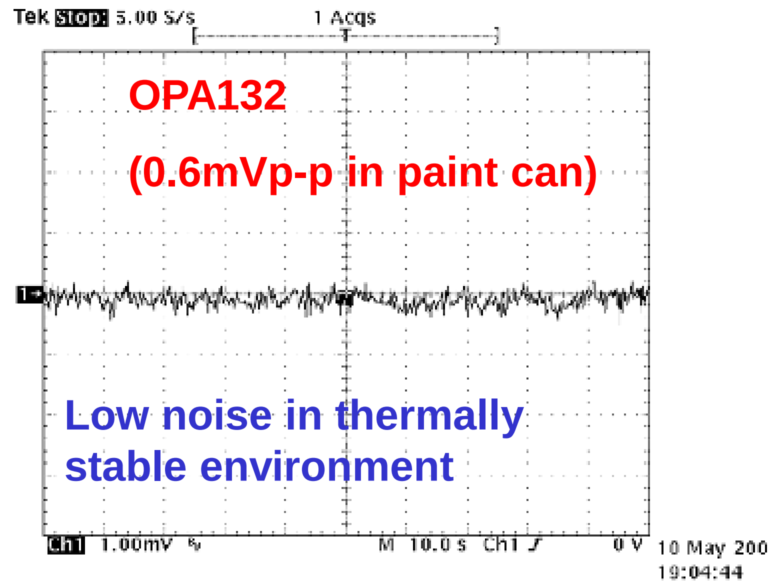


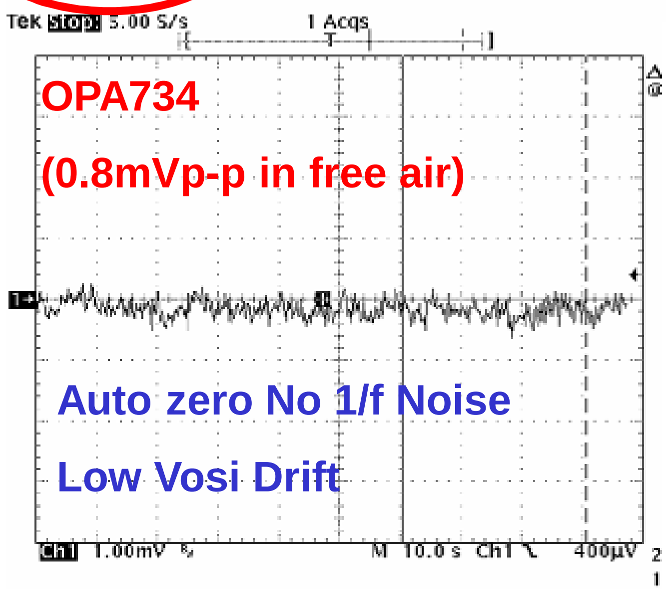
Measure Noise Floor of 1/f Filter

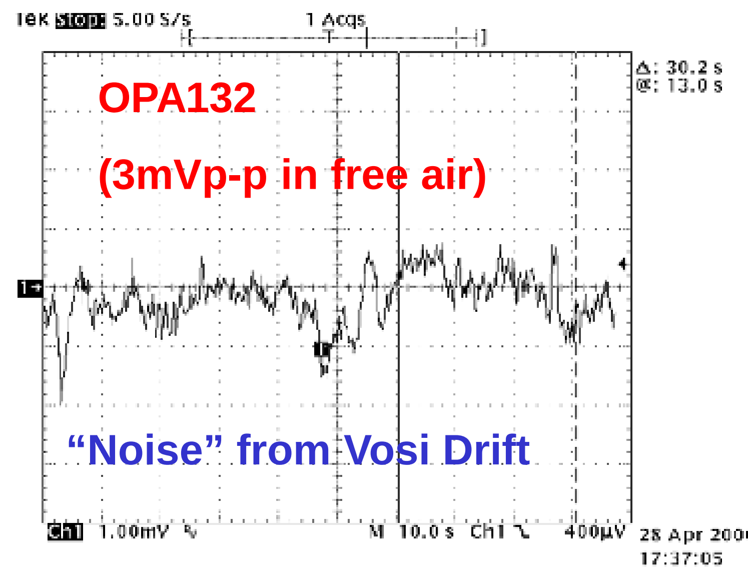
What Op-Amp will give us the lowest noise Floor?

Op-Amp	General Description	V_n (nV/rt-Hz)	I_n (fA/rt-Hz)
OPA227	Low noise Bipolar	3.5 @10Hz 6 @1Hz 20 @0.1Hz	2,000 @10Hz 6,000 @1Hz 20,000 @0.1Hz
OPA132	Low noise CMOS	23 @10Hz 80 @1Hz 228 @0.1Hz	3
OPA735	Auto Zero CMOS	135	40









Note: measurements in paint can minimize thermal drift.