Analysis And Measurement Of Intrinsic Noise In Op Amp Circuits
Part I: Introduction And Review Of Statistics
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Noise can be defined as any unwanted signal in an electronic system. Noise is responsible for reducing the quality of audio signals or introducing errors into precision measurements. Board and system level electrical design engineers are interested in determining the worst case noise they can expect in their design and design methods for reducing noise and measurement techniques to accurately verify their design.

Intrinsic and extrinsic noise are the two fundamental types of noise that affect electrical circuits. Extrinsic noise is generated by external sources. Digital switching, 60 Hz noise and power supply switching are common examples of extrinsic noise. Intrinsic noise is generated by the circuit element itself. Broadband noise, thermal noise and flicker noise are the most common examples of intrinsic noise. This article series will describe how to predict the level of intrinsic noise in a circuit with calculations, and using SPICE simulations. Noise measurement techniques will be discussed also.

Thermal Noise

Thermal noise is generated by the random motion of electrons in a conductor. Because this motion increases with temperature so does the magnitude of thermal noise. Thermal noise can be viewed as a random variation in the voltage present across a component (eg a resistor). Fig. 1.1 shows what thermal noise looks like in the time domain (standard oscilloscope measurement). It also shows that if you look at this random signal statistically, it can be represented as a Gaussian distribution. The distribution is drawn sideways to help show its relationship with the time domain signal.

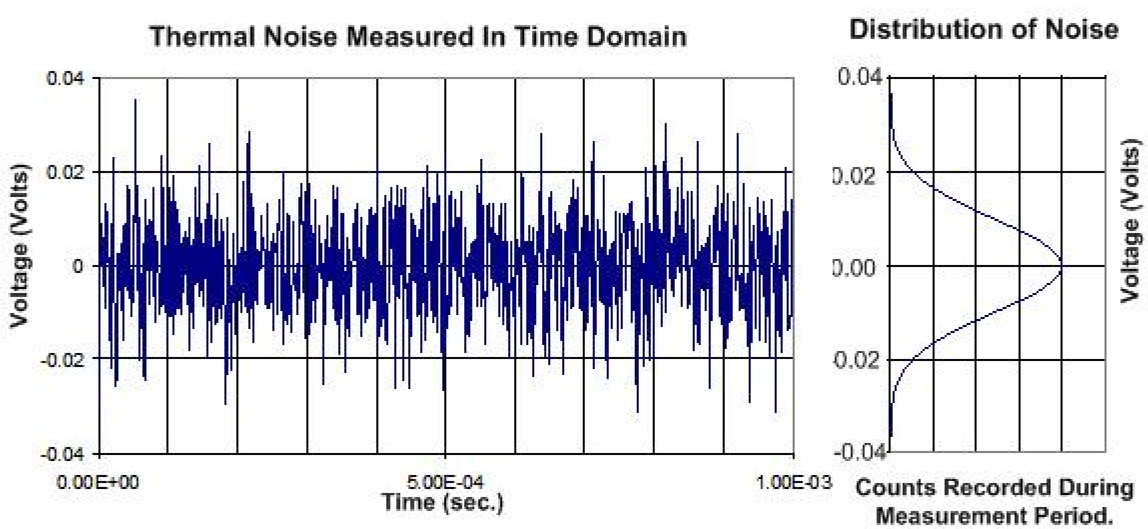

Fig. 1.1: White noise Shown In Time Domain And Statistically

The power contained within a thermal noise signal is directly proportional to temperature and bandwidth. Note that a simple application of the power formula can be used to express the relationship in terms of voltage and resistance (see Equation 1.1). This expression is useful because it allows you to estimate the root-mean square (rms) noise in the circuit. Furthermore, it illustrates the importance of using low resistance components when possible in low noise circuits.

$e_n = \sqrt{4kTR\Delta f}$ where e is the rms noise voltage
T is Temperature in Kelvin (K)
R is Resistance in Ohms (Ω)
f is noise bandiwdth frequency in Hertz (Hz)
k is Boltzmann's Constant 1.381E-23 joule/K

Note to convert degrees Celsius to Kelvin

$T_K = 273.15^oC + T_C$

Equation 1.1: Rms Thermal Noise Voltage

The important thing to know about Equation 1.1 is that it allows you to find an rms noise voltage. In most cases, engineers want to know, "What is the worst case noise scenario?" In other words, they are most interested in the peak-to-peak voltage. When attempting to translate an rms thermal noise voltage to peak-to-peak noise, it is important to remember the thermal noise has Gaussian distribution. There are some simple rules of thumb that are based on statistical relationships that can be used to convert rms to peak-to-peak value. Before presenting these, however, we will discuss some of the mathematical background. The focus of this article is to review this statistical background; subsequent articles will cover the measurement and analysis of practical analog circuits.

Probability Density Function

The mathematical equation that forms the normal distribution function is called the *Probability Density Function* (see Equation 1.2). Plotting a histogram of noise voltage measured over a time interval will approximate the shape of this function. Fig. 1.2 shows a measured noise histogram with the probability distribution function superimposed on it.

$$f(x) = \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{\left[\frac{-(x-\mu)^2}{2\sigma^2}\right]}$$

Where
f(x) -- Probability **Density** function for Gaussian Distribution
x-- the random variable. In this case noise voltage.

x-- the random variable. In this case noise voltage.
μ -- the mean value
σ -- the standard deviation

Equation 1.2: Probability Density Function For Gaussian Distribution

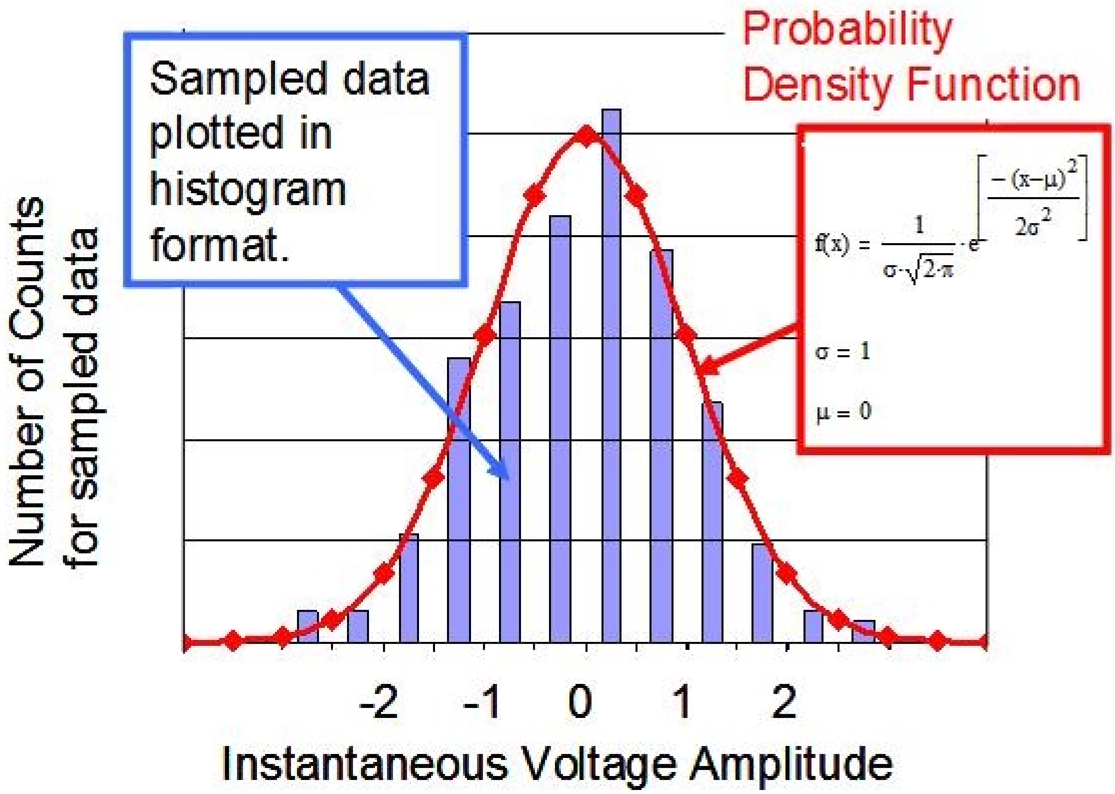

Fig. 1.2: Measured Distribution With Superimposed Probability Density Function

Probability Distribution Function

The *Probability Distribution Function* is the integral of the probability density function. This function is very useful because it tells us what the probability is that an event will occur in a given interval (see Equation 1.3, and Figure 1.3). For example, assume that Figure 1.3 is a noise probability distribution function. The function tells us that there is a 30% chance that you will measure a noise voltage between -1 V and +1 V (ie the interval [-1, 1]) at any instant in time.

$$P(a < x < b) = \int_a^b f(x)\,dx = \int_a^b \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{\left[\frac{-(x-\mu)^2}{2\sigma^2}\right]} dx$$

Where
$P(a < x < b)$ -- the probability that x will be in the interval (a, b)
x-- the random variable. In this case noise voltage.
μ -- the mean value
σ -- the standard deviation

Equation 1.3: Probability Distribution Function

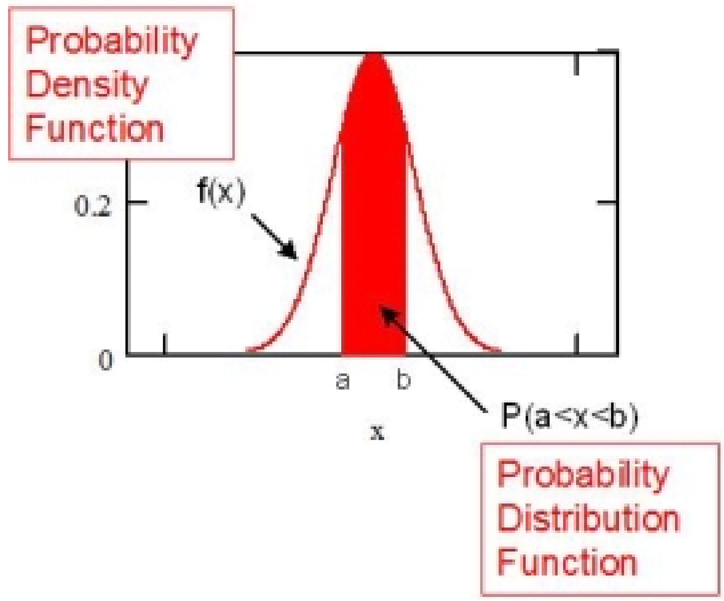

Fig. 1.3: Probability Density Function And Probability Distribution Function

This Probability Distribution Function is instrumental in helping us translate rms to peak-to-peak. Note that the tails of the Gaussian distribution are infinite. This implies that any noise voltage is possible. While this is theoretically true, in practical terms the probability that extremely large instantaneous noise voltages are generated is very small. For example, the probability that we measure a noise voltage between -3σ and +3σ is 99.7 %. In other words, there is only a 0.3 % chance of measuring a voltage outside of this interval. So for this reason, ±3σ (ie 6σ) is often used to estimate the peak-to-peak value for a noise signal. Note that some engineers use 6.6σ to estimate the peak-to-peak value of noise. There is no agreed upon standard for this estimation. Figure 1.4 graphically shows how 2σ will catch 68 % of the noise. Table 1.1 summarizes the relationship between standard deviation and probability of measuring a noise voltage.

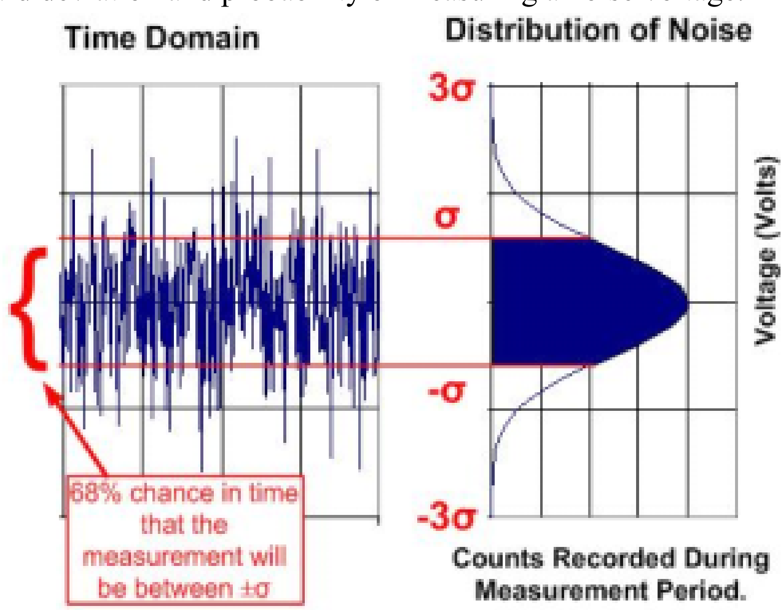

Fig. 1.4: Illustrates How Standard Deviation Relates To Peak-To-Peak

NUMBER OF STANDARD DEVIATIONS	CHANCE OF MEASURING VOLTAGE
2σ (same as $\pm\sigma$)	68.3 %
3σ (same as $\pm 1.5\sigma$)	86.6 %
4σ (same as $\pm 2\sigma$)	95.4 %
5σ (same as $\pm 2.5\sigma$)	98.8 %
6σ (same as $\pm 3\sigma$)	99.7 %
6.6σ (same as $\pm 3.3\sigma$)	99.9 %

Table 1.1: Number Of Standard Deviations And Percentage Chance Of Measuring

Thus, we have a relationship that allows us to estimate peak-to-peak noise given the standard deviation. In general, however, we want to convert rms to peak-to-peak. Often people assume that the rms and standard deviation are the same. This is not always the case. The two values are equal only when there is no dc component (the dc component is the average value μ). In the case of thermal noise, there is no dc component so the standard deviation and rms values are equal. Two examples are in the Appendix showing cases where the standard deviation is equal to rms and where it is not.

At the start of this article the formula for computing rms thermal noise voltage was introduced. Another way of computing the rms noise voltage is to measure a large number of discrete points and use statistics to estimate the standard deviation. For example, if you have a large number of samples from an ADC you could use Equation 1.4, 1.5 and 1.6 to compute the mean, standard deviation and rms of the noise signal. Example 1.3 in the Appendix illustrates how these formulae could be used in a simple Basic program. The Appendix also lists a more comprehensive set of useful statistical equations for your reference.

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \text{(1.4) Mean Value}$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} \left(x_i - \mu\right)^2} \qquad \text{(1.5) Standard Deviation}$$

$$RMS = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2} \qquad \text{(1.6) RMS}$$

Equations 1.4, 1.5, 1.6: Statistical Equations For A Discrete Population

One final concept to cover in this review is the addition of noise signals. In order to add two noise signals, you must know if the signals are correlated or uncorrelated. Noise signals from two independent sources are uncorrelated. For example, the noise from two independent resistors or two op amps is uncorrelated. A noise source can become correlated through a feedback mechanism. Noise-canceling headphones are a good example of the addition of correlated noise sources. They cancel acoustic noise by summing inversely-correlated noise. Equation 1.7 shows how to add correlated noise signals. Note that in the case of the noise-canceling headphones the correlation factor would be C= -1.

$$e_{nT} = \sqrt{e_{n1}^2 + e_{n2}^2 + 2 \cdot C \cdot e_{n1} \cdot e_{n2}}$$

Addition of two correlated noise sources

Equation 1.7: Addition Of Random Correlated Signals

$$e_{nT} = \sqrt{e_{n1}^2 + e_{n2}^2}$$

Normally noise sources are uncorrelated (i.e. C=0)
This equation is used for uncorrelated noise source

Equation 1.8: Addition Of Random Uncorrelated Signals

In most cases we will add uncorrelated noise sources (see Equation 1.8). Adding noise in this form is effectively summing two vectors using the Pythagorean Theorem. Figure 1.5 shows the addition graphically. A useful approximation is that if one of these sources is one-third the amplitude of the other, the smaller source can be ignored.

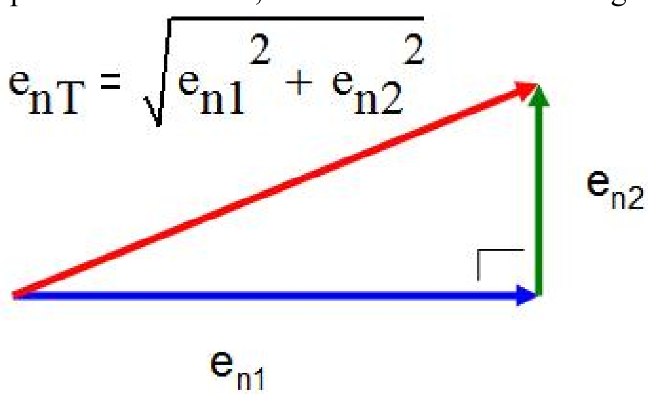

Fig. 1.5: Pythagorean Theorem For Noise

Summary And Preview

This part of the noise series introduced the concept of noise and reviewed some of the statistical fundamentals necessary to perform noise analysis. These fundamentals will be used throughout this series. Part II of this series will introduce the noise model of the op amp and describe some methods for calculating total output noise.

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Appendix 1.1

Example 1: This example shows a mathematical calculation where rms is not equal to standard deviation. In general, the standard deviation and rms are not equal when there is a dc component (ie non-zero mean value).

Example where $RMS \neq STDEV$

$$g(t) := \sin(t) + 0.3$$

$$\mu := \frac{1}{2\pi - 0}\int_0^{2\pi} g(t)\,dt \qquad \mu = 0.3$$

Variance for a Discrete Statistical Population

$$var := \frac{1}{2\pi - 0}\int_0^{2\pi} (g(t) - \mu)^2\,dt \qquad var = 0.5$$

Standard deviation for a Discrete Statistical Population

$$\sigma := \sqrt{var} \qquad \sigma = 0.707$$

Root Mean Squared (RMS) for a Discrete Statistical Population
This is the same as σ if $\mu = 0$

$$RMS := \sqrt{\frac{1}{2\pi - 0}\left(\int_0^{2\pi} g(t)^2\,dt\right)} \qquad RMS = 0.768$$

$$RMS = \sqrt{\sigma^2 + \mu^2} \quad \text{So} \quad \sigma = \sqrt{RMS^2 - \mu^2}$$

$$\sigma := \sqrt{RMS^2 - \mu^2} \qquad \sigma = 0.707$$

Appendix 1.2

Example 2: This example shows a mathematical calculation where rms is equal to standard deviation. In general, the standard deviation and rms are equal when there is a no DC component (i.e., zero mean value).

Example where RMS = STDEV

$g(t) := \sin(t)$

$$\mu := \frac{1}{2\pi - 0}\int_0^{2\pi} g(t)\,dt \qquad \mu = 0$$

Variance defined for a Probability Distribution Function

$$var := \frac{1}{2\pi - 0}\int_0^{2\pi} (g(t) - \mu)^2\,dt \qquad var = 0.5$$

Standard deviation defined for a Probability Distribution Function

$$\sigma := \sqrt{var} \qquad \sigma = 0.707$$

Root Mean Squared (RMS) defined for a Probability Distribution Function
This is the same as σ if $\mu = 0$

$$RMS := \sqrt{\frac{1}{2\pi - 0}\left(\int_0^{2\pi} g(t)^2\,dt\right)} \qquad RMS = 0.707$$

Appendix 1.3

```
Dim x(5) as double                          'x() is an array of measured voltages
Dim N as integer                            'N is the size of the population
Dim Sum, Sum_Sqr, Sum_Sigma as double       'collects the sum
Dim Average, RMS, Sigma as double           'results we are calculating

x(1) = 1.2: x(2) = 0.8: x(3) = 1.8: x(4) = 0.7: x(5) = 1.2: N = 5

For i = 1 to N
   Sum = Sum + x(i)
   Sum_Sqr = Sum_Sqr + (x(i)) ^ 2
Next i
Average = Sum / N
RMS = (Sum_Sqr / N) ^ 0.5
For i = 1 to N
   Sum_Sigma = Sum_Sigma + (x(i) - Average) ^ 2
Next i
Sigma = (Sum_Sigma / N) ^ 0.5

Print "Average= "; Average
Print "Standard Deviation= "; Sigma
Print "RMS= "; RMS
```

```
Result of run
Average=   1.14
Standard Deviation=   0.387814
RMS=   1.20416
```

Example 3: Basic Program Used To Implement Mean, Standard Deviation And Rms

Mean defined for a Probability Distribution Function

$$\mu = \int_{-\infty}^{\infty} (x)f(x)\,dx$$ (1) Continuous form

$$\mu = \sum_{x=-\infty}^{\infty} [(x)\cdot f(x)]$$ (2) Discrete form

Variance defined for a Probability Distribution Function

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$$ (3) Continuous form

$$\sigma^2 = \sum_{x=-\infty}^{\infty} \left[(x-\mu)^2\cdot f(x)\right]$$ (4) Discrete form

Standard deviation defined for a Probability Distribution Function

$$\sigma = \sqrt{\sigma^2} = \sqrt{\int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx}$$ (5) Continuous form

$$\sigma = \sqrt{\sigma^2} = \sqrt{\sum_{x=-\infty}^{\infty} \left[(x-\mu)^2\cdot f(x)\right]}$$ (6) Discrete form

Root Mean Squared (RMS) defined for a Probability Distribution Function
This is the same as σ if $\mu = 0$

$$RMS = \sqrt{\int_{-\infty}^{\infty} x^2 f(x)\,dx}$$ (7) Continuous form

$$RMS = \sqrt{\sum_{x=-\infty}^{\infty} \left(x^2\cdot f(x)\right)}$$ (8) Discrete form

Appendix 1.4

Statistical Equations Using the Probability Distribution Function:

Mean defined for a Probability Distribution Function

$$\mu = \int_{-\infty}^{\infty} (x) f(x)\, dx$$ (1) Continuous form

$$\mu = \sum_{x=-\infty}^{\infty} [(x) \cdot f(x)]$$ (2) Discrete form

Variance defined for a Probability Distribution Function

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$ (3) Continuous form

$$\sigma^2 = \sum_{x=-\infty}^{\infty} \left[(x-\mu)^2 \cdot f(x)\right]$$ (4) Discrete form

Standard deviation defined for a Probability Distribution Function

$$\sigma = \sqrt{\sigma^2} = \sqrt{\int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx}$$ (5) Continuous form

$$\sigma = \sqrt{\sigma^2} = \sqrt{\sum_{x=-\infty}^{\infty} \left[(x-\mu)^2 \cdot f(x)\right]}$$ (6) Discrete form

Root Mean Squared (RMS) defined for a Probability Distribution Function
This is the same as σ if $\mu = 0$

$$\text{RMS} = \sqrt{\int_{-\infty}^{\infty} x^2 f(x)\, dx}$$ (7) Continuous form

$$\text{RMS} = \sqrt{\sum_{x=-\infty}^{\infty} \left(x^2 \cdot f(x)\right)}$$ (8) Discrete form

Appendix 1.5

Statistical Equations Using For Measured Data

Mean defined for a Discrete Statistical Population

$$\mu = \frac{1}{b-a}\int_a^b g(t)\,dt$$ (9) Continuous form

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$$ (10) Discrete form

Variance defined for a Probability Distribution Function

$$\sigma^2 = \frac{1}{b-a}\int_a^b (g(t)-\mu)^2\,dt$$ (11) Continuous form

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} (x_i-\mu)^2$$ (12) Discrete form

Standard deviation defined for a Probability Distribution Function

$$\sigma = \sqrt{\frac{1}{b-a}\int_a^b (g(t)-\mu)^2\,dt}$$ (13) Continuous form

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (x_i-\mu)^2}$$ (14) Discrete form

Root Mean Squared (RMS) defined for a Probability Distribution Function
This is the same as σ if $\mu = 0$

$$RMS = \sqrt{\frac{1}{b-a}\left(\int_a^b g(t)^2\,dt\right)}$$ (15) Continuous form

$$RMS = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}$$ (16) Discrete form