Analysis And Measurement Of Intrinsic Noise In Op Amp Circuits
Part III: Resistor Noise And Sample Calculations

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In part II we developed a method for converting the noise spectral density curves from a product data sheet to noise sources in an op amp model. In this part we will learn how to use the model to compute the total output noise for a simple op amp circuit. The total noise referred-to-input (RTI) will contain noise from the op amp voltage noise source, noise from the op amp current noise source, and resistor noise. This combined noise source will be multiplied by the op amp *noise gain*. Fig. 3.1 shows all the different sources, to be combined and multiplied by the noise gain.

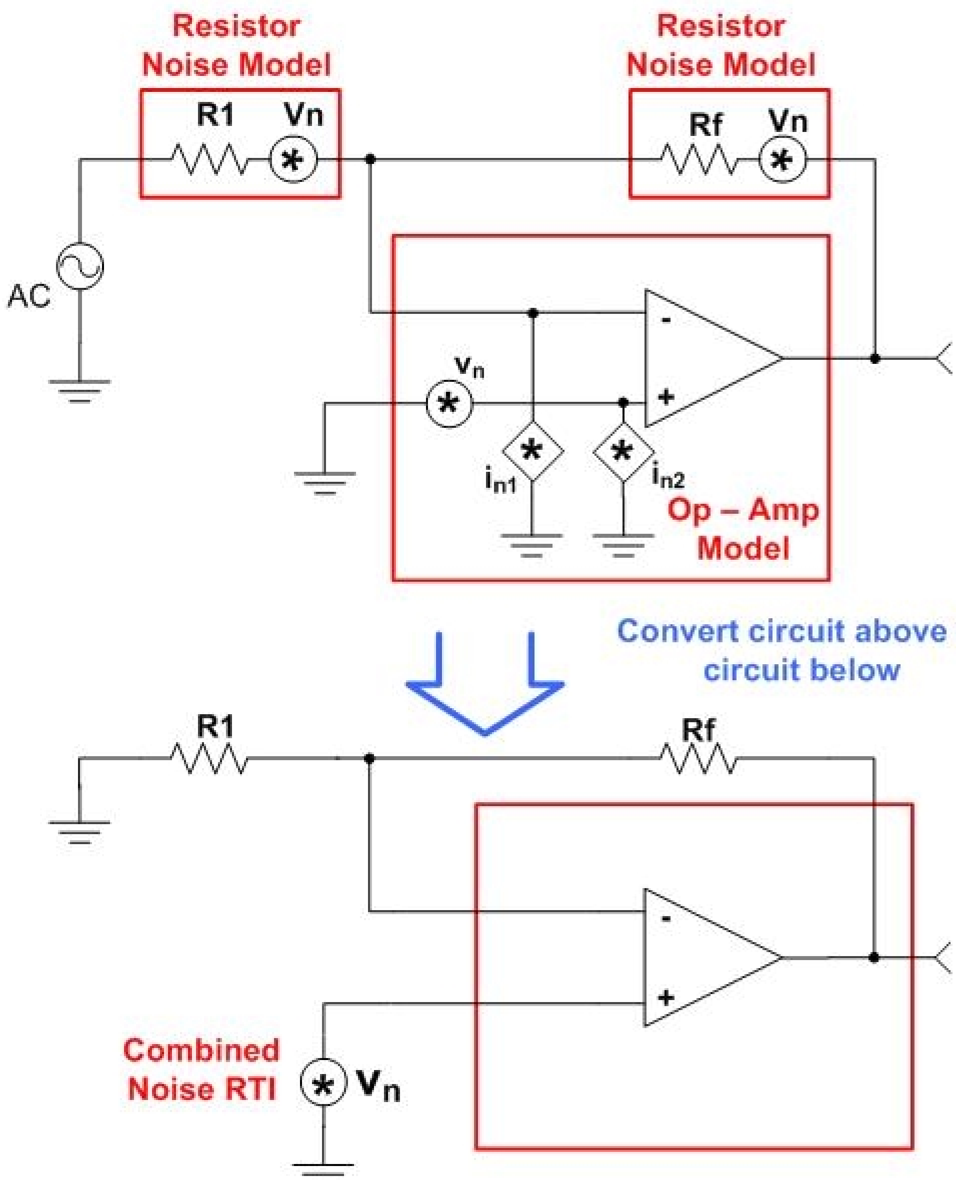

Fig. 3.1: Combine The Noise Sources

Noise gain is the gain that the op amp circuit sees to the total noise referred-to-input (RTI). In some cases this is not equivalent to the signal gain. Fig. 3.2 shows an example where the signal gain is one and the noise gain is two. The Vn source represents contributions of noise from several sources. Note that it is common engineering practice to lump all of the noise sources to a common source at the non-inverting input. Our end goal is to compute noise referred-to-output (RTO) of our op amp circuit.

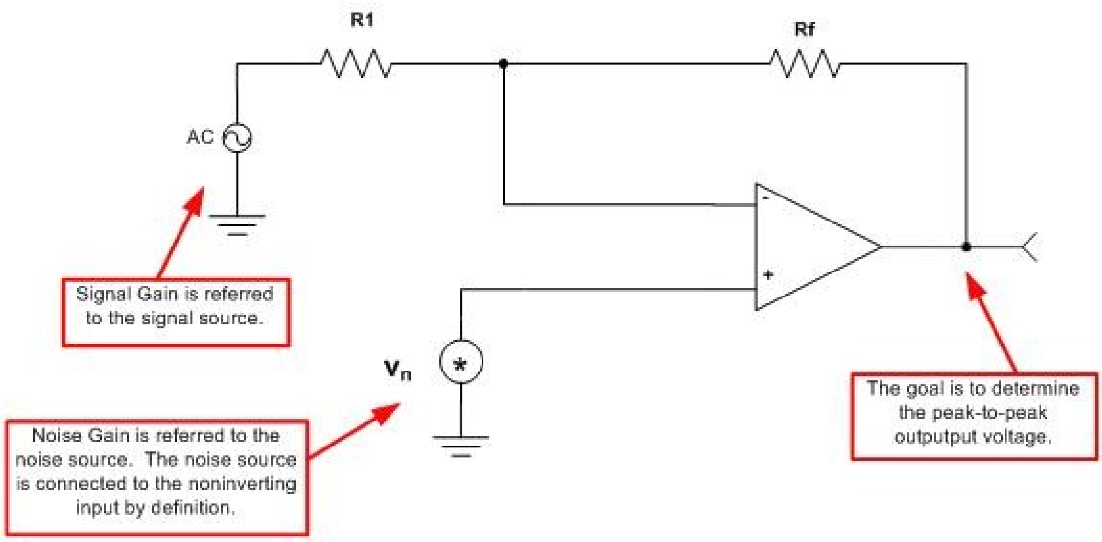

Fig. 3.2: Noise Gain Vs Signal Gain

$$\text{Noise\_Gain} = \frac{R_f}{R_1} + 1$$

Eq 3.1

From the previous article we know how to compute the voltage noise input, but how do we convert the current noise sources to a voltage noise source? One way to do this is to do an independent nodal analysis for each current source and use superposition to sum the results. Be careful to make sure that the results from each current source is added using the root sum of the squares (RSS). Equations 3.2 and 3.3 allow you to convert current noise to an equivalent voltage noise source for a simple op amp circuit. Fig. 3.3 shows this graphically. The full derivation for this circuit is given in the Appendix 3.1.

$$e_{n\_i} = i_n \cdot R_{eq}$$

$$R_{eq} = R_1 \parallel R_f$$

Eqs 3.2 - 3.3

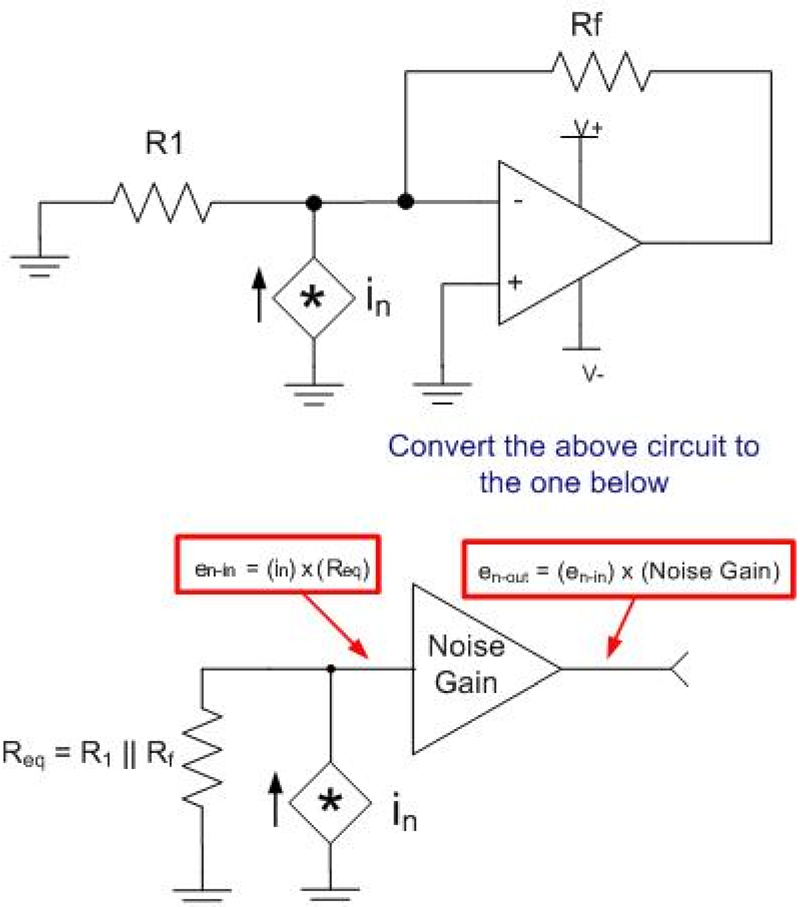

Fig. 3.3: Convert Current Noise To Voltage Noise (Equivalent Circuit)

Another thing that must be considered is the thermal voltage noise from the resistors in the op amp circuit. These voltage sources can be independently analyzed using a nodal analysis. The results are combined using superposition and RSS addition. Equations 3.4 and 3.5 allow you to combine all the thermal noise sources into a single noise source referred to the input. This noise input referred thermal noise source is expressed as an equivalent resistor. Fig. 3.4 shows this graphically. The full derivation for this circuit is given in Appendix 3.2.

$$R_{eq} = R_1 \parallel R_f$$

$$e_{nr} = \sqrt{4 \cdot k \cdot T \cdot R_{eq} \cdot \Delta f}$$

Eqs 3.4 - 3.5

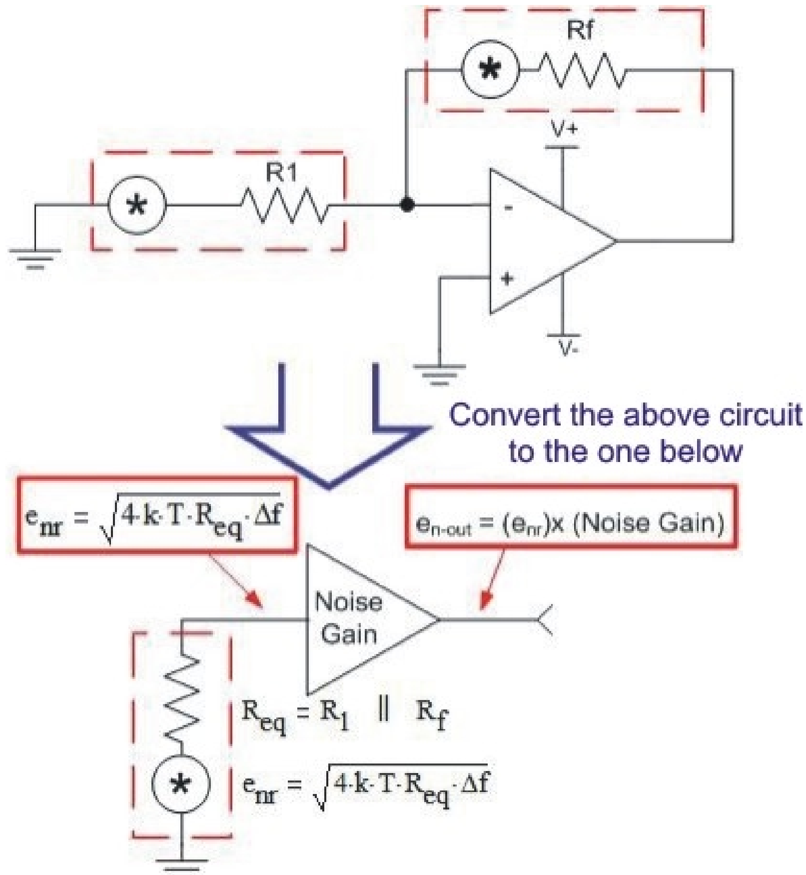

Fig. 3.4: Thermal Noise RTI For Simple Op Amp Circuit (Equivalent Circuit)

The final step to computing noise is to combine all these noise sources and multiply by the noise gain to compute the output noise. This rms noise is typically used to estimate the peak-to-peak value by multiplying by six. Recall from Part I that there is a 99.7% chance that any instantaneous noise measurement will be less then six times the rms noise. Equations 3.6, 3.7, and 3.8 summarize this final step.

$$e_{n\_in} = \sqrt{e_{n\_i}^{\;2} + e_{n\_v}^{\;2} + e_{n\_r}^{\;2}} \qquad \text{Eq 3.6}$$

$$e_{n\_out} = e_{n\_in} \cdot Noise\_Gain \qquad \text{Eq 3.7}$$

$$e_{n\_out\_pp} = e_{n\_out} \cdot 6.0 \quad \text{For +/- } 3\sigma \qquad \text{Eq 3.8}$$

Example Calculation

At this point, finally, we are ready to go through a real world example. Sometimes engineers are overwhelmed by the amount of work required to get to this point. In fact, it is possible to use simulation software to do some of this difficult work for you. However, it is important to have an understanding of the theoretical background because it will give you a more intuitive understanding of how noise works. Furthermore, you should always do a quick *back-of-the-envelope* calculation before you simulate a circuit so that you know if your simulation result is correct. In Part 4 we will discuss how to do this analysis using a SPICE simulator package.

Fig. 3.5 illustrates the simple op amp configuration that will be used for this analysis example. Note that the specifications used in this example were taken from the OPA627 data sheet, downloadable at the Texas Instruments web site. http://www.ti.com

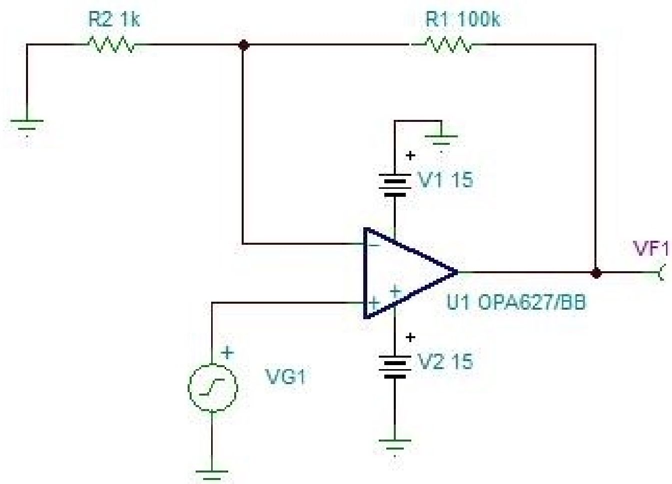

Fig. 3.5: Example Circuit

The first step to this analysis is to determine the noise gain and noise bandwidth for this circuit. The noise gain is given by Equation 3.2 (Noise_Gain = Rf/R1 + 1 = 100 kΩ/1 kΩ +1 = 101). The signal bandwidth is limited by the closed-loop bandwidth of the op amp. Using the unity-gain bandwidth from the data sheet, the closed-loop bandwidth can be determined using Equation 3.9. If the gain-bandwidth product is not listed in the data sheet, use the unity-gain bandwidth specification -- which is the same for unity-gain stable amplifiers.

$$\text{Closed\_Loop\_Bandwidth} = \frac{\text{Gain\_Bandwidth\_Product}}{\text{Noise\_Gain}} \qquad (3.9)$$

$$\text{Closed\_Loop\_Bandwidth} = \frac{16\text{MHz}}{101} = 158\text{kHz}$$

Eq 3.9

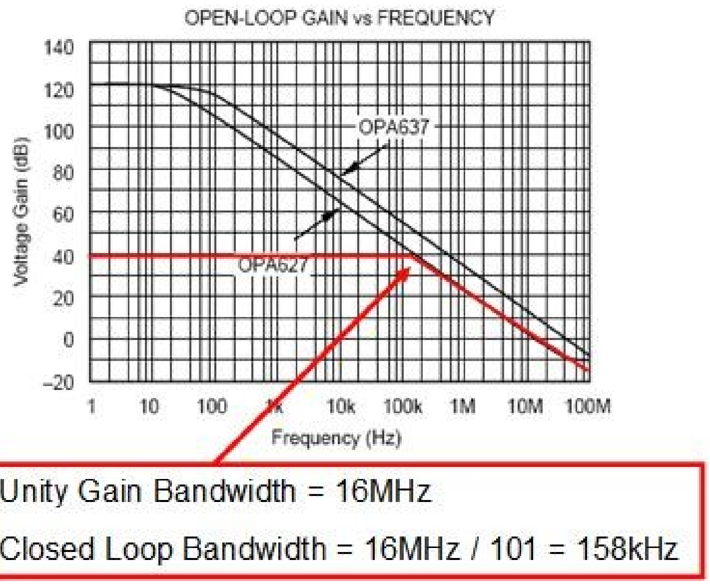

Fig. 3.6: Closed-Loop Bandwidth For Simple Non-Inverting Amp

The next part of the analysis is to get the broadband and 1/f noise spectral density specification from the data sheet. The specification is sometimes shown graphically (see Fig. 3.7) or in a table format (see Fig. 3.8). The spectral density values and the closed-loop bandwidth are used to compute the total input voltage noise. Example 3.1 shows how the total input noise is computed using the formulas introduced previously.

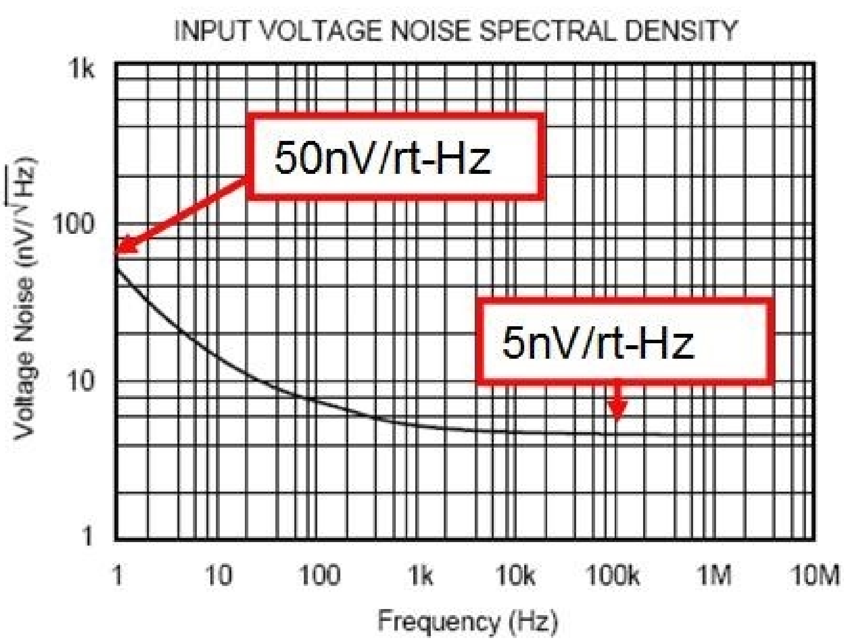

Fig. 3.7: OPA627 Noise Spectral Density Specification To Be Used In Calculations

	OPA627AM, AP, AU OPA637AM, AP, AU			
	MIN	TYP	MAX	UNITS
NOISE				
Input Voltage Noise				
Noise Density: f = 10Hz		20		nV/√Hz
f = 100Hz		10		nV/√Hz
f = 1kHz		5.6		nV/√Hz
f = 10kHz		4.8		nV/√Hz
Voltage Noise, BW = 0.1Hz to 10Hz		0.8		µVp-p
Input Bias Current Noise				
Noise Density, f = 100Hz		2.5		fA/√Hz
Current Noise, BW = 0.1Hz to 10Hz		48		fAp-p

Fig. 3.8: OPA627 Noise Spectral Density Specifications

Broadband Voltage Noise Component:

$$BW_n = f_H \cdot K_n \qquad (2.2)$$

$$BW_n = (158kHz)\cdot(1.57) = 248kHz$$ From Table 2.1 and Figure 3.6 (1.57 for single pole roll-off)

$$e_{nBB} = e_{BB} \cdot \sqrt{BW_n} \qquad (2.3)$$

$$e_{nBB} = \left(5\,\frac{nV}{\sqrt{Hz}}\right)\cdot\sqrt{248kHz} = 2490nVrms$$ From Figure 3.8

1/f Voltage Noise Component:

$$e_{fnorm} = e_{at\_f} \cdot \sqrt{f} \qquad (2.4)$$

$$e_{fnorm} = \left(50\,\frac{nV}{\sqrt{Hz}}\right)\cdot\sqrt{1Hz} = 50nV$$ From Figure 3.8

$$e_{nf} = e_{fnorm} \cdot \sqrt{\ln\left(\frac{f_H}{f_L}\right)} \qquad (2.5)$$

$$e_{nf} = (50nV)\cdot\sqrt{\ln\left(\frac{248kHz}{0.1Hz}\right)} = 191.8nVrms$$ We use f_H = BWN and f_L = 0.1Hz (typical number)

Total Voltage Noise (referred to the input of the amplifier):

$$e_{n\_v} = \sqrt{{e_{nf}}^2 + {e_{nBB}}^2} \qquad (2.6)$$ From calculations above

$$e_{n\_v} = \sqrt{(2490nVrms)^2 + (191.8nVrms)^2} = 2497nVrms$$

Example 3.1: Compute Magnitude Of Voltage Noise Referred To Input

Next we need to convert the current noise to an equivalent input referred voltage noise. First we will convert the current noise spectral density to a current source which is multiplied by an equivalent input resistance to compute input voltage noise. It should be noted that the 1/f calculation is not required for this example because the amplifier is a J-FET input. J-FET amplifiers generally do not have 1/f current noise. This procedure is summarized in Example 3.2. Note that the equations for all parts of this sample calculation are summarized in Appendix 3.1. The summary in the appendix shows the case where current noise does have a 1/f region.

Broadband Current Noise Component:

$$BW_n = f_H \cdot K_n \qquad (2.2)$$

$$BW_n = (158kHz)\cdot(1.57) = 248kHz$$ From Table 2.1 and Figure 3.6

$$i_{nBB} = i_{BB}\cdot\sqrt{BW_n} \qquad (2.3)$$

$$i_{nBB} = \left(2.5\frac{fA}{\sqrt{Hz}}\right)\cdot\sqrt{248kHz} = 1.244pArms$$ From Figure 3.8

$i_n = i_{nBB}$ For this case there is no 1/f noise

$$R_{eq} = R_1 \parallel R_f \qquad (3.3)$$

$$R_{eq} = \frac{R_f \cdot R_1}{R_f + R_1} = \frac{(100k\Omega)\cdot(1k\Omega)}{100k\Omega + 1k\Omega} = 0.99k\Omega$$

$$e_{n\_i} = i_n \cdot R_{eq} \qquad (3.2)$$

$$e_{n\_i} = (1.244pArms)\cdot(0.99k\Omega) = 1.23nVrms$$

Since the total Voltage noise is e_{n_v} = 2497nVrms, the current noise voltage (1.23nVrms) will be negligible.

Example 3.2: Convert Current Noise Spectral Density to Equivalent Input Noise Voltage

Example 3.3 illustrates how input referred resistor noise is calculated. Note that for this example the resistor noise is similar in magnitude to the op amp noise and so it will significantly contribute to the output noise.

Thermal Noise (Resistor Noise) Component:

$$BW_n = f_H \cdot K_n \qquad (2.2)$$

$$BW_n = (158kHz)\cdot(1.57) = 248kHz$$ From Table 2.1 and Figure 3.6

$$R_{eq} = R_1 \,||\, R_f \qquad (3.3)$$

$$R_{eq} = \frac{R_f R_1}{R_f + R_1} = \frac{(100k\Omega)\cdot(1k\Omega)}{100k\Omega + 1k\Omega} = 0.99k\Omega$$

$$e_{n\_r} = \sqrt{4kT \cdot R_{eq} \cdot \Delta f} \qquad (3.5 \text{ or } 1.1)$$

$$e_{n\_r} = \sqrt{4\left(1.38\cdot 10^{-23}\right)\cdot(273 + 25)\cdot(0.99k)(248kHz)} = 2010nVrms$$

Since the total Voltage noise is e_{n_v} = 2497nVrms, the resister noise voltage (2010nVrms) is significant.

Example 3.3: Convert Resistor Noise To Equivalent Input Noise Voltage

Now that we have computed all the noise components we can determine the total noise referred-to-input (RTI). This result will be multiplied by the noise gain to compute the noise referred to the output. Finally, the conversion factor from Table 1.1 will be used to estimate the peak-to-peak output. Example 3.4 shows the details.

Voltage Noise from Op-Amp RTI:

$e_{n\_v} = 2497nVrms$ (From Example 3.1)

Current Noise from Op-Amp RTI:

$e_{n\_i} = 1.23nVrms$ (From Example 3.2)

Resistor Noise RTI:

$e_{n\_r} = 2010nVrms$ (From Example 3.3)

Compute Total RMS Noise RTI:

$$e_{n\_in} = \sqrt{e_{n\_v}^2 + e_{n\_i}^2 + e_{n\_r}^2} \quad (3.6)$$

$$e_{n\_in} = \sqrt{(2497nVrms)^2 + (1.23nVrms)^2 + (2010nVrms)^2} = 3205nVrms$$

Compute Total RMS Noise RTO:

$$e_{n\_out} = e_{n\_in} \cdot Noise\_Gain \quad (3.7)$$

$$e_{n\_out} = (3205nVrms) \cdot (101) = 324\mu Vrms$$

Estimate Total Peak-to-Peak Noise RTO:

$$e_{n\_out\_pp} = e_{n\_out} \cdot 6.0 \quad (3.8)$$

$$e_{n\_out\_pp} = (324\mu Vrms) \cdot 6.0 = 1.94mVpp$$ Final result!

Example 3.4: Compute Total Peak-To-Peak Output Noise

Summary And Preview

This part of the noise series completes the hand calculations for a simple op amp circuit. Using this technique we are able to predict the peak-to-peak output noise based on data sheet specifications. For the example circuit in this configuration we estimate that the peak-to-peak output noise will be 1.94 mVpp. We will return to this example in upcoming articles and verify that this is indeed an accurate estimate of the output noise through measurement and SPICE analysis.

Although the calculations shown were for a simple configuration, this technique can be used for more complex circuits. In the next section we will show how a circuit simulation software package (TINA SPICE) can be used to do noise analysis. It should be noted, however, that the hand analysis technique should always be performed before doing circuit simulation to give confidence that the simulation was done properly.

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- Tim Green, Applications Engineering Manager
- Neil Albaugh, Senior Applications Engineer

References

Robert V Hogg, and Elliot A Tanis, *Probability and Statistical Inference*, 3rd Edition, Macmillan Publishing Co

C. D. Motchenbacher, and J. A. Connelly, *Low-Noise Electronic System Design*, A Wiley-Interscience Publication

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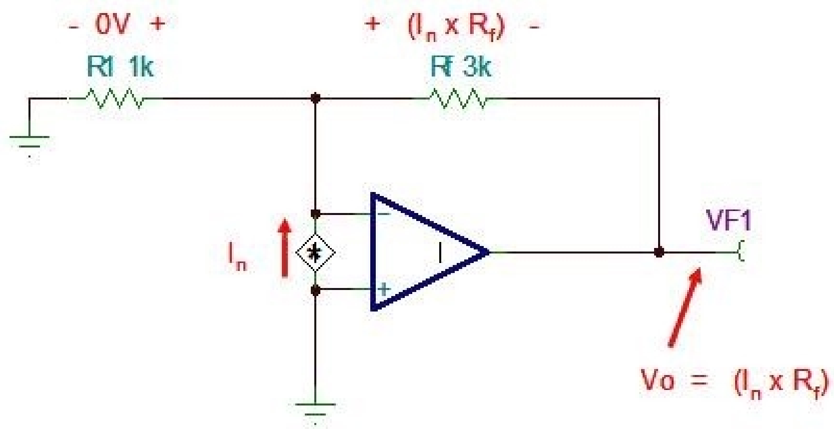
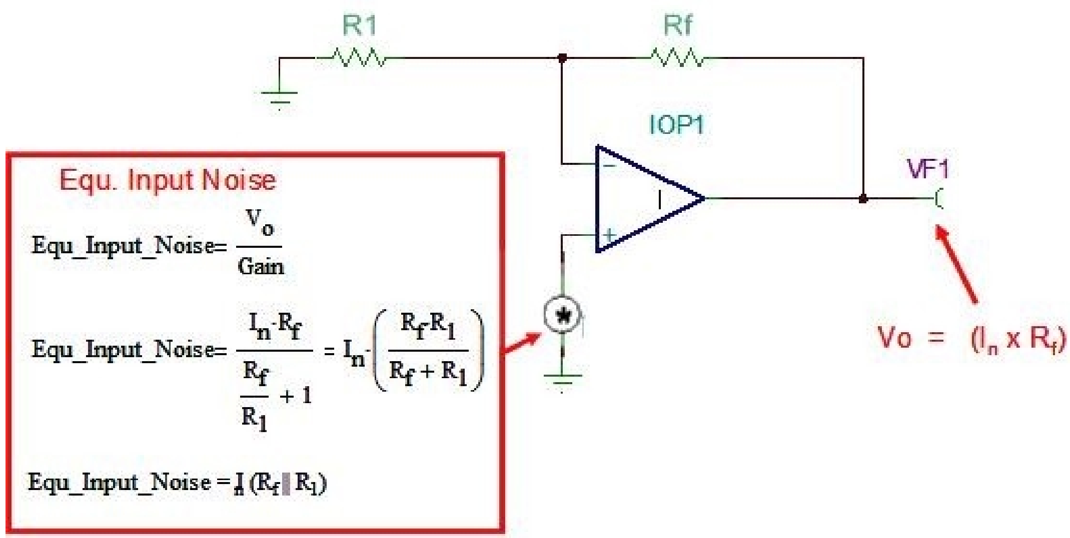
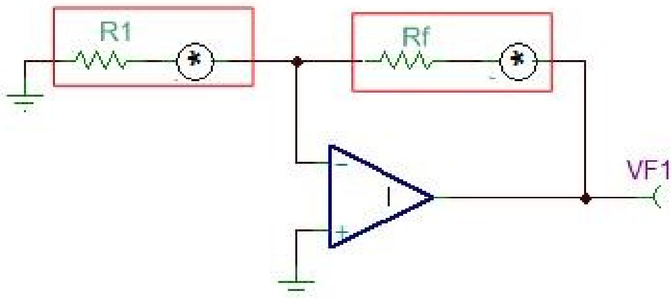
Appendix 3.1: Derivation Of Conversion Of Current Noise To Voltage Noise



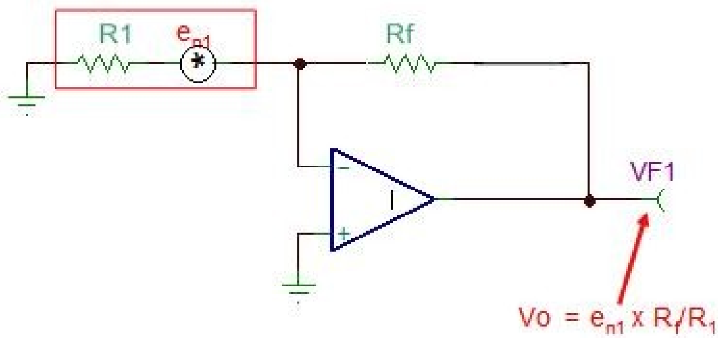


Appendix 3.2: Derivation Of Resistor Noise To Voltage Noise For Simple Op Amp

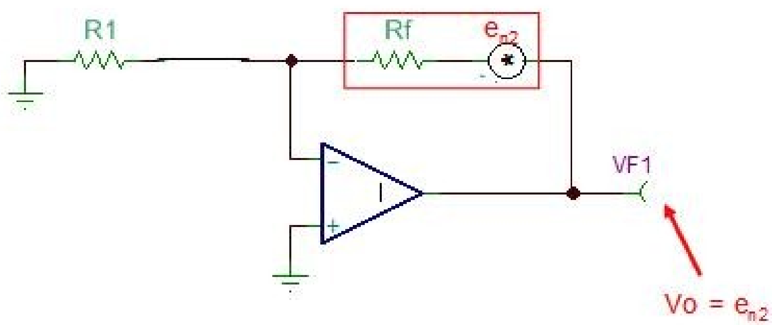
Equivalent Circuit



Use Superposition (R1)



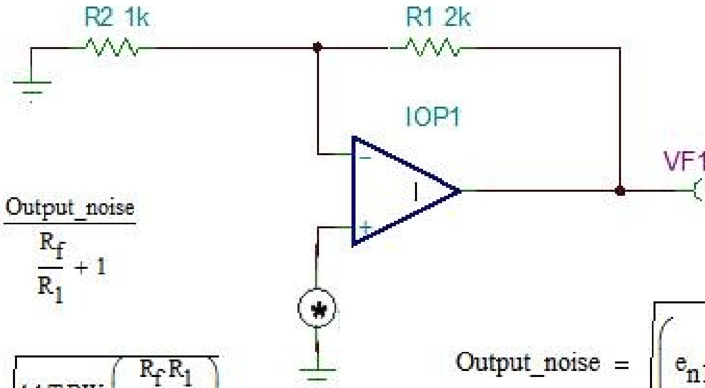
Use Superposition (Rf)



Appendix 3.2: Derivation Of Resistor Noise To Voltage Noise (Continued)

Add noise components and refer to the input.

Note: the input noise is equivalent to Rf || R1 (proof on next page)



$$\text{Input\_noise} = \frac{\text{Output\_noise}}{\frac{R_f}{R_1} + 1}$$

$$\text{Output\_noise} = \sqrt{\left(e_{n1} \cdot \frac{R_f}{R_1}\right)^2 + e_{n2}^{\,2}}$$

$$\text{Input\_noise} = \sqrt{4 \cdot k T \cdot BW \cdot \left(\frac{R_f \cdot R_1}{R_f + R_1}\right)}$$

Equivalent to noise of Rf || R1

Appendix 3.2: Derivation Of Resistor Noise To Voltage Noise (Continued)

$$\text{Output\_Noise}^2 = \sqrt{e_{n1}^2 + e_{n2}^2} = \sqrt{\left(e_{n1}\cdot\frac{R_f}{R_1}\right)^2 + e_{n2}^2}$$

Let $\beta = 4\cdot kT\cdot BW$

$$\text{Output\_noise} = \sqrt{\left(\sqrt{\beta\cdot R_1}\cdot\frac{R_f}{R_1}\right)^2 + \left(\sqrt{\beta\cdot R_f}\right)^2}$$

$$\text{Input\_noise} = \frac{\sqrt{\left(\sqrt{\beta\cdot R_1}\cdot\frac{R_f}{R_1}\right)^2 + \left(\sqrt{\beta\cdot R_f}\right)^2}}{\frac{R_f}{R_1} + 1}$$

$$\text{Input\_noise}^2 = \frac{\beta\cdot\frac{R_f^2}{R_1} + \beta\cdot R_f}{\frac{\left(R_f + R_1\right)^2}{R_1^2}} = \frac{\beta\cdot R_f^2\cdot R_1 + \beta\cdot R_f\cdot R_1^2}{\left(R_f + R_1\right)^2}$$

$$\text{Input\_noise} = \sqrt{\frac{\beta\cdot R_f^2\cdot R_1 + \beta\cdot R_f\cdot R_1^2}{\left(R_f + R_1\right)^2}} = \sqrt{\beta\cdot\frac{R_f\cdot R_1}{R_f + R_1}}$$

$$\text{Input\_noise} = \sqrt{4\cdot kT\cdot BW\cdot\left(\frac{R_f\cdot R_1}{R_f + R_1}\right)}$$ Equ to noise of Rf || R1

Appendix 3.3: Equations For Simple Op Amp Circuit (Voltage Noise)

Noise Gain for Simple Op-Amp:

$$\text{Noise\_Gain} = \frac{R_f}{R_1} + 1 \qquad (3.2)$$

Broadband / Gain Relationship:

$$\text{Closed\_Loop\_Bandwidth} = \frac{\text{Unitity\_Gain\_Bandwidth}}{\text{Noise\_Gain}} \qquad (3.8)$$

Broadband Voltage Noise Component:

$$BW_n = f_H \cdot K_n \qquad (2.2)$$

$$e_{nBB} = e_{BB} \cdot \sqrt{BW_n} \qquad (2.3)$$

1/f Voltage Noise Component:

$$e_{fnorm} = e_{at\_f} \cdot \sqrt{f} \qquad (2.4)$$

$$e_{nf} = e_{fnorm} \cdot \sqrt{\ln\left(\frac{f_H}{f_L}\right)} \qquad (2.5)$$

Total Voltage Noise (referred to the input of the amplifier):

$$e_{n\_v} = \sqrt{e_{nf}^{2} + e_{nBB}^{2}} \qquad (2.6)$$

From calculations above

Appendix 3.4: Equations For Simple Op Amp Circuit (Current Noise)

Broadband Current Noise Component:

$$i_{nBB} = i_{BB} \cdot \sqrt{BW_n} \qquad (2.3)$$

1/f Current Noise Component:

$$i_{fnorm} = i_{at\_f} \cdot \sqrt{f} \qquad (2.4)$$

$$i_{nf} = i_{fnorm} \cdot \sqrt{\ln\left(\frac{f_H}{f_L}\right)} \qquad (2.5)$$

Total Current Noise (referred to the input of the amplifier):

$$i_n = \sqrt{i_{nf}^2 + i_{nBB}^2} \qquad (2.6)$$

From calculations above

Convert Current to Voltage Noise

$$R_{eq} = R_1 \parallel R_f \qquad (3.3)$$

$$e_{n\_i} = i_n \cdot R_{eq} \qquad (3.2)$$

Appendix 3.5: Equations For Simple Op Amp Circuit (Resistor And Total Noise)

Thermal Noise (Resistor Noise) Component:

$$R_{eq} = R_1 \parallel R_f \qquad (3.3)$$

$$e_{n\_r} = \sqrt{4kT \cdot R_{eq} \cdot \Delta f} \qquad (3.5 \text{ or } 1.1)$$

where
k = 1.381E-23 joule/K
T = temperature in Kelvin $T_k = 273.15C + T_c$
Req = equivalent resistance for simple Op-Amp Circuit
Δf = Noise Bandwidth

Compute Total RMS Noise RTI:

$$e_{n\_in} = \sqrt{{e_{n\_v}}^2 + {e_{n\_i}}^2 + {e_{n\_r}}^2} \qquad (3.6)$$

Compute Total RMS Noise RTO:

$$e_{n\_out} = e_{n\_in} \cdot Noise\_Gain \qquad (3.7)$$

Estimate Total Peak-to-Peak Noise RTO:

$$e_{n\_out\_pp} = e_{n\_out} \cdot 6.0 \qquad (3.8)$$