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Space Vector Pulse Width Modulation Technique

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Abstract— This paper studies the space vector pulse width modulation technique (SVPWM) for the three-phase two position six switches voltage source inverter. Space vector pulse width modulation (SVPWM) provides a superior technique compared to the other PWM techniques. The SVPWM is easier digital realization, reduced harmonics, reduced switching losses and better dc bus utilization. In SVPWM the three phase quantities can be represented by a single complex vector. In this paper, the required parameters of SVPWM implementation such as time duration and the switching patterns of the inverter switches are discussed.

Keywords— SVPWM, Voltage source inverter, Complex reference voltage, Sectors, Switching states

I. INTRODUCTION

A completely different approach results from representing the three phase inverter output voltages in space vector system. In space vector system, the voltage is formed by a space vector of defined quantity and angle in the complex plane (x-y plane). The space vector modulation is recently reported digital PWM generation technique which is based on the concept of Park (space) vector representation of three phase waveforms [1]. On other words, the space vector method of analysis enables representation of the three phase quantities (voltages or currents) by a single complex vector. This method of analysis has been effectively used in analysis of the three phase machines and also of the three phase inverters. It also gives the possibility of several schemes of waveform optimization during transient and steady state operation. The advantages of space vector pulse width modulation are:

- 1) Higher value of the maximum fundamental output voltage as compared with
- 2) the suboscillation technique and various sampling techniques.
- 3) It does not carry much weight in high frequency PWM control.
- 4) Produces less harmonic distortion in the critical range where the ratio of switching frequency to fundamental frequency is low.

Generally, the space vector technique is essentially based around the decomposition of a reference voltage vector into voltage vectors realizable on a six pulse inverter although, space vector modulation is actually a special case of the triangulation technique [2].

II. DEFINITION OF 3-PHASE COMPOSITE VECTOR

When the space vector method is applied to three output voltages of the inverter bridge, a single vector of fixed length

is obtained. This vector occupies one of eight positions in x-y plane according to the switching state of the inverter. Fig. 1 shown a three-phase voltage source inverter model with six power transistors Q1 to Q6 that shape the output voltage exits across a balanced three phase star connected load such as AC motor.

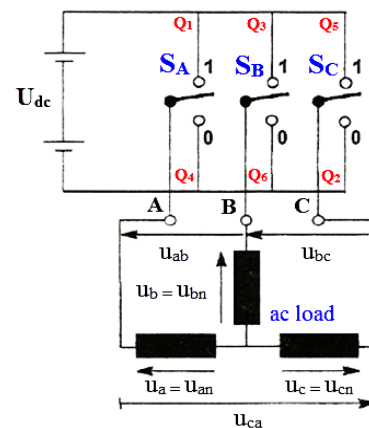


Fig. 1 Three phase star connected load

The individual phase-to-phase voltages U_{an} , U_{bn} , and U_{cn} appearing across each of the phase loads can be written as:

$$u_{an} = U_s \cos \omega t \quad (1)$$

$$u_{bn} = U_s \cos(\omega t - 2\pi/3) \quad (2)$$

$$u_{cn} = U_s \cos(\omega t - 4\pi/3) \quad (3)$$

It is well known that any three phase system may be transformed into an equivalent two phase system as follow:

$$u_\alpha = (2/3)[u_a - (1/2)u_b - (1/2)u_c] \quad (4)$$

and

$$u_{\beta} = (2/3)[(\sqrt{3}/2)u_b - (\sqrt{3}/2)u_c] \quad (5)$$

Where u_{α} and u_{β} together form an orthogonal instantaneous two phase ac set whose vector sum is equal to the vector \underline{u}_s as shown in Fig. 2.

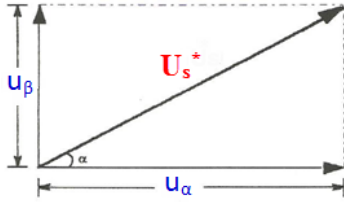


Fig. 2 Decomposition of space vector to u_{α} and u_{β} .

Where; \underline{u}_s is the composite vector and given by:

$$\underline{u}_s = u_{\alpha} - ju_{\beta} \quad (6)$$

Substituting from Equ. 4 and Equ. 5 in Equ. 6

$$\underline{u}_s = (2/3)[u_a - (1/2)u_b - (1/2)u_c] \quad (7)$$

$$+ j(\sqrt{3}/2)u_b - j(\sqrt{3}/2)u_c]$$

$$\underline{u}_s = (2/3)[u_a - au_b - a^2u_c] \quad (8)$$

Where

$$a = -(1/2) + j(\sqrt{3}/2) = e^{j\frac{2\pi}{3}} \quad (9)$$

$$a^2 = -(1/2) - j(\sqrt{3}/2) = e^{j\frac{4\pi}{3}} \quad (10)$$

By using Equ. 9 and Equ. 10, the composite vector becomes:

$$\underline{u}_s = (2/3)(u_a + u_b(t)e^{j\frac{2\pi}{3}} + u_c(t)e^{j\frac{4\pi}{3}}) \quad (11)$$

Considering the states for the inverter switches (inverter transistors) shown in Fig. 1 and combining them yields the eight possible switching states listed in Table 1, each of these eight states is defined by the positions of the three transistors (or switches) according to the switching sequence. When an upper transistor is switched ON, the corresponding lower transistor is switched OFF. Therefore, the ON and OFF states of the upper transistors Q1, Q3 and Q5 can be used to determine the output voltage.

TABLE I

List of switching states according to the conducting transistors

| \underline{u}_s | Q ₁ | Q ₃ | Q ₅ |
|-------------------|----------------|----------------|----------------|
|-------------------|----------------|----------------|----------------|

| | | | |
|-------------------|---|---|---|
| \underline{u}_0 | 4 | 6 | 2 |
| \underline{u}_1 | 1 | 6 | 2 |
| \underline{u}_2 | 1 | 3 | 2 |
| \underline{u}_3 | 4 | 3 | 2 |
| \underline{u}_4 | 4 | 3 | 5 |
| \underline{u}_5 | 4 | 6 | 5 |
| \underline{u}_6 | 1 | 6 | 5 |
| \underline{u}_7 | 1 | 3 | 5 |

Thus, The inverter has six states when a voltage is applied to the motor and two states when the motor is shorted through the upper or lower transistors resulting in zero volts being applied to the motor as shown in Fig. 3.

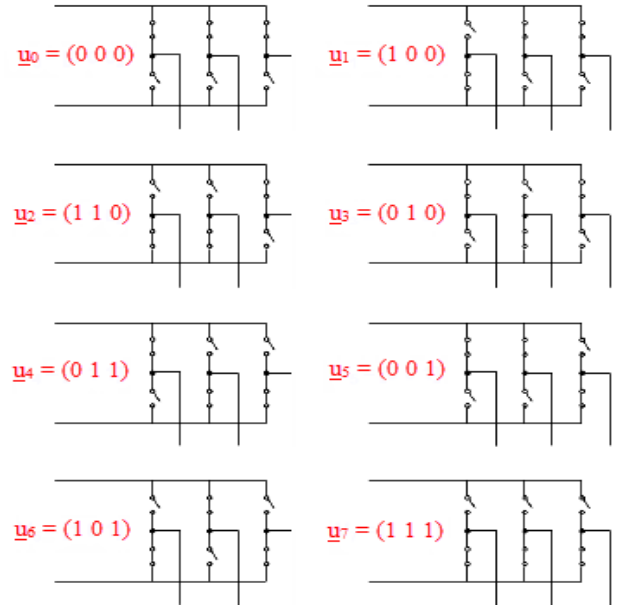


Fig. 3 The eight inverter switching states

III. POSSIBLE SPACE VECTOR POSITIONS

As shown in Fig.(3), the possible space vector positions of three phase inverter output can be evaluated according to the states of inverter switches and the corresponding values of phase voltages (u_a , u_b , and u_c) by using Equ. 9 as follow:

- For $u_1 = [100]$ i.e. transistors 612 ON:

$$\underline{u}_1 = (2/3)[(2/3)U_{dc} - (a/3)U_{dc} - (a^2/3)u_{dc}] \quad (12)$$

By using the values of a and a^2 from Equ. 9 and Equ. 10, the vector u_1 becomes:

$$\underline{u}_1 = \frac{2}{3}U_{dc} \angle 0^\circ \quad (13)$$

-For $u_2 = [110]$ i.e. transistors 132 ON :

$$\underline{u}_2 = (2/3)[(1/3)U_{dc} - (a/3)U_{dc} - (2a^2/3)u_{dc}] \quad (14)$$

Hence

$$\underline{u}_2 = \frac{2}{3}U_{dc} \angle 60^\circ \quad (15)$$

Similarly the magnitude and angle of space vector for all possible switching states becomes:

-For $u_3 = [010]$ i.e. transistors 432 ON:

$$\underline{u}_3 = \frac{2}{3}U_{dc} \angle 120^\circ \quad (16)$$

-For $u_4 = [011]$ i.e. transistors 435 ON:

$$\underline{u}_4 = \frac{2}{3}U_{dc} \angle 180^\circ = -\underline{u}_1 \quad (17)$$

-For $u_5 = [001]$ i.e. transistors 465 ON:

$$\underline{u}_5 = \frac{2}{3}U_{dc} \angle 240^\circ = -\underline{u}_2 \quad (18)$$

-For $u_6 = [101]$ i.e. transistors 165 ON:

$$\underline{u}_6 = \frac{2}{3}U_{dc} \angle 300^\circ = -\underline{u}_3 \quad (19)$$

-For $u_0 = [000]$ and $u_7 = [111]$ i.e. transistors 462 and 135 ON respectively:

$$\underline{u}_0 = \underline{u}_7 = 0 \quad (20)$$

Then, the output of the 3-phase inverter can be represented by one vector occupy six position (u_1 to u_6) in the space according to the time instant and two zero vectors (u_0 and u_7) as shown in Table 2, which summaries the switching vectors along with the corresponding line to neutral voltage and line to line voltages applied to the motor.

TABLE III
Switching vectors, Phase voltages and Output
Line to Line voltages

| Voltage Vectors | Switching Vectors | | | Line to neutral voltage | | | Line to line voltage | | |
|-------------------|-------------------|----------------|----------------|-------------------------|-----------------|-----------------|----------------------|-----------------|-----------------|
| | S ₁ | S ₂ | S ₃ | U _{an} | U _{bn} | U _{cn} | U _{ab} | U _{bc} | U _{ca} |
| \underline{u}_0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \underline{u}_1 | 1 | 0 | 0 | 2/3 | -1/3 | -1/3 | 1 | 0 | -1 |
| \underline{u}_2 | 1 | 1 | 0 | 1/3 | 1/3 | -2/3 | 0 | 1 | -1 |
| \underline{u}_3 | 0 | 1 | 0 | -1/3 | 2/3 | -1/3 | -1 | 1 | 0 |
| \underline{u}_4 | 0 | 1 | 1 | -2/3 | 1/3 | 1/3 | -1 | 0 | 1 |
| \underline{u}_5 | 0 | 0 | 1 | -1/3 | -1/3 | 2/3 | 0 | -1 | 1 |
| \underline{u}_6 | 1 | 0 | 1 | 1/3 | -2/3 | 1/3 | 1 | -1 | 0 |
| \underline{u}_7 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |

IV. PRINCIPLE OF SPACE VECTOR

The object of the three phase inverter bridge is to synthesize a balanced set of three phase ac voltage at its output terminals from a constant dc voltage supply. This subsection describes how the inverter switches are operated upon to produce different values for the space vector of stator voltage thereby ensuring a desired three phase output waveforms. In the three phase two position inverter, each leg consists of two switches which may not be closed (or opened) simultaneously. The output of each inverter leg may therefore be either at the dc link voltage (when the top switch is on) or zero (when the bottom switch is on) [3]. Considering these two states for each inverter leg and combining them yields the eight possible switching states. Each of these eight states is defined by the positions of the switches (S_1 , S_2 , and S_3) and a "1" indicates a closed position while a "0" indicates an open position as shown in Fig. 4. On the other hand, the eight states is defined by the case of each transistor in each arm of the inverter bridge, switched-on or switched-off.

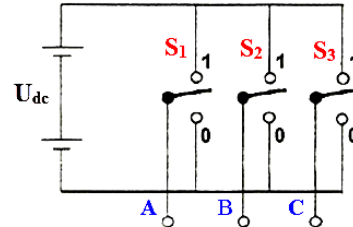


Fig. 4 Switching representation of three phase two position inverter

Consequently, the output of the six pulse three phase inverter bridge has eight possible output vectors depending on the possible switching states as shown in Table 3.

TABLE III
List of switching states

| \underline{u}_s | S ₁ | S ₂ | S ₃ |
|-------------------|----------------|----------------|----------------|
| \underline{u}_0 | 0 | 0 | 0 |
| \underline{u}_1 | 1 | 0 | 0 |
| \underline{u}_2 | 1 | 1 | 0 |
| \underline{u}_3 | 0 | 1 | 0 |
| \underline{u}_4 | 0 | 1 | 1 |

| | | | |
|-------------------|---|---|---|
| \underline{u}_5 | 0 | 0 | 1 |
| \underline{u}_6 | 1 | 0 | 1 |
| \underline{u}_7 | 1 | 1 | 1 |

In other words, Two of these possible output voltage vectors (\underline{u}_0 and \underline{u}_7) are null or zero vectors in which all three phases are equal to (000 or 111), while the six remaining vectors ($\underline{u}_1, \dots, \underline{u}_6$) are non-zero values and all spatially separated by 60° as shown in Fig. 5.

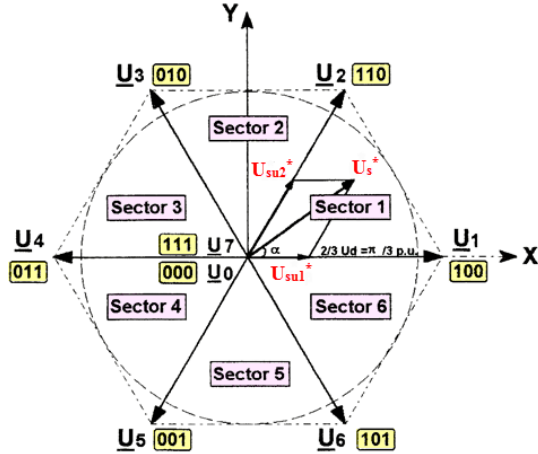


Fig. 5 The Desired space vector \underline{u}_s^* and the possible space vectors of the inverter output
Each of these space vectors of the stator voltage is defined by the formula:

$$\underline{U}_s(t) = \frac{2}{3} (u_{s1}(t) + u_{s2}(t)e^{j\frac{2\pi}{3}} + u_{s3}(t)e^{j\frac{4\pi}{3}}) \quad (21)$$

The Stepped waveform of the output phase voltage of the three phase voltage source inverter driving three phase load without any further modulation is shown in Fig. 6.

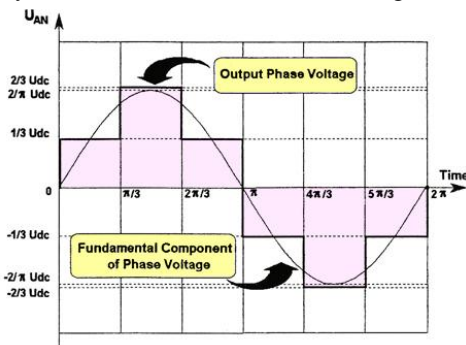


Fig. 6 Output phase voltage of two position six pulses inverter.

This waveform gives amplitude of the fundamental phase voltage of $(2/\pi) U_{dc}$, where U_{dc} is the total dc voltage produced from the rectifier bridge. This value $(2/\pi) U_{dc}$ is used as the base value in normalizing the voltage quantities included in the system. As a consequence the length of every output space vector in per unit is given by:

$$U_{1 \text{ p.u.}} = (2/\pi U_{dc}) / (2/\pi U_{dc}) = \pi/3 \quad (22)$$

To ensure that, the output of the inverter is a sinusoidal waveform, the values of u_{s*} must be lying inside the maximum circular locus shown by the dotted line in Fig. 5. This circle has radius of $(1/\sqrt{3}) U_{dc}$ or $(\pi/2\sqrt{3}) \text{ p.u.}$

Then, the maximum fundamental phase component that can be obtained with a circular trajectory of the stator voltage vector using the definition of space vector modulator is given by:

$$U_{1 \text{ max}} = (1/\sqrt{3}) U_{dc} = 0.577 U_{dc} \quad (23)$$

and the per unit value of this voltage is given by:

$$U_{1 \text{ max p.u.}} = \pi/2\sqrt{3} = 0.906 \quad (24)$$

which is defined as the **maximum modulation index** of space vector modulator. Generally, the normal three phase sinusoidal voltage can be represented by a space vector occupies any point in the x-y plane. The length of this vector depends upon the amplitude of the sinusoidal voltage while its direction depends on the phase angle.

V. SPACE VECTOR MODULATOR STRATEGY

The space vector modulator strategy is based on geometrically approximating the reference space vector \underline{u}_s^* by the two adjacent inverter vectors together with the zero vector. The problem in trying to synthesize a set of three phase voltages from an inverter lies in the inherently discrete nature. To overcome this problem, the inverter must modulate the widths of its output pulses such that their instantaneous time average becomes equal that of the vector \underline{u}_s^* [3].

In the case of a three phase inverter this requires the inverter to produce the switching voltage vectors ($\underline{u}_1, \dots, \underline{u}_6$) in such a way that their instantaneous average value equals that of the desired composite voltage vectors \underline{u}_s^* over an entire period. Fig. 5 illustrates how this may be done using SVM.

In Fig. 5 the desired composite voltage vector has been superimposed upon the inverter switching voltage vectors. Note that, the switching voltage vectors are symmetrically spaced at 60° intervals and that the area enclosed between any of these vectors is called Sector or Sextant.

It is clear that, the three phase inverter produces only six discrete (non zero) switching voltage vectors ($\underline{u}_1, \dots, \underline{u}_6$) and the remaining two (\underline{u}_0 and \underline{u}_7) are of zero voltage as shown in Fig. 5. As an example sector 2 in Fig. 5 is the area flanked by the switching voltage vectors \underline{u}_2 and \underline{u}_3 and so on.

The importance of defining such sectors is that should the desired voltage vector at any stage fall into a particular sector then only the flanking switching voltage vectors are used to

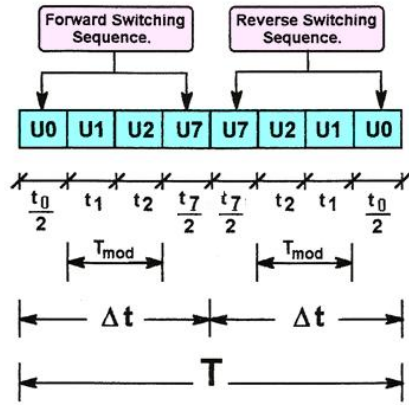


Fig. 10 Switching Sequence of Voltage Vector.

Where:

T_0 , t_1 , t_2 , and t_7 : are the time interval.

T_{mod} : is the modulation time.

Δt : is the sampling interval.

T : is the switching time (switching cycle).

During each switching cycle T shown in Fig. 10, the inverter output must give \underline{u}_1 , \underline{u}_2 and \underline{u}_0 or \underline{u}_7 for times t_1 , t_2 and t_0 or t_7 respectively. Equating the volt-second intervals over half of a PWM switching cycle gives:

$$\underline{u}_s^* \Delta t = \underline{u}_0 (t_0/2) + \underline{u}_1 t_1 + \underline{u}_2 t_2 + \underline{u}_7 (t_7/2) \quad (25)$$

For the traditional space vector technique, it was arbitrarily assumed that, the null voltage vector time was equally divided between t_0 and t_7 , then

$$t_0 = t_7 \quad (26)$$

Therefore, Equ. 25 becomes:

$$\underline{u}_s^* \Delta t = \underline{u}_0 t_0 + \underline{u}_1 t_1 + \underline{u}_2 t_2 \quad (27)$$

$$\text{And } \Delta t = t_0 + t_1 + t_2 \quad (28)$$

or

$$\Delta t = t_0 + T_{mod} \quad (29)$$

where

$$T_{mod} = t_0 + t_2 \quad (30)$$

In order to determine the time intervals (t_1 , t_2 , and t_0), the reference space vector \underline{u}_s^* must be resolved into two components \underline{u}_{su1} and \underline{u}_{su2} in the direction of \underline{u}_1 and \underline{u}_2 respectively as shown in Fig. 11.

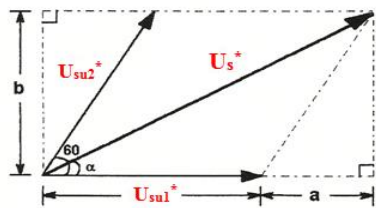


Fig. 11 Decomposition of reference space vector.

From Fig. 11, the distance a and b can be found from:

$$a = u_{su2}^* \cos 60^\circ = (1/2) u_{su2}^* \quad (31)$$

$$b = u_s^* \sin \alpha \quad (32)$$

and

$$u_{su1}^* = u_s^* \cos \alpha - a \quad (33)$$

$$u_{su2}^* = b / \sin 60 = (2/\sqrt{3}) b \quad (34)$$

Substituting from Equ. 32 in Equ. 34, we get:

$$u_{su2}^* = (2/\sqrt{3}) u_s^* \sin \alpha \quad (35)$$

Substituting from Equ. 31 in Equ. (33), then:

$$u_{su1}^* = u_s^* \cos \alpha - (1/2) u_{su2}^* \quad (36)$$

Substituting from Equ. 35 in Equ. 36, we get:

$$u_{su1}^* = u_s^* [\cos \alpha - (1/\sqrt{3}) \sin \alpha] \quad (37)$$

The times t_1 and t_2 are proportional to the components u_{su1}^* and u_{su2}^* respectively and the switching time T is proportional to $(2/3) U_{dc}$. Then, the time intervals t_1 and t_2 can be found as:

$$t_1 = (2/3) u_{su1}^* T / U_{dc} \quad (38)$$

and

$$t_2 = (2/3) u_{su2}^* T / U_{dc} \quad (39)$$

Substituting from Equ. 35 and Equ. 37 in Equ. 38) and (39) and taking into account the values of u_s^* inside the maximum circular locus shown by the dotted circle in Fig. 5 of radius $(1/\sqrt{3})$ or $(\pi/2\sqrt{3})$ per unit, then:

$$t_1 = (\sqrt{3}/2) [u_s^* / (\pi/3)] T [\cos \alpha - (1/\sqrt{3}) \sin \alpha] \quad (40)$$

$$t_2 = (\sqrt{3}/2) [u_s^* / (\pi/3)] T [(2/\sqrt{3}) \sin \alpha] \quad (41)$$

and

$$t_0 = T - T_{mod} \quad (42)$$

Equ. 40 and Equ. 41 can be rewritten in a simplified form as:

$$t_1 = A \cdot u_s^* \cos \alpha - (t_2/2) \quad (43)$$

$$t_2 = B \cdot u_s^* \sin \alpha \quad (44)$$

where

A and B are constants and given by:

$$A = (3\sqrt{3}/2\pi) T \quad (45)$$

$$B = (3/\pi) T \quad (46)$$

Thus, the on-time for each different inverter switching voltage is given by Equ. 43 and Equ. 44. The switching sequence for

the upper switches in the six sectors defined in Fig. 5 and the three phase output waveforms of the space vector modulator are shown in Fig 12. The switching period T is divided into two equally sampling intervals. The sequence of switching in the first interval is reversed in the second interval.

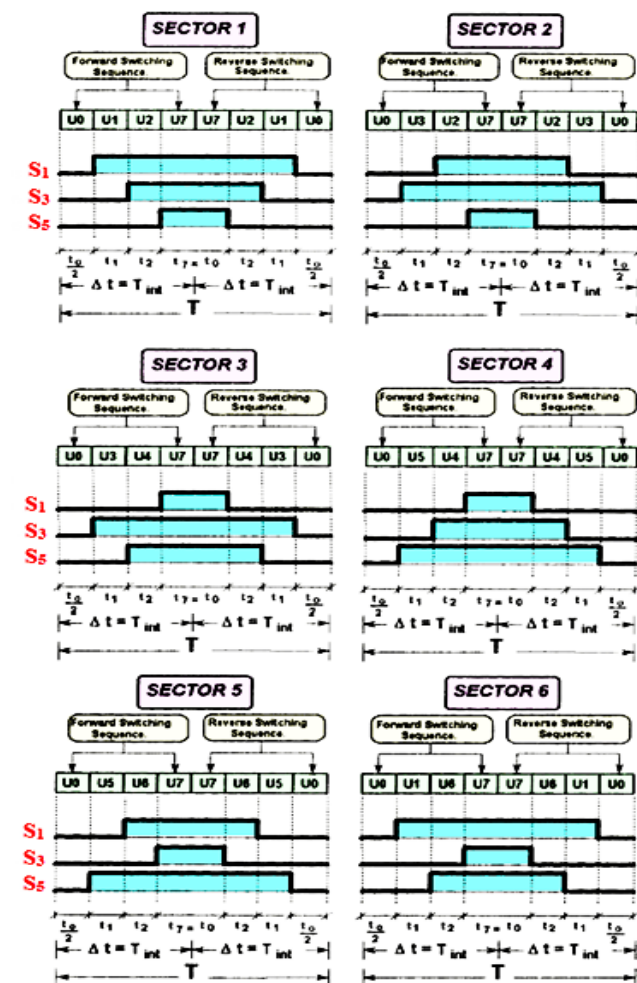


Fig. 12 Switching sequence of SVM output in different sectors
According to Fig. 12, the switching time at each sector for the upper and lower switches is summarized in Table 5.

TABLE V
Calculation of Switching Time at each Sector

| Sectors | Upper Switches (S ₁ , S ₃ , S ₅) | Lower Switches (S ₄ , S ₆ , S ₂) |
|----------|---|---|
| Sector 1 | S ₁ = t ₁ + t ₂ + t ₀ /2 S ₃ = t ₂ + t ₀ /2 S ₅ = t ₀ /2 | S ₄ = t ₀ /2 S ₆ = t ₁ + t ₀ /2 S ₂ = t ₁ + t ₂ + t ₀ /2 |
| Sector 2 | S ₁ = t ₁ + t ₀ /2 S ₃ = t ₁ + t ₂ + t ₀ /2 S ₅ = t ₀ /2 | S ₄ = t ₁ + t ₀ /2 S ₆ = t ₀ /2 S ₂ = t ₁ + t ₂ + t ₀ /2 |
| Sector 3 | S ₁ = t ₀ /2 S ₃ = t ₁ + t ₂ + t ₀ /2 S ₅ = t ₂ + t ₀ /2 | S ₄ = t ₁ + t ₂ + t ₀ /2 S ₆ = t ₀ /2 S ₂ = t ₁ + t ₀ /2 |

| Sectors | Upper Switches (S ₁ , S ₃ , S ₅) | Lower Switches (S ₄ , S ₆ , S ₂) |
|----------|---|---|
| Sector 4 | S ₁ = t ₀ /2 S ₃ = t ₁ + t ₀ /2 S ₅ = t ₁ + t ₂ + t ₀ /2 | S ₄ = t ₁ + t ₂ + t ₀ /2 S ₆ = t ₂ + t ₀ /2 S ₂ = t ₀ /2 |
| Sector 5 | S ₁ = t ₂ + t ₀ /2 S ₃ = t ₀ /2 S ₅ = t ₁ + t ₂ + t ₀ /2 | S ₄ = t ₁ + t ₀ /2 S ₆ = t ₁ + t ₂ + t ₀ /2 S ₂ = t ₀ /2 |
| Sector 6 | S ₁ = t ₁ + t ₂ + t ₀ /2 S ₃ = t ₀ /2 S ₅ = t ₁ + t ₀ /2 | S ₄ = t ₀ /2 S ₆ = t ₁ + t ₂ + t ₀ /2 S ₂ = t ₂ + t ₀ /2 |

VII. CONCLUSIONS

A SVPWM technique which is a digital modulating technique based on a reduced computation method was presented. In SVPWM technique, the inverter gating signals derived from the sampled amplitudes of the reference phase voltages. The SVPWM scheme drive the inverter with eight switching states, this switching states can be represented by a state vector in the two-axis space formed a hexagon shape with six sectors. The time interval of switching the state vectors in each sector calculated in a sampled time T in order to implement the required modulation procedure. The modulation index approaches to (90.6%) and the maximum output fundamental is (0.577U_{dc}) because the linear region in SVPWM is larger than other types of PWM technique. SVPWM technique provides a constant switching frequency and gives an excellent harmonic reduction in output voltage and current.

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