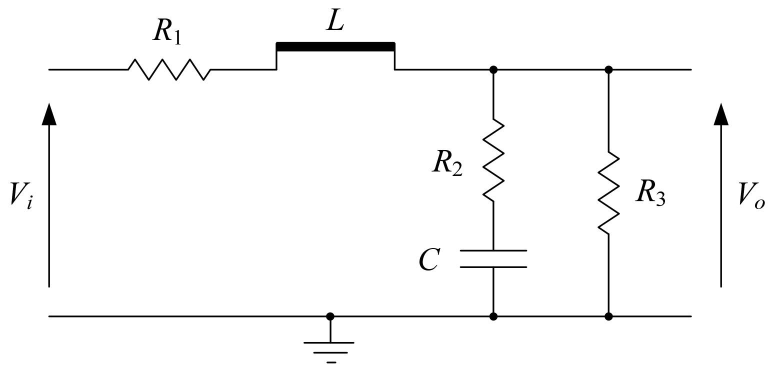
Design Example - 1

Step 1: Determine the plant transfer function



$L = 0.9\ \mu\text{H}$
$C = 471\ \mu\text{F}$
$R_1 = 2.2\ \text{m}\Omega$
$R_2 = 0.6\ \text{m}\Omega$
$R_3 = 1\ \Omega$

The switching frequency is 1 MHz.

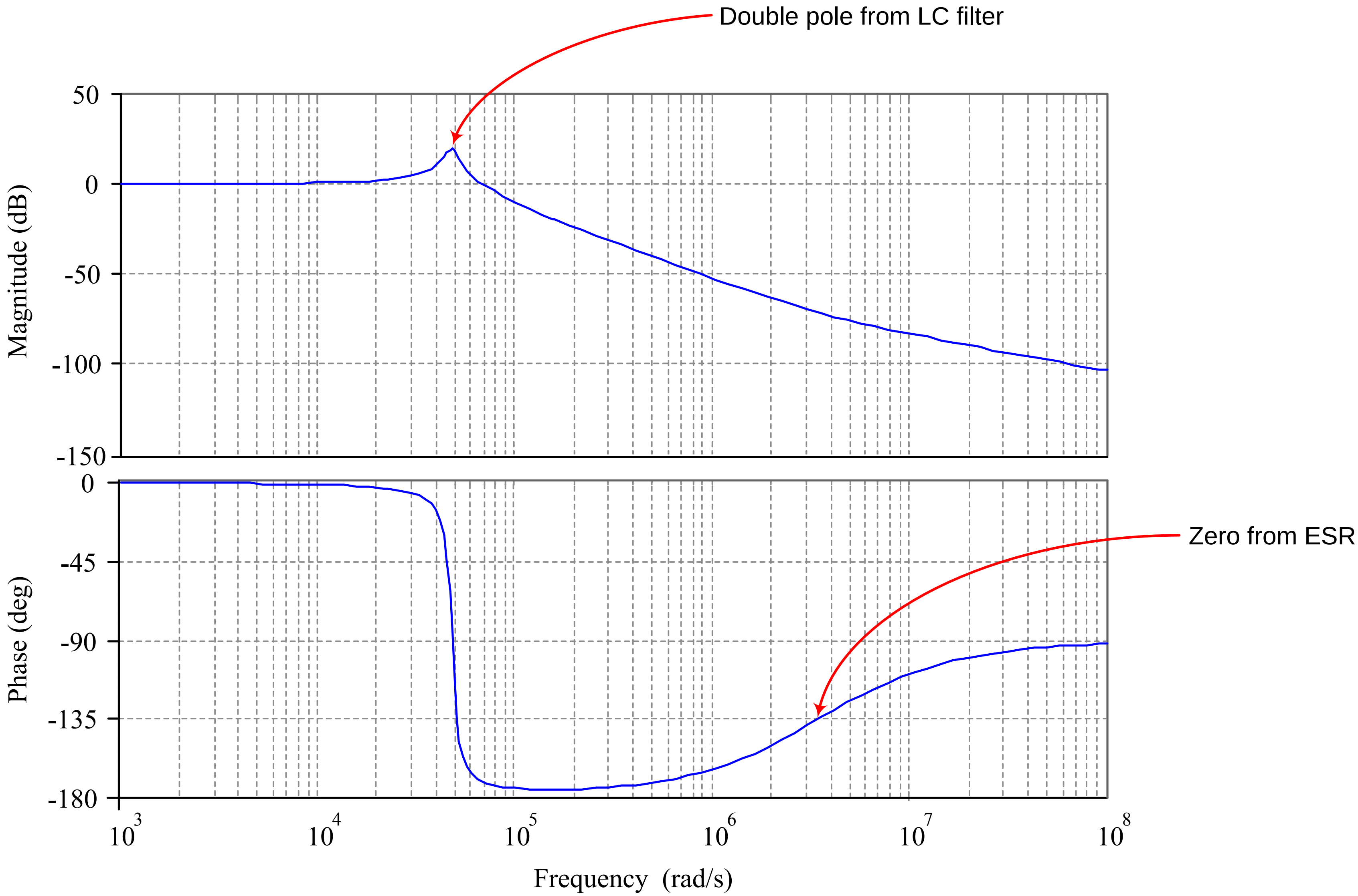
In Continuous Conduction Mode (CCM) the buck output stage has transfer function

$$G(s) = \frac{sCR_2R_3 + R_3}{s^2LC(R_2+R_3) + s(CR_1(R_2+R_3) + CR_2R_3 + L) + (R_1+R_3)}$$

Inserting the passive component values for the workshop board, we obtain

$$G(s) = \frac{2.826\times10^{-7}s + 1}{4.252\times10^{-10}s^2 + 2.219\times10^{-6}s + 1.002}$$

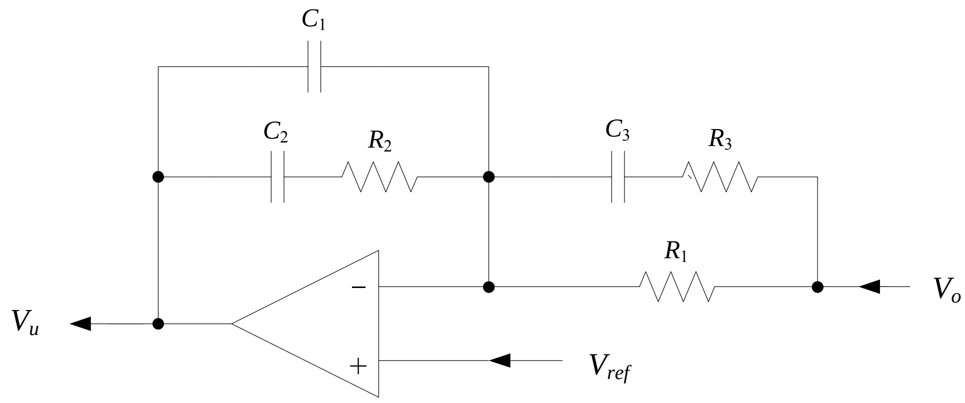
Design Example - 1a



The plant frequency response exhibits a well-defined peak in the magnitude curve from the LC filter, and a high-frequency zero from ESR of the output capacitor.

Design Example - 2

Step 2: Design the analog compensator



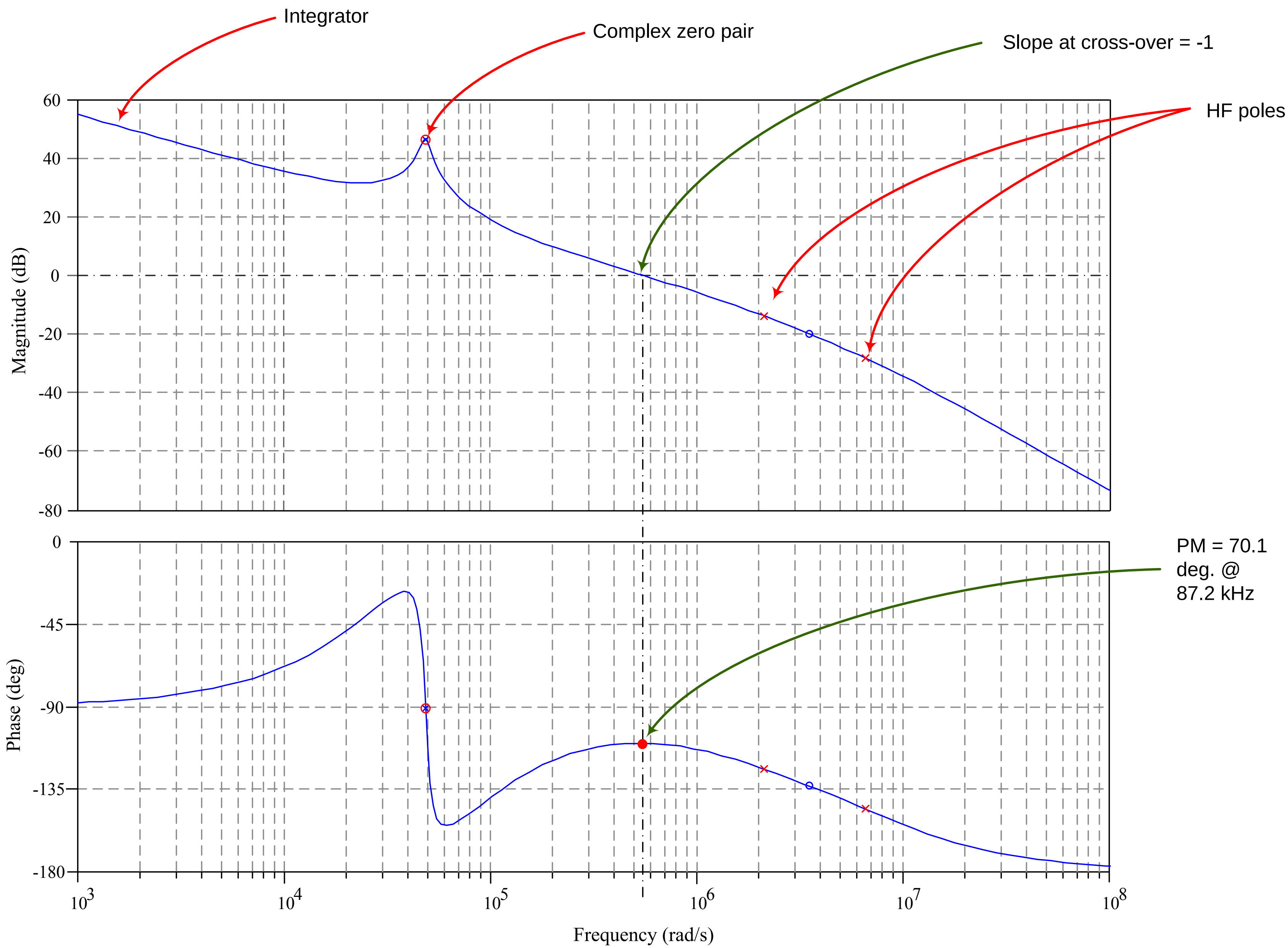
The transfer function of the classical type III compensator is

$$F(s) = \frac{R_1 + R_3}{R_1 R_3 C_1} \frac{\left(s + \frac{1}{R_2 C_2}\right)\left(s + \frac{1}{(R_1 + R_3) C_3}\right)}{s\left(s + \frac{C_1 + C_2}{R_2 C_1 C_2}\right)\left(s + \frac{1}{R_3 C_3}\right)}$$

Pole & zero frequencies are selected using any of the established design techniques.

$$F(s) = A_s \frac{(s + \omega_{z1})(s + \omega_{z2})}{s(s + \omega_{p1})(s + \omega_{p2})}$$

Design Example - 2a



Frequency response plot of open loop with analogue compensator.

Design Example - 3

Step 3: Transform the compensator into discrete time

$$F(s) = A_s \frac{(s+\omega_{z1})(s+\omega_{z2})}{s(s+\omega_{p1})(s+\omega_{p2})}$$

The digital transfer function can be formed using a Tustin transformation: $s \leftarrow \frac{2}{T}\frac{z-1}{z+1}$

$$F(z) = A_s \frac{\left(\frac{2}{T}\frac{z-1}{z+1}+\omega_{z1}\right)\left(\frac{2}{T}\frac{z-1}{z+1}+\omega_{z2}\right)}{\frac{2}{T}\frac{z-1}{z+1}\left(\frac{2}{T}\frac{z-1}{z+1}+\omega_{p1}\right)\left(\frac{2}{T}\frac{z-1}{z+1}+\omega_{p2}\right)}$$

The 0Hz gains of the transfer functions must be matched: $\left|F(s)\right|_{s=0} = \left|F(z)\right|_{z=1}$

After some algebra, we arrive at the third order digital transfer function

$$F(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}$$

Design Example - 3a

Using the Tustin method, we have transformed a transfer function in s to one in z.

$$F(s)=\frac{\omega_{p1}}{s}\frac{\left(\frac{s}{\omega_{z1}}+1\right)\left(\frac{s}{\omega_{z2}}+1\right)}{\left(\frac{s}{\omega_{p2}}+1\right)\left(\frac{s}{\omega_{p3}}+1\right)}$$

$$F(z)=\frac{b_0+b_1z^{-1}+b_2z^{-2}+b_3z^{-3}}{1+a_1z^{-1}+a_2z^{-2}+a_3z^{-3}}$$

After a lot of algebra, the coefficients turn out to be:

$$a_1=-\frac{-12+T^2\omega_{p2}\omega_{p3}-2T\left(\omega_{p2}+\omega_{p3}\right)}{\left(2+T\omega_{p2}\right)\left(2+T\omega_{p3}\right)}$$

$$a_2=-\frac{12+T^2\omega_{p2}\omega_{p3}-2T\left(\omega_{p2}+\omega_{p3}\right)}{\left(2+T\omega_{p2}\right)\left(2+T\omega_{p3}\right)}$$

$$a_3=-\frac{\left(-2+T\omega_{p2}\right)\left(-2+T\omega_{p3}\right)}{\left(2+T\omega_{p2}\right)\left(2+T\omega_{p3}\right)}$$

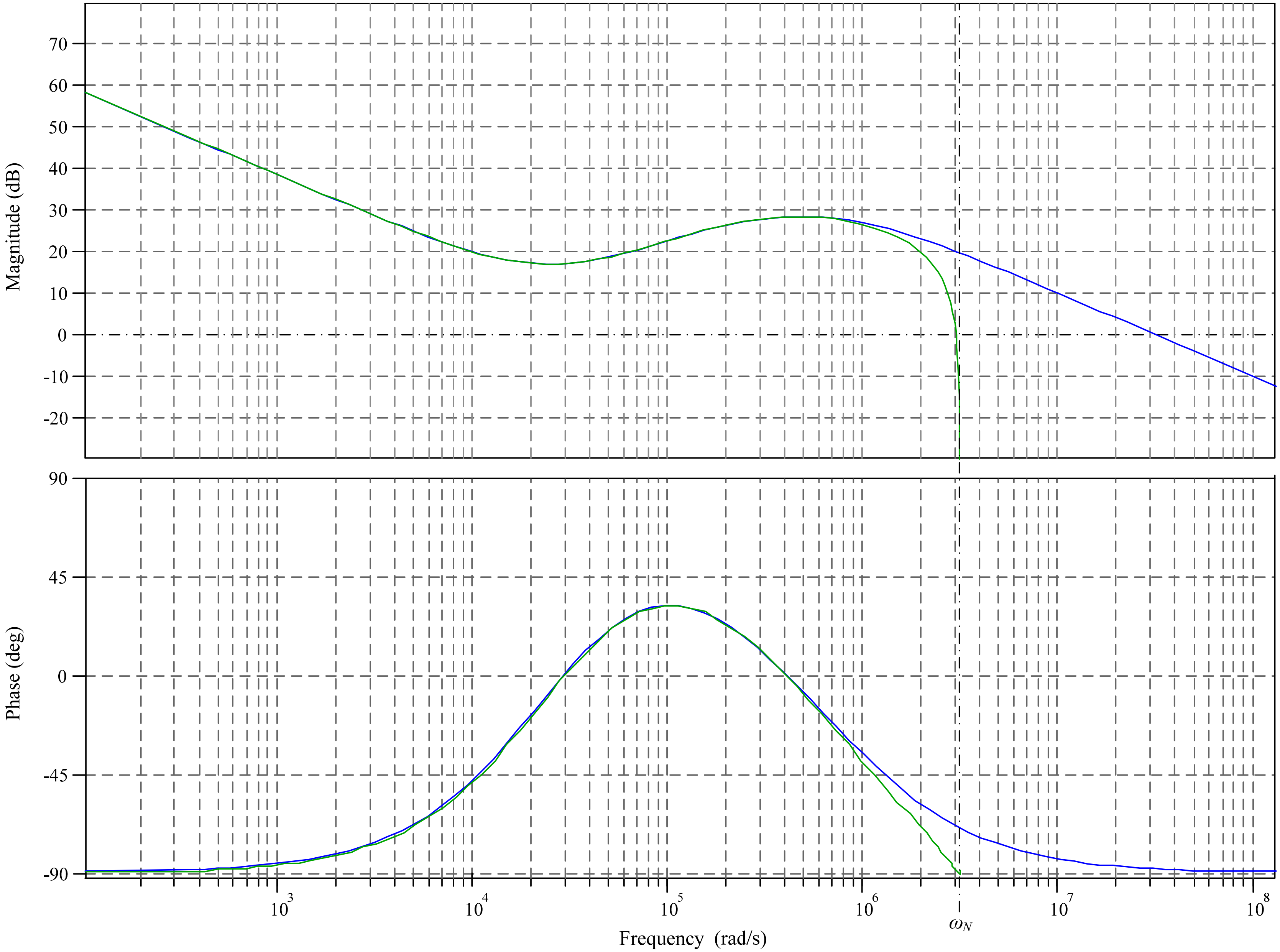
$$b_0=\frac{T\omega_{p1}\omega_{p2}\omega_{p3}\left(2+T\omega_{z1}\right)\left(2+T\omega_{z1}\right)}{2\omega_{z1}\omega_{z2}\left(2+T\omega_{p2}\right)\left(2+T\omega_{p3}\right)}$$

$$b_1=\frac{T\omega_{p1}\omega_{p2}\omega_{p3}\left(-4+3T^2\omega_{z1}\omega_{z2}+2T\left(\omega_{z1}+\omega_{z2}\right)\right)}{2\omega_{z1}\omega_{z2}\left(2+T\omega_{p2}\right)\left(2+T\omega_{p3}\right)}$$

$$b_2=\frac{T\omega_{p1}\omega_{p2}\omega_{p3}\left(-4+3T^2\omega_{z1}\omega_{z2}-2T\left(\omega_{z1}+\omega_{z2}\right)\right)}{2\omega_{z1}\omega_{z2}\left(2+T\omega_{p2}\right)\left(2+T\omega_{p3}\right)}$$

$$b_3=\frac{T\omega_{p1}\omega_{p2}\omega_{p3}\left(-2+T\omega_{z1}\right)\left(-2+T\omega_{z2}\right)}{2\omega_{z1}\omega_{z2}\left(2+T\omega_{p2}\right)\left(2+T\omega_{p3}\right)}$$

Design Example - 3b



Frequency response comparison of analogue (blue) and digital (green) compensators. f_s = 1MHz.

Design Example - 4

Step 4: Determine the difference equation for the compensator

Normalizing to the highest power of z in the denominator

$$F(z) = \frac{u(z)}{e(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}$$

$$u(z) = b_0\, e(z) + b_1 z^{-1} e(z) + b_2 z^{-2} e(z) + b_3 z^{-3} e(z) - a_1 z^{-1} u(z) - a_2 z^{-2} u(z) - a_3 z^{-3} u(z)$$

To find the difference equation, we simply apply the shifting property of the z transform

$$\mathcal{Z}\{f(k-n)\} = z^{-n} f(z)$$

$$u(k) = b_0\, e(k) + b_1\, e(k-1) + b_2\, e(k-2) + b_3\, e(k-3) - a_1\, u(k-1) - a_2\, u(k-2) - a_3\, u(k-3)$$

All that remains is to code and test the controller algorithm.