Discrete Time PID Controller for DPEK

This controller is based on the "ideal" PID, which in Laplace form is written

$$G(s) = K_P + \frac{K_I}{s} + s\,K_D$$

Discrete approximation by numerical integration is used for the integral and derivative terms. The derivative term uses backwards Euler approximation which leads to the substitution

$$s \leftarrow \frac{z-1}{Tz}$$

The integral term uses trapezoidal approximation, with the substitution

$$s \leftarrow \frac{2}{T}\frac{z-1}{z+1}$$

The z-transform of the equivalent PID controller becomes

$$G(z) = K_P + \frac{T}{2}\frac{z+1}{z-1}K_I + \frac{z-1}{Tz}K_D$$

Re-arranging to express each terem as a power of z:

$$2Tz(z-1)\,G(z) = K_P\,2Tz(z-1) + K_I T^2 z(z+1) + K_D\,2(z-1)^2$$

$$\begin{aligned}(2Tz^2 - 2Tz)\,G(z) = {} & z^2(2TK_P + T^2K_I + 2K_D) \\ & + z(-2TK_P + T^2K_I - 4K_D) + 2K_D\end{aligned}$$

The controller gains are re-defined by:

$$\begin{aligned}K_P{}' &= K_P \\ K_I{}' &= \frac{T}{2}K_I \\ K_D{}' &= \frac{1}{T}K_D\end{aligned}$$

Inserting these into the controller equation gives

$$\begin{aligned}(2Tz^2 - 2Tz)\,G(z) = {} & z^2(2TK_P{}' + 2TK_I{}' + 2TK_D{}') \\ & + z(-2TK_P{}' + 2TK_I{}' - 4TK_D{}') + 2TK_D{}'\end{aligned}$$

A common factor of $2T$ can be removed and the equation re-arranged to find the transfer function

$$G(z) = \frac{b_0 z^2 + b_1 z + b_2}{z^2 - z}$$

where…

$$b_0 = K_P{}' + K_I{}' + K_D{}'$$
$$b_1 = -K_P{}' + K_I{}' - 2K_D{}'$$
$$b_2 = K_D{}'$$