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Designing Compensators for the Control of Switching Power Supplies

Presented by Christophe Basso

Senior Scientist
IEEE Senior Member

Agenda

- ❑ Feedback generalities
- ❑ The divider and the virtual ground
- ❑ Phase margin and crossover
- ❑ Poles and zeros
- ❑ Boosting the phase at crossover
- ❑ Compensator types
- ❑ Practical implementations: the op amp
- ❑ Practical implementations: the OTA
- ❑ Practical implementations: the TL431
- ❑ Design examples
- ❑ A real case study
- ❑ Conclusion

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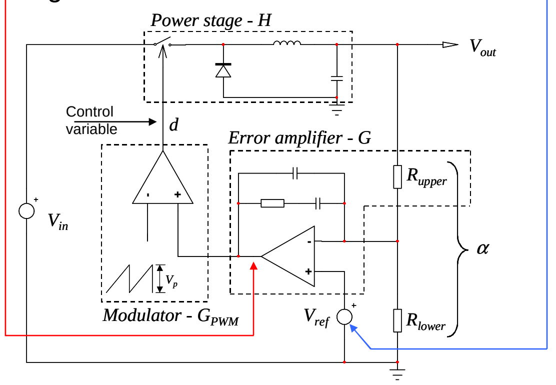
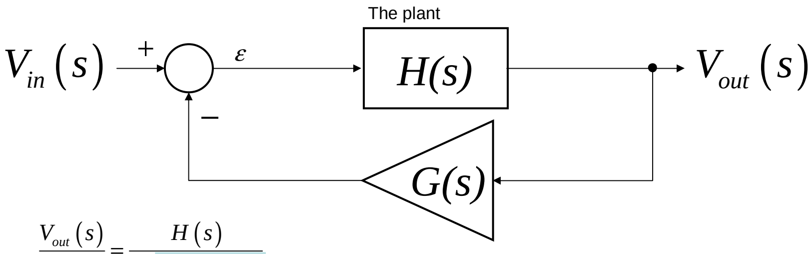
What is a regulated power supply?

- ❑ V_{out} is permanently compared to a reference voltage V_{ref}.
- ❑ The reference voltage V_{ref} is precise and stable over temperature.
- ❑ The error $\varepsilon = V_{ref} - \alpha V_{out}$, is amplified and sent to the control input.
- ❑ The power stage reacts to reduce ε as much as it can.



How to build an oscillator?

❑ How to keep self-sustained oscillations?



$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{H(s)}{1+H(s)G(s)}$$ → Open-loop gain *T(s)*

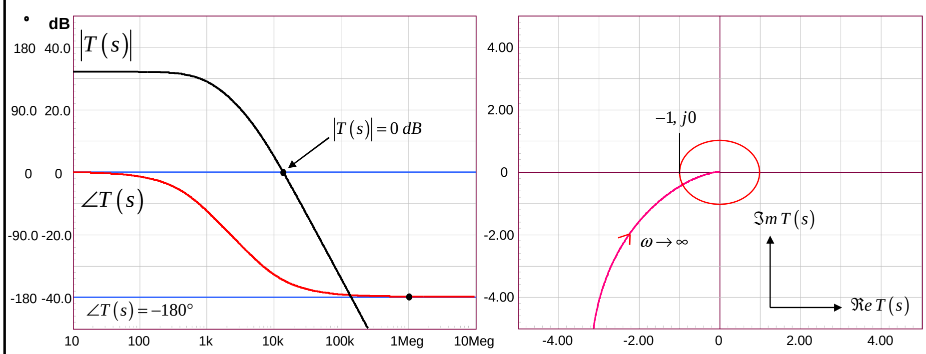
$$V_{out}(s) = \lim_{V_{in}(s)\to 0}\left[\frac{H(s)}{1+G(s)H(s)}V_{in}(s)\right]$$

To sustain self-oscillations, as $V_{in}(s)$ goes to zero, quotient must go infinite

$$1+G(s)H(s)=0 \longrightarrow \begin{cases} \left|G(s)H(s)\right| = 1 = 0\ dB \\ \angle G(s)H(s) = -180° \end{cases} \longrightarrow$$ *Nyquist* $-1, j0$

Where is the point -1,j0?

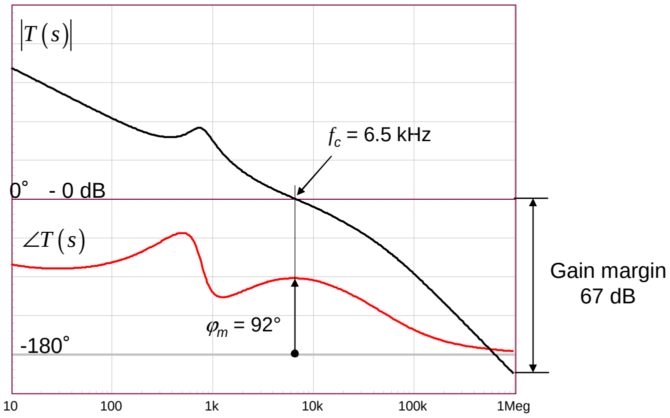
❑ In a Bode plot, we deal with both magnitude and argument:
- when |*T(s)*| crosses the 0-dB axis, this is the "1" point
- when arg *T(s)* crosses the -180° axis, this is the "-" sign



❑ In a Nyquist plot, we deal with the argument and real part of *T(s)*
- the point -1,j0 represents the 0-dB gain and the sign reversal

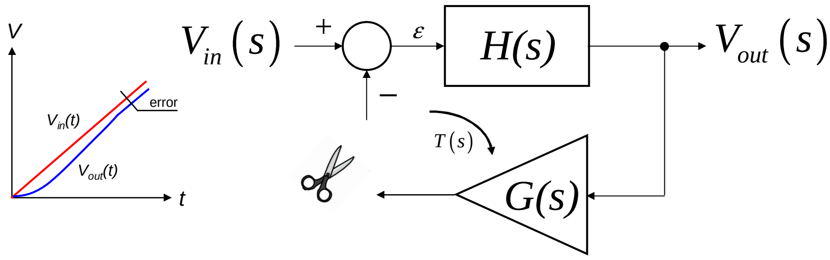
If you fear oscillations, build phase margin!

- ❑ The frequency at which $|T(s)|$ = 0 dB is the crossover frequency, f_c
- ❑ The distance between arg $T(f_c)$ and the -180° limit is called:
  - ➢ the phase margin, noted φ_m



In the literature, V_{out} must follow V_{in}

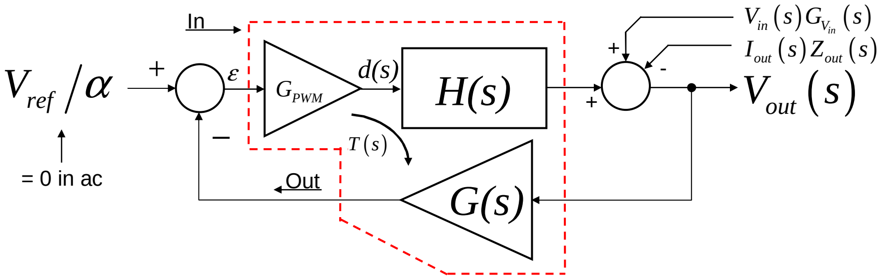
- ❑ Text books cover loop control theory assuming V_{out} follows V_{in}:
  - ➢ If V_{in} imposes a ramp, V_{out} must follow with the least error
- ❑ The loop is then open to check V_{out} over V_{in}



- ❑ In our converters, V_{in} is V_{ref}/α and is fixed!
- ❑ If the loop gain is high enough, we should have: $V_{out} = V_{ref}/\alpha$
- ❑ The perturbations are V_{in} and I_{out}
  - ➢ The model must be updated

How does this translate to our converter?

❑ The loop gain $T(s)$ includes $H(s)$, $G(s)$ and G_{PWM}



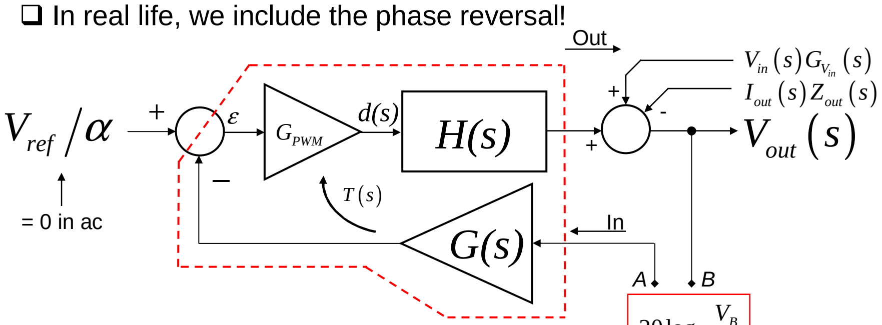
$$T(s) = G_{PWM} H(s) G(s)$$

❑ In the literature, $T(s)$ is considered *without* the phase reversal brought by the negative feedback:

$\longrightarrow$ $\angle T(s) = -180°$ brings instability

How does this translate to our converter?

❑ In real life, we include the phase reversal!



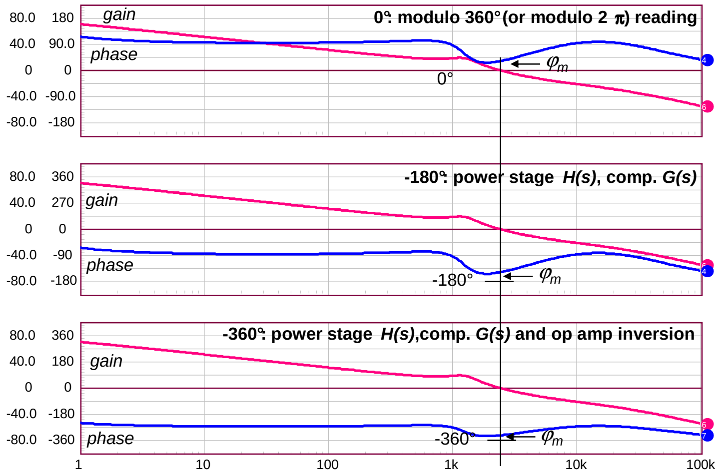
$$T(s) = -G_{PWM} H(s) G(s)$$

❑ In the real life, $T(s)$ *includes* the phase reversal brought by the negative feedback:

$\longrightarrow$ $\angle T(s) = -360°$ brings instability

These plots are identical

❑ all these plots read the same phase margin!



SPICE Network analyzer

Literature

-360° is a complete turn!

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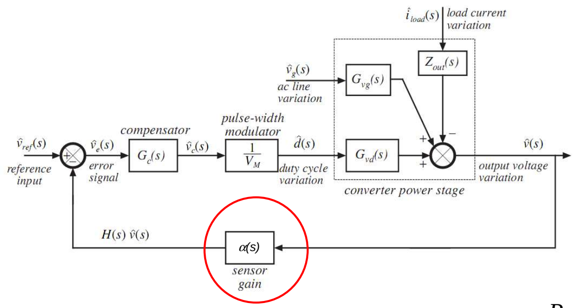
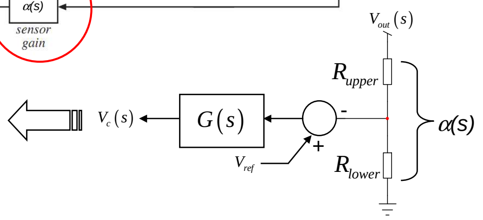
Hey, where is the divider network?

- ❑ In some text books, the divider network enters the picture



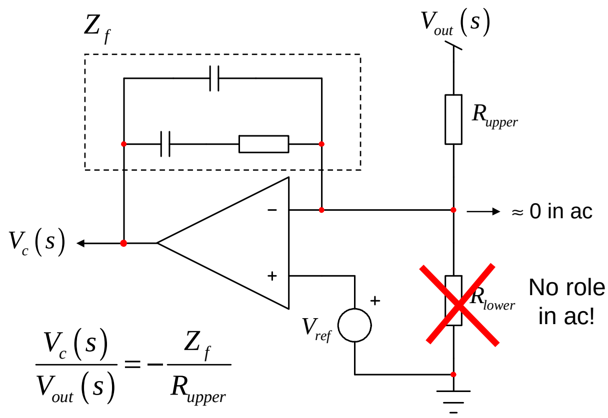
$$\frac{V_c(s)}{V_{out}(s)} = -\frac{R_{lower}}{R_{lower}+R_{upper}} G(s)$$



R. Erickson, D. Masksimovic, “Fundamentals of Power Electronics”, Kluwers, 2001

The virtual ground excludes R_{lower}

- ❑ In reality, the feedback is often made with an op amp

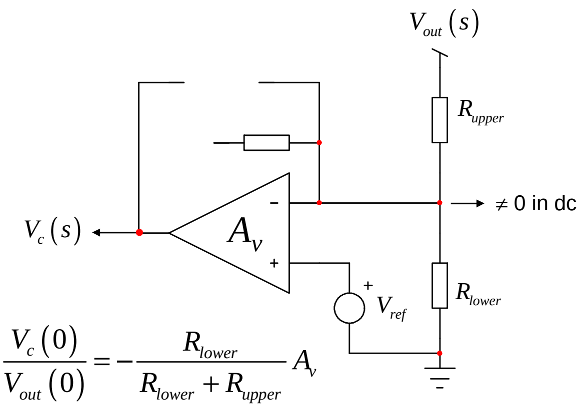


$$\frac{V_c(s)}{V_{out}(s)} = -\frac{Z_f}{R_{upper}}$$

- ❑ Because of the local feedback via Z_f, we have a virtual ground

Looks like the divider in back…

- In a type 1, 2 or 3, the local feedback is lost for $s = 0$

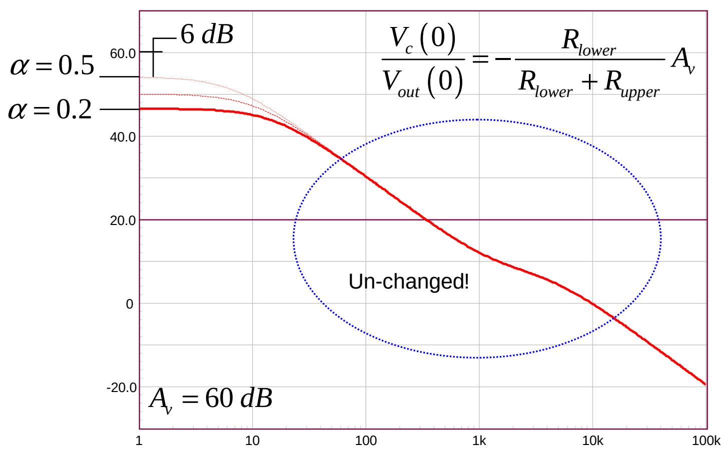


$$\frac{V_c(0)}{V_{out}(0)} = -\frac{R_{lower}}{R_{lower} + R_{upper}} A_v$$

- The 0-Hz gain is indeed changed but not f_c!

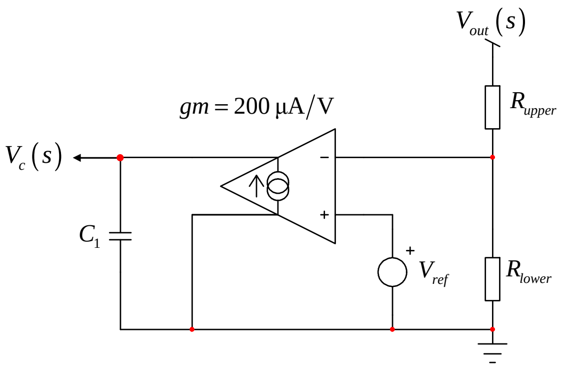
Looks like the divider in back…

- With an op amp, only the dc gain is affected



You don't have a virtual ground in an OTA!

- ❑ R_{lower} enters the picture in all equations



$$R_{equ} = \frac{R_{upper} + R_{lower}}{R_{lower}\, gm}$$

$$f_{po} = \frac{1}{2\pi C_1 R_{equ}}$$

- ❑ Unless an OTA is used, the divider plays <u>no</u> role in ac!

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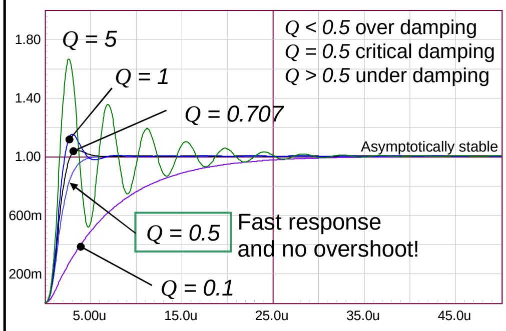
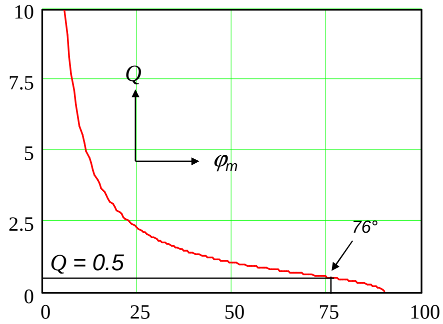
How much phase margin to chose?

- ❑ a Q factor of 0.5 (critical response) implies a φ_m of 76°
- ❑ a 45° φ_m corresponds to a Q of 1.2: oscillatory response!





- ❑ phase margin depends on the needed response: fast, no overshoot…
- ❑ good practice is to shoot for 60° and make sure φ_m always > 45°

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Which crossover frequency to select?

- ❑ crossover frequency selection depends on several factors:
- ▪ *switching frequency*: theoretical limit is $F_{sw}/2$
- ➢ in practice, stay below 1/5 of F_{sw} for noise concerns
- ▪ *output ripple*: if ripple pollutes feedback, «tail chasing» can occur.
- ➢ crossover frequency rolloff is mandatory, e.g. in PFC circuits
- ▪ *presence of a Right-Half Plane Zero* (RHPZ):
- ➢ you cannot cross over beyond 30% of the lowest RHPZ position
- ▪ *output undershoot specification*:
- ➢ select crossover frequency based on undershoot specs

$$V_p \approx \frac{\Delta I_{out}}{2\pi f_c C_{out}}$$

Don't push the crossover frequency too far!!

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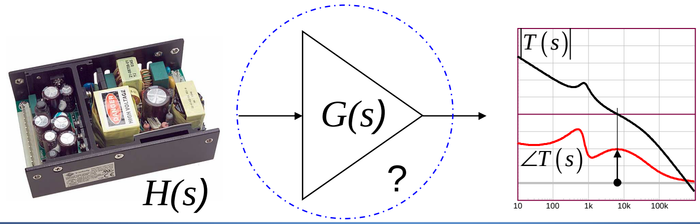
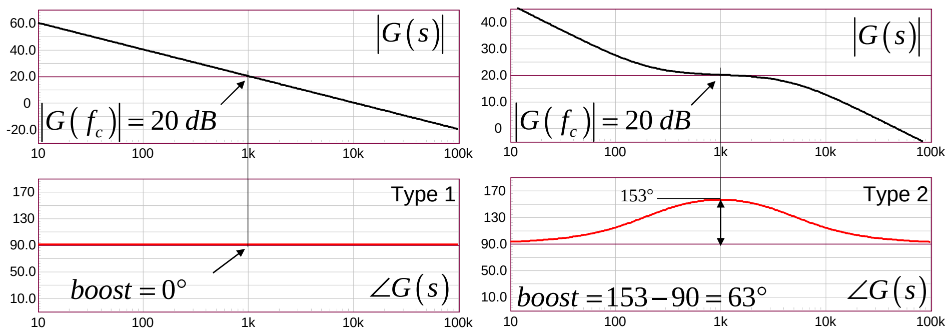
How to force crossover and phase margin?

- ❑ The converter we want to compensate exhibits a transfer function
- ➢ This is the power stage *open-loop* transfer function noted $H(s)$
- ❑ On this plot, a crossover frequency is identified, f_c
- ❑ The designer reads the gain deficiency and the phase rotation at f_c
- ➢ it can sometimes be a gain excess, in PFC stages for instance
- ❑ A *compensator* transfer function $G(s)$ is inserted so that it:
- ➢ provides gain/attenuation at the crossover frequency: $\left|H(f_c)G(f_c)\right| = 1$
- ➢ *boosts* the phase at the crossover: $\arg H(f_c) + \arg G(f_c) = -360° + \varphi_m$



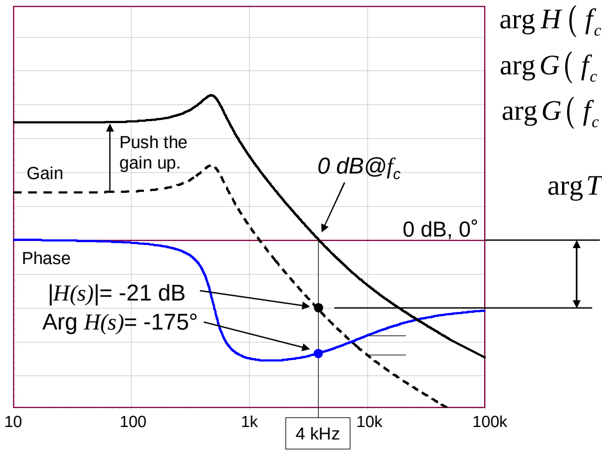
What do we mean by “phase boost”?

- ❑ Control theory instructs to keep $T(s)$ away from the point -1,j0
- ❑ At the frequency where $|T(s)| = 1$, arg $T(s)$ should be less than -360°
- ❑ To generate phase margin, we need to improve arg $T(s)$ at crossover
- ❑ The compensator G is tailored to provide phase correction at f_c
- ❑ The amount of needed phase correction is called the *phase boost*



How to force crossover and phase margin?

- ❑ Here, we want a 4-kHz crossover point and a 60° phase margin
- ❑ Build *G(s)* so that |*G(4kHz)*| = +21 dB and arg *G(4kHz)* = -125°



$$\arg H(f_c) + \arg G(f_c) = -360° + \varphi_m$$

$$\arg G(f_c) = -360° + \varphi_m - \arg H(f_c)$$

$$\arg G(f_c) = -125°$$

$$\arg T(f_c) = -175 - 125 = -300°$$

Tailor *G(s)* to exhibit a gain of +21 dB@ f_c.

$$\varphi_m = 60°$$

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Poles, zeros and RHPZ

❑ A loop gain can be put under the following form:

$$H(s)=\frac{N(s)}{D(s)}$$

$N(s)$ → numerator

$D(s)$ → denominator

❑ solving for $N(s)$ = 0, the roots are called the **zeros**

❑ solving for $D(s)$ = 0, the roots are called the **poles**

$$H(s)=\frac{(s+5k)(s+30k)}{s+1k}$$

Numerator roots

$s_{z_1}=-5k$

$s_{z_2}=-30k$

$s_{p_1}=-1k$

Denominator root

$$f_{z_1}=\frac{5k}{2\pi}=796\ Hz$$

$$f_{z_2}=\frac{30k}{2\pi}=4.77\ kHz$$

$$f_{p_1}=\frac{1k}{2\pi}=159\ Hz$$

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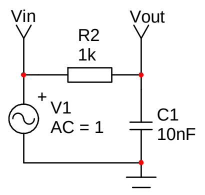
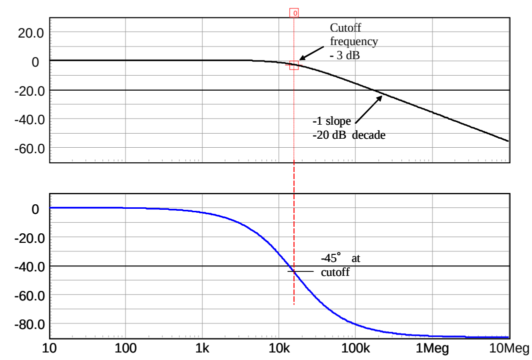
The pole

❑ A pole creates a phase lag of -45° at its cutoff frequency





$$\frac{V_{out}(s)}{V_{in}(s)}=\frac{1}{1+sRC}$$

We can write the equation in different forms:

$$\frac{V_{out}(s)}{V_{in}(s)}=\frac{1}{1+\frac{s}{\omega_0}}=\frac{1}{1+\frac{s}{s_{p_1}}}\quad \text{where}\quad \omega_0=s_{p_1}=\frac{1}{RC}$$

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The Pole

- ❑ Its magnitude at the cutoff frequency is -3 dB
- ❑ Its asymptotic phase, when in the LHP, at $f = \infty$ is -90°
- ❑ The pole "lags" the phase

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{1+sRC} = \frac{1}{1+\frac{s}{\omega_0}}$$

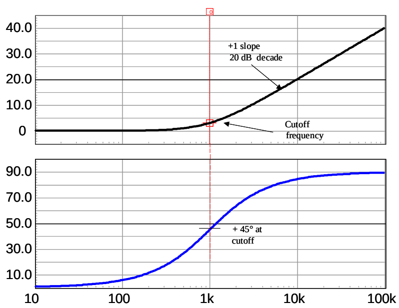
$$20\log_{10}\left|\frac{V_{out}(s_{p_1})}{V_{in}(s_{p_1})}\right| = 20\log_{10}\frac{|1|}{\left|1+\frac{s_{p_1}}{s_{p_1}}\right|} = 20\log_{10}\frac{1}{\sqrt{2}} = -3\,dB \qquad \text{At } f = f_{p1}$$

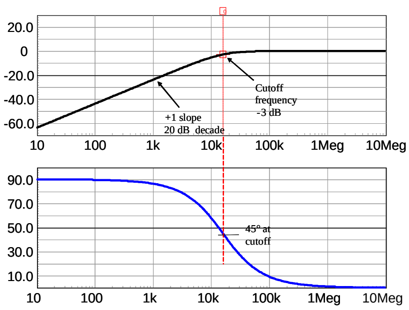
$$\arg\frac{V_{out}(s_{p_1})}{V_{in}(s_{p_1})} = \arg(1) - \arg\left(1+\frac{s_{p_1}}{s_{p_1}}\right) = -\arctan(1) = -\frac{\pi}{4} \qquad \text{At } f = f_{p1}$$

$$\arg\frac{V_{out}(\infty)}{V_{in}(\infty)} = \arg(1) - \arg\left(1+\frac{\infty}{s_{p_1}}\right) = -\arctan(\infty) = -\frac{\pi}{2} \qquad \text{At } f = \infty$$

The Zero

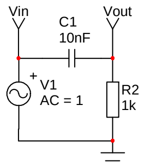
- ❑ A zero boosts the phase by +45° at its cutoff frequency





The general form of a zero:

$$G(s) = 1+\frac{s}{\omega_0}$$



$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{sRC}{1+sRC} = \frac{\frac{s}{\omega_0}}{1+\frac{s}{\omega_0}}$$

$$\omega_0 = \frac{1}{RC}$$

The zero

- Its magnitude at the cuttoff frequency is +3 dB
- Its asymptotic phase, when in LHP, at $f = \infty$ is +90°
- The zero "boosts" the phase

$$\frac{V_{out}(s)}{V_{in}(s)} = 1 + \frac{s}{\omega_0}$$

$$20\log_{10}\left|\frac{V_{out}(s_{z_1})}{V_{in}(s_{z_1})}\right| = 20\log_{10}\left|1+\frac{s_{z_1}}{s_{z_1}}\right| = 20\log_{10}\sqrt{2} = +3\,dB$$ At $f = f_{z1}$

$$\arg\frac{V_{out}(s_{p_1})}{V_{in}(s_{p_1})} = \arg\left(1+\frac{s_{z_1}}{s_{z_1}}\right) = \arctan(1) = +\frac{\pi}{4}$$ At $f = f_{z1}$

$$\arg\frac{V_{out}(\infty)}{V_{in}(\infty)} = \arg\left(1+\frac{\infty}{s_{p_1}}\right) = \arctan(\infty) = +\frac{\pi}{2}$$ At $f = \infty$

Poles and zeros at the origin

- Poles and zeros can sometimes appear "at the origin"

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{\frac{s}{\omega_0}}{D(s)}$$ Zero for s = 0: zero at the origin → As f increases the gain increases with a +1 slope (+20 dB/decade)

$$\arg\frac{V_{out}(s)}{V_{in}(s)} = \arg\left(\frac{\frac{s}{s_{zo}}}{0}\right) = \arctan(\infty) = +\frac{\pi}{2}$$ For $f > f_{zo}$

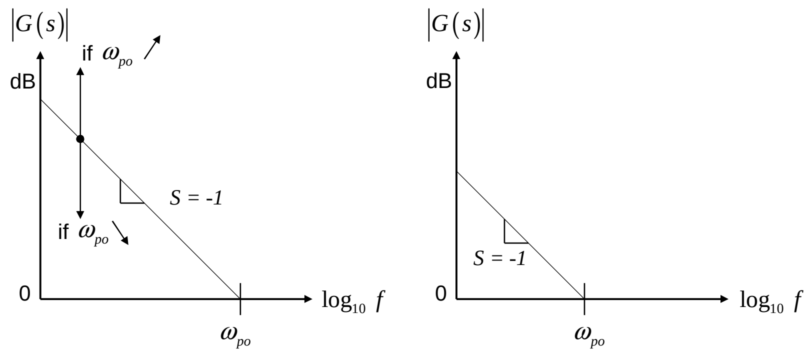
$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{N(s)}{\frac{s}{\omega_0}}$$ Pole for s = 0: pole at the origin → As f increases the gain decreases with a -1 slope (-20 dB/decade)

$$\arg\frac{V_{out}(s)}{V_{in}(s)} = \arg(1) - \arg\left(\frac{\frac{s}{s_{po}}}{0}\right) = -\arctan(\infty) = -\frac{\pi}{2}$$ For $f > f_{po}$

A pole at the origin introduces a fixed phase lag of -90°

Poles and zeros at the origin

- ❑ The intregration time constant changes the 0-dB crossover frequency



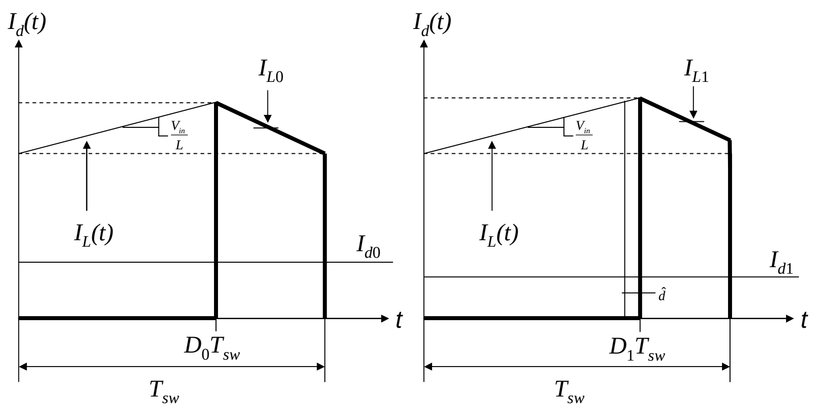
$$G(s)=\frac{1}{sRC}=\frac{1}{s/\omega_{po}}$$

$s = 0$ is the origin pole

$\omega_{po}=\frac{1}{RC}$ is the <u>0-dB crossover pole frequency</u>

The Right-Half-Plane Zero

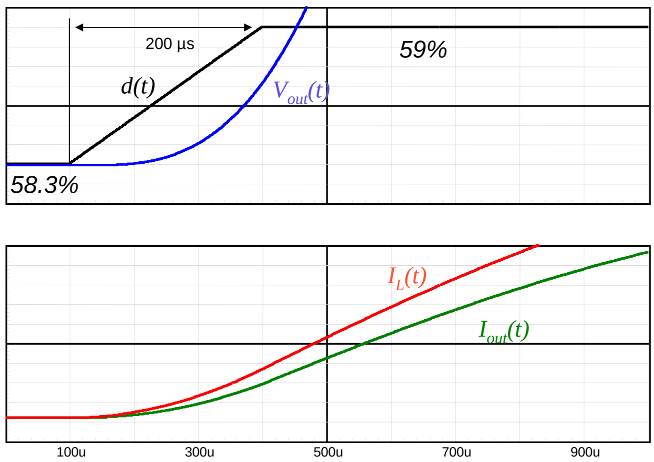
- ❑ In a CCM boost, I_{out} is delivered during the off time: $I_{out}=I_d=I_L(1-D)$



- ❑ If D <u>brutally</u> increases, D' reduces and I_{out} drops!
- ❑ What matters is the inductor current slew-rate → $\frac{d\langle V_L\rangle(t)}{dt}$

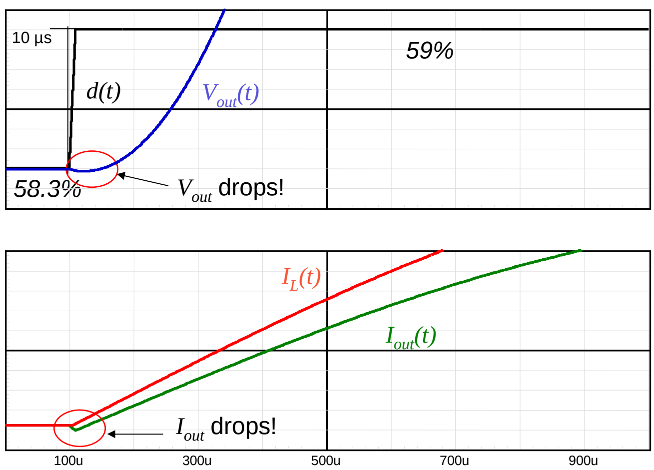
The Right-Half-Plane Zero

❑ If $I_L(t)$ can rapidly change, I_{out} increases when D goes up



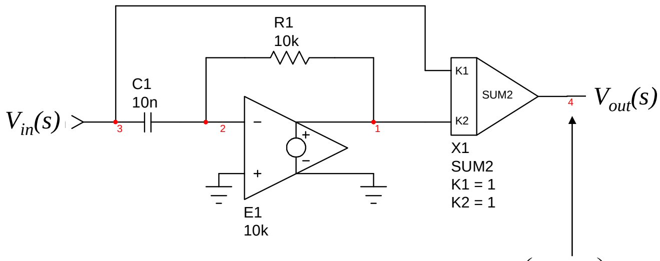
The Right-Half-Plane Zero

❑ If $I_L(t)$ is limited because of a big L, I_{out} drops when D increases



The Right-Half-Plane Zero

- ❑ To limit the effects of the RHPZ, limit the duty ratio slew-rate
- ❑ Chose a crossover frequency equal to 20-30% of RHPZ position
- ▪ A simple RHPZ can be easily simulated:

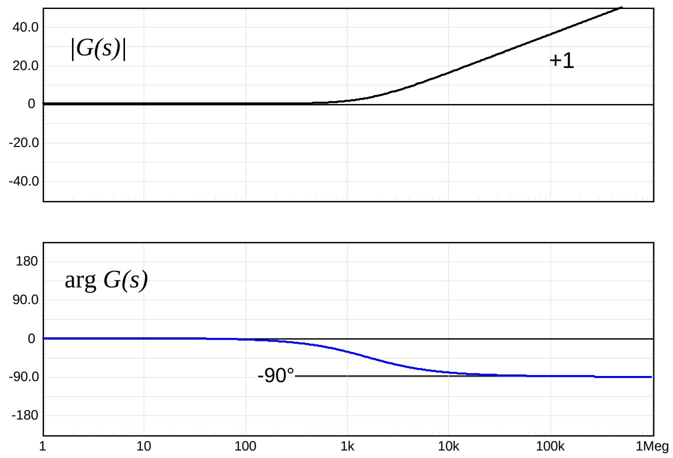


$$V_{out}(s) = V_{in}(s) - V_{in}(s)\frac{R_1}{\frac{1}{sC_1}} = V_{in}(s)\left(1 - \frac{s}{\omega_0}\right)$$

The neg. sign confirms for the RHPZ presence

The Right-Half-Plane Zero

- ❑ With a RHPZ we have a boost in gain but a lag in phase!



LHPZ

$$G(s) = 1 + \frac{s}{\omega_0}$$

RHPZ

$$G(s) = 1 - \frac{s}{\omega_0}$$

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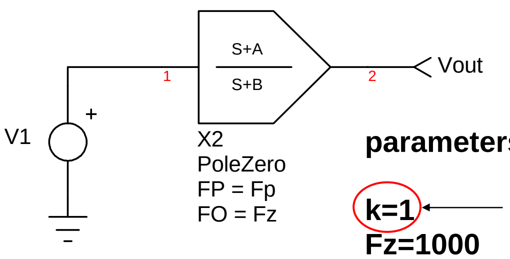
Combining poles and zeros

- ❑ We know that a pole lags the signal phase by -90°
- ❑ A zero boosts the signal phase by +90°
- ➢ if we combine both in *G(s)*, we can control the phase from:
- ➢ 0° if the pole and the zero are coincident
- ➢ +90° if the pole and the zero are split far away from e ach other

up to 180° with double pole/zero pair!



parameters

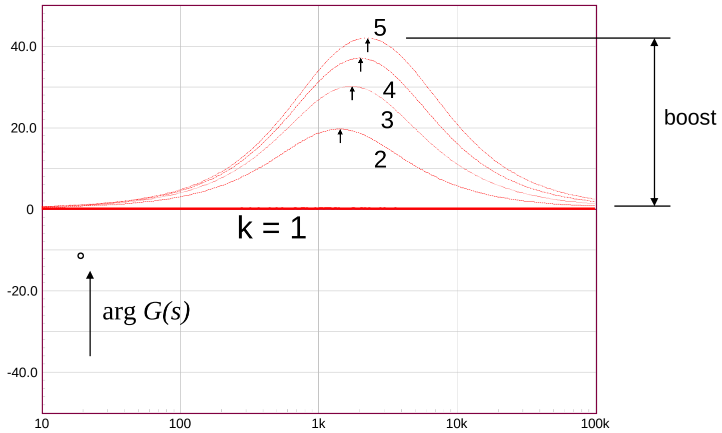
k=1 ← k is swept from 1 to 10

Fz=1000

Fp=1000*k

Combining poles and zeros

- ❑ When the pole and zero are coincident (k = 1), no boost
- ❑ The farther they are, the greater the boost
- ❑ As the pole/zero split appart, *f* at which the boost peaks, changes



Combining poles and zeros

- ❑ The equation where a pole and a zero are combined is:

$$G(s)=\frac{\left(1+\frac{s}{s_{z_1}}\right)}{\left(1+\frac{s}{s_{p_1}}\right)}=\frac{N}{D}$$

- ❑ The argument of a quotient is: arg *N* – arg *D*

$$\arg G(f)=\arctan\left(\frac{f}{f_{z_1}}\right)-\arctan\left(\frac{f}{f_{p_1}}\right)$$

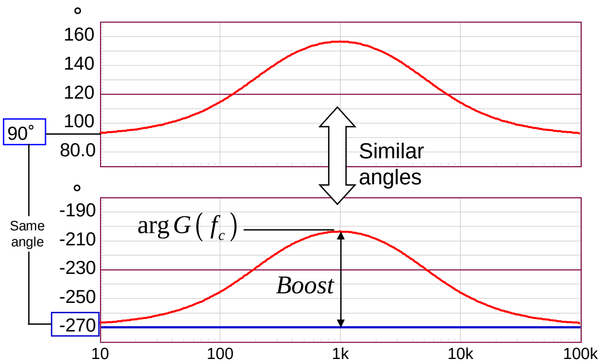
- ❑ Where does the phase peak (the boost) occur?

$$\frac{d\left(\arctan\left(\frac{f}{f_{z_1}}\right)-\arctan\left(\frac{f}{f_{p_1}}\right)\right)}{df}=0$$

Max boost occurs at: $f=\sqrt{f_{z_1} f_{p_1}}$

Do not forget the op amp

- ❑ In reality, poles and zeros are combined with an op amp
- ❑ To reduce the static error, we need a high dc gain
- ➢ A pole at the origin is almost always part of $G(s)$ → integrator
- ➢ An origin pole permanently lags the phase by -90°
- ➢ With the op amp, the minimum phase lag is: -90-180 = -270°



SPICE shows a +90° phase rotation rather than a -270° value. Why?

Because of the modulo 2π representation:

$$\theta = -\frac{3\pi}{2} \pm k2\pi$$

k =1

$$\theta = \frac{-3\pi + k4\pi}{2} = \frac{\pi}{2}$$

How to calculate the necessary boost?

- ❑ We know that arg $G(s)$ lags by -270° for $s = 0$
- ❑ The arguments sum of $G(f_c)$ and $H(f_c)$ must stay away from -360°
- ➢ φ_m is the distance between [arg $G(f_c)$ + arg $H(f_c)$] and -360°

$$\arg H(f_c) - 270° + BOOST - \varphi_m = -360°$$

$$\boxed{BOOST = \varphi_m - \arg H(f_c) - 90°}$$

- ❑ Assume a 4-kHz crossover frequency is wanted
- ❑ arg $H(4k)$ = -68°, how much boost for a 70° phase margin?

$$BOOST = 70 + 68 - 90 = 48° \Longrightarrow \arg G(4k) = -270 + 48 = -222°$$

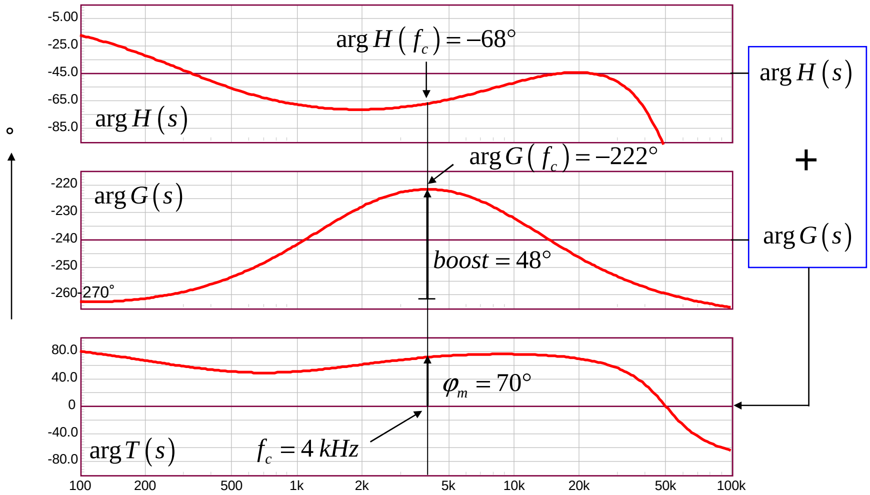
- ❑ Combining the previous equations, we have:

$$f_p = \left[\tan(boost) + \sqrt{\tan^2(boost) + 1}\right] f_c = 2.6 \times 4k = 10.4\,kHz$$

$$f_z = \frac{f_c^{\,2}}{f_p} = \frac{16k}{10.4k} \approx 1.54\,kHz$$

How to calculate the necessary boost?

❑ The pole has been placed at 10.4 kHz and the zero at 1.5 kHz



How to crossover at f_c then?

❑ We know how to create the boost by placing 1 pole and 1 zero

❑ How do we now create the right gain at crossover?

❑ The final formula for $G(s)$ must include the 0-dB crossover pole:

$$G(s)=\frac{\left(1+\frac{s}{s_{z_1}}\right)}{\frac{s}{s_{p_o}}\left(1+\frac{s}{s_{p_1}}\right)}=\frac{\frac{s}{s_{z_1}}\left(1+\frac{s_{z_1}}{s}\right)}{\frac{s}{s_{p_o}}\left(1+\frac{s}{s_{p_1}}\right)}=\frac{s_{p_o}}{s_{z_1}}\frac{\left(1+\frac{s_{z_1}}{s}\right)}{\left(1+\frac{s}{s_{p_1}}\right)}$$

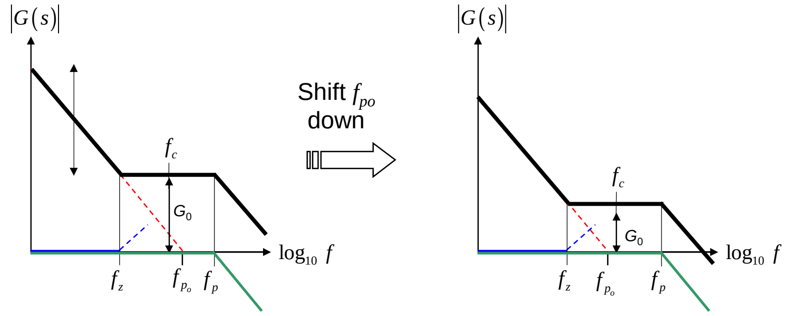
❑ The ratio s_{p_o}/s_{z_1} can be expressed as G_0:

$$G(s)=G_0\frac{\left(1+\frac{s_{z_1}}{s}\right)}{\left(1+\frac{s}{s_{p_1}}\right)}$$

By adjusting the 0-dB crossover pole frequency f_{po}, you can tailor the gain at crossover.

Shift s_{po} to adjust the crossover gain

- ❑ The zero is fixed to get the proper phase boost
- ❑ By adjusting the 0-dB crossover pole position, you adjust the gain at f_c



- ❑ This so-called <u>mid-band gain</u> makes *T(s)* crossover at f_c
- ❑ Always write compensator transfer function with G_0: $G(s) = G_0 A(s)$

Agenda

What is a type 1 amplifier?

- ❑ In some cases, you do not need phase boost at all
  - ➢ If arg *H(s)* is smaller than -45° within the band of interest:

$$\arg H(f_c) - 270° \le -315° \Longrightarrow \varphi_m \ge 45°$$

  - ➢ A 45° phase margin is guaranteed
- ❑ There is an origin pole at $s = 0$

$$G(s) = -\frac{1}{\frac{s}{s_{p_o}}} = -\frac{s_{p_o}}{s} \Longrightarrow G(j\omega) = j\frac{\omega_{p_o}}{\omega} \Longrightarrow |G(f)| = \frac{f_{p_o}}{f}$$

$$\Longrightarrow \arg G(s) = \arg(-1) - \arg\left(\frac{s/s_{p_o}}{0}\right) = -\pi - \arctan(\infty) = -\frac{3\pi}{2}$$

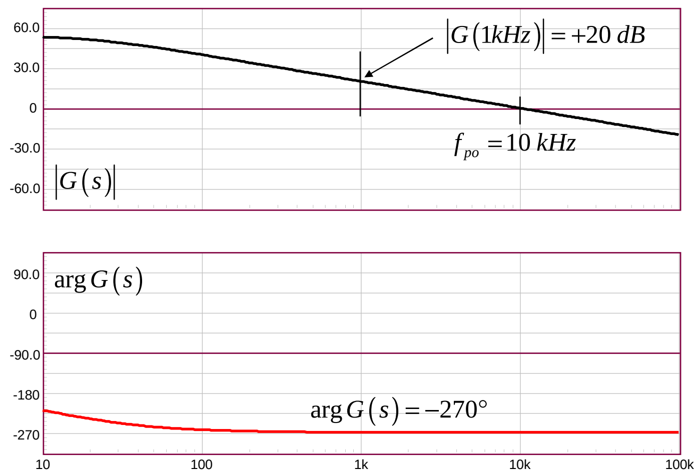
- ❑ Select f_{po} depending on the wanted gain at crossover:

$$|G(1kHz)| = 20\ dB \longrightarrow |G(1kHz)| = 10^{|G|/20} = 10^1 = 10$$

$$f_{po} = 10 f_c = 10\ kHz$$

What Bode plot for the type 1?

- ❑ The type 1 does not provide phase boost at all



What is a type 2 amplifier?

❑ In the vast majority of cases, phase boost is needed
❑ If the needed phase boost is less than 90°, a type 2 can do the job
➢ an origin pole plus a zero and a pole:

$$G(s) = -\frac{1+\frac{s}{s_{z_1}}}{\frac{s}{s_{p_o}}\left(1+\frac{s}{s_{p_1}}\right)} \overset{\text{factor } s/s_{z_1}}{=} -G_0\frac{1+\frac{s_{z_1}}{s}}{1+\frac{s}{s_{p_1}}} \quad \text{with } G_0 = \frac{s_{p_o}}{s_{z_1}}$$

❑ The magnitude is derived as:

$$|G(f)| = G_0\frac{\sqrt{1+\left(\frac{f_{z_1}}{f}\right)^2}}{\sqrt{1+\left(\frac{f}{f_{p_1}}\right)^2}}$$

❑ The argument is found to be:

$$\arg G(f) = \arctan\left(-\frac{f_{z_1}}{f}\right) - \pi - \arctan\left(\frac{f}{f_{p_1}}\right)$$

$$boost = \arctan\left(f/f_{z_1}\right) - \arctan\left(f/f_{p_1}\right)$$

❑ The 0-dB crossover pole frequency is placed at: $f_{po} = G_0 f_{z_1}$

Where to place the poles and zero?

❑ First, place the pole and zero for the needed phase boost
❑ Then adjust the origin pole 0-dB frequency at the right value
➢ 5-kHz crossover gain deficiency is -18 dB, required boost is +68°

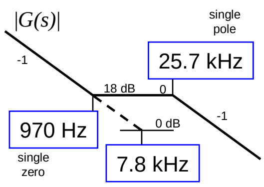
$$f_{p_1} = \left[\tan(boost) + \sqrt{\tan^2(boost)+1}\right] f_c = 5.14 \times 5k = 25.7\ kHz$$

$$f_{z_1} = \frac{f_c^{\,2}}{f_{p_1}} = \frac{25k}{25.7k} \approx 970\ Hz$$

❑ A +18-dB gain is necessary at 5 kHz:

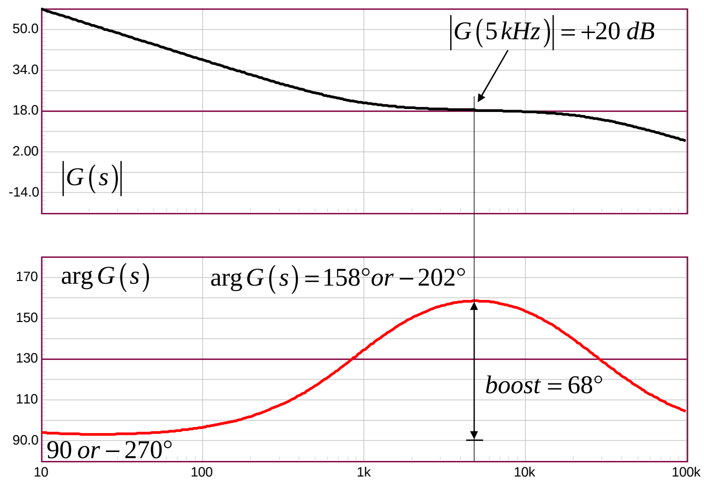
$$|G(5kHz)| = 18\ dB \longrightarrow |G(5kHz)| = 10^{|G|/20} \approx 8$$

$$f_{po} = 8 f_{z_1} = 7.8\ kHz$$



What Bode plot for a type 2?

- ❑ The type 2 provides phase boost up to +90°



What is a type 3 amplifier?

- ❑ Sometimes, a phase boost greater than 90° is needed
- ❑ By doubling the pole and zero, we can boost up to 180°

$$G(s) = -\frac{\left(1+\frac{s}{s_{z_1}}\right)\left(1+\frac{s}{s_{z_2}}\right)}{\frac{s}{s_{p_o}}\left(1+\frac{s}{s_{p_1}}\right)\left(1+\frac{s}{s_{p_2}}\right)} = -\frac{s_{p_o}}{s_{z_1}}\frac{\left(1+\frac{s_{z_1}}{s}\right)\left(1+\frac{s}{s_{z_2}}\right)}{\left(1+\frac{s}{s_{p_1}}\right)\left(1+\frac{s}{s_{p_2}}\right)}$$

- ❑ The magnitude is derived as:

$$|G(f)| = \frac{f_{p_o}}{f_{z_1}}\frac{\sqrt{1+\left(\frac{f_{z_1}}{f}\right)^2}\sqrt{1+\left(\frac{f}{f_{z_2}}\right)^2}}{\sqrt{1+\left(\frac{f}{f_{p_1}}\right)^2}\sqrt{1+\left(\frac{f}{f_{p_2}}\right)^2}}$$

- ❑ The argument is found to be:

$$\arg G(f) = \arg N - \arg D$$

$$\arg N = \arctan\left(-\frac{f_{z_1}}{f}\right) - \pi + \arctan\left(\frac{f}{f_{z_2}}\right)$$

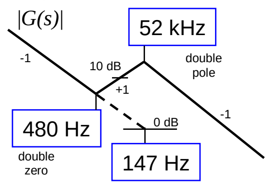
$$\arg D = \arctan\left(\frac{f}{f_{p_1}}\right) + \arctan\left(\frac{f}{f_{p_2}}\right)$$

Where to place the poles and zeros?

- ❑ Poles and zeros can be coincident (*k* factor) or split
- ❑ Place the double pole and the double zero to get the boost
- ❑ Then adjust the origin pole 0-dB frequency at the right value
- ➢ 5-kHz crossover gain deficiency is +10 dB, required boost is +158°
- ❑ If we consider coincident poles and zeros:

$$Boost = 2\left[\arctan\left(\frac{f_p}{f_c}\right) - \arctan\left(\frac{f_c}{f_p}\right)\right] \qquad f_c = 5\ kHz = \sqrt{f_z f_p}$$

$$f_{p_{1,2}} = \frac{f_c}{\tan\left(45 - \frac{Boost}{4}\right)} = \frac{5k}{96.3m} \approx 52\ kHz \qquad f_{z_{1,2}} = \frac{f_c^{\ 2}}{f_{p_{1,2}}} = \frac{25k}{52k} \approx 480\ Hz$$



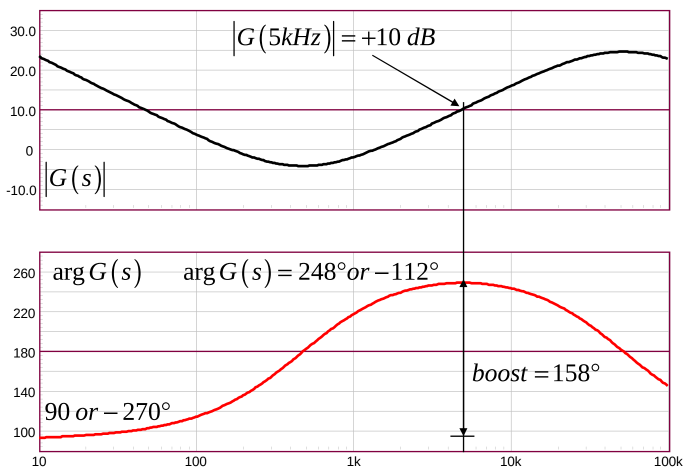
- ❑ A +10-dB gain is necessary at 5 kHz:

$$\left|G(5kHz)\right| = 10\ dB \longrightarrow \left|G(5kHz)\right| = 10^{|G|/20} \approx 3.2$$

$$f_{po} = \frac{G_{f_c}}{f_c}\left(f_{z_{1,2}}\right)^2 \approx 147\ Hz$$

What Bode plot for a type 3?

- ❑ The type 3 provides phase boost up to +180°



When to use these compensators?

❑ **Type 1** is used where no phase boost is necessary at crossover
❑ If a 45° φ_m is ok, a type 1 can be used where arg $H(f_c)$ < 45°:
➢ Power Factor Correction circuits
➢ Current mode power supplies in CCM, DCM or CrM (BCM)
➢ Voltage-mode power supplies in DCM
❖ Pure integrator, brings output overshoot

❑ **Type 2** is targetting applications where a phase boost is necessary
❑ In the above examples where a φ_m larger than 45° is requested
❖ Most popular choice for current mode converters

❑ **Type 3** is selected where a large phase boost is mandatory
❑ This is the case for CCM voltage-mode converters
❑ Generally, 2nd order and beyond types of transfer functions

How to implement these compensators?

❑ **Operational Amplifier:** most documented architecture
➢ virtual ground arrangement excludes the resistive divider ratio
➢ high open-loop gain for reduced static error
➢ best flexibility for poles/zeros arrangement

❑ **Transconductance Amplifier:** mainly used in PFC circuits
➢ offers a means to sense the output voltage on the feedback pin
❖ less flexibility for type 3 arrangement
❖ transconductance value appears in the poles/zeros equations

❑ **TL431:** the most popular architecture
➢ combines an op amp and a reference voltage: cheapest approach
➢ easy interface with an optocoupler
❖ low open-loop gain
❖ biasing requirements hamper its flexibility

Agenda

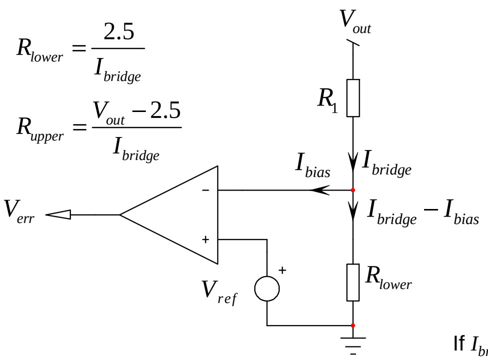
- ❑ Feedback generalities
- ❑ The divider and the virtual ground
- ❑ Phase margin and crossover
- ❑ Poles and zeros
- ❑ Boosting the phase at crossover
- ❑ Compensator types
- ❑ **Practical implementations: the op amp**
- ❑ Practical implementations: the OTA
- ❑ Practical implementations: the TL431
- ❑ Design examples
- ❑ A real case study
- ❑ Conclusion

Designing the divider network

- ❑ The design starts with the divider network ratio
- ❑ Make sure enough current circulates in the bridge:
  - ➢ it improves noise immunity
  - ➢ it shields you against offset current in the op amp
  - ❖ it degrades the no-load consumption…

$$R_{lower} = \frac{2.5}{I_{bridge}}$$

$$R_{upper} = \frac{V_{out} - 2.5}{I_{bridge}}$$



$$V_{lower} = V_{ref} = \left(I_{bridge} - I_{bias}\right) R_{lower}$$

$$I_{bridge} = \left(V_{out} - V_{ref}\right) / R_1$$

±500 nA NCP1608
6.5 µA for a TL431

$$V_{out} = V_{ref}\left(\frac{R_1}{R_{lower}} + 1\right) + R_1 I_{bias}$$

Watch out with PFC!

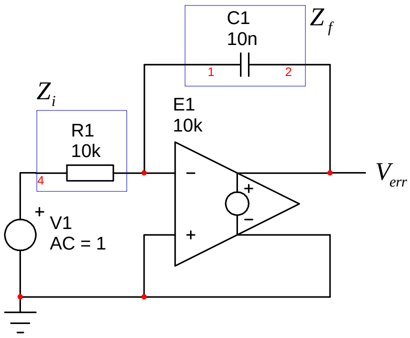
If $I_{bias} << I_{bridge}$:

$$V_{out} \approx V_{ref}\left(\frac{R_1}{R_{lower}} + 1\right)$$

If I_{bridge} = 250 µA → $R_{lower} = 2.5/250u = 10\ k\Omega$

Type 1 with an op amp

❑ Type 1 is an inverting integrator providing one pole at the origin



$$G(s) = \frac{Z_f}{Z_i} = \frac{\frac{1}{sC_1}}{R_1} = \frac{1}{sC_1R_1}$$

$$\omega_{po} = \frac{1}{R_1C_1} \Rightarrow G(s) = -\frac{1}{\frac{s}{\omega_{po}}}$$

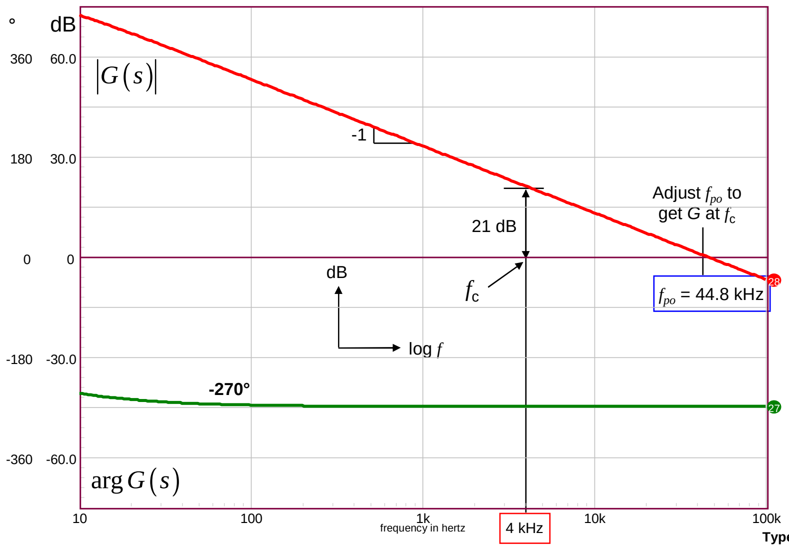
$$|G(j\omega)| = \left| j\frac{\omega_{po}}{\omega} \right| = \sqrt{\left(\frac{\omega_{po}}{\omega}\right)^2} = \frac{\omega_{po}}{\omega}$$

If you need a +21-dB gain to crossover at 4 kHz, where to place the 0-dB crossover pole?

$$f_{po} = G_{f_c} f_c = 10^{21/20} \times 4k = 44.8\,kHz$$

Type 1 – op amp

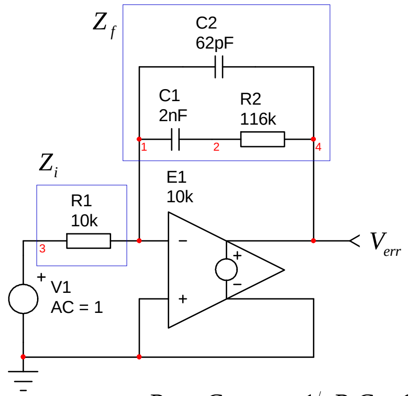
Type 1 with an op amp – Bode plot



Type 1 – op amp

Type 2 with an op amp (full analysis)

❑ Type 2 keeps the origin pole but adds one zero and one extra pole



$$\frac{Z_f}{Z_i} = \frac{\left(\frac{1}{sC_1}+R_2\right)\frac{1}{sC_2} \Big/ \left(\frac{1}{sC_1}+R_2\right)+\frac{1}{sC_2}}{R_1}$$

Re-arrange

$$G(s) = -\frac{1+sR_2C_1}{sR_1(C_1+C_2)\left(1+sR_2\left[\frac{C_1C_2}{C_1+C_2}\right]\right)}$$

Factor sR_2C_1

$$G(s) = -\frac{R_2}{R_1}\frac{C_1}{C_1+C_2}\frac{1/sR_2C_1+1}{\left(1+sR_2\left[\frac{C_1C_2}{C_1+C_2}\right]\right)} = -G_0\frac{1+s_z/s}{1+s/s_p}$$

Type 2 – op amp

Type 2 with an op amp (full analysis)

❑ In the gain expression, we have:

$$G_0 = \frac{R_2}{R_1}\frac{C_1}{C_1+C_2} \qquad \omega_z = \frac{1}{R_2C_1} \qquad \omega_p = \frac{1}{R_2\frac{C_1C_2}{C_1+C_2}}$$

❑ As ω_z, ω_p, G_0 and R_1 are given (boost, V_{out} etc.) how to get R_2?

$$\left|G(f_c)\right| = G_0\frac{\sqrt{1+\left(\frac{f_z}{f_c}\right)^2}}{\sqrt{1+\left(\frac{f_c}{f_p}\right)^2}} \Longrightarrow R_2 = \frac{G_{f_c}R_1f_p}{f_p-f_z}\frac{\sqrt{\left(\frac{f_c}{f_p}\right)^2+1}}{\sqrt{\left(\frac{f_z}{f_c}\right)^2+1}}$$

❑ Other component values are then extracted:

$$C_1 = \frac{1}{2\pi R_2 f_z} \qquad C_2 = \frac{C_1}{2\pi f_p C_1 R_2 - 1}$$

Type 2 – op amp

Type 2 with an op amp (simplified analysis)

- In most cases, C_2 is much smaller than C_1. Therefore:

$$G(s) \approx -\frac{R_2}{R_1}\frac{1/sR_2C_1+1}{(1+sR_2C_2)} = -G_0\frac{1+s_z/s}{1+s/s_p}$$

$$G_0 \approx \frac{R_2}{R_1} \qquad \omega_z = \frac{1}{R_2C_1} \qquad \omega_p \approx \frac{1}{R_2C_2}$$

$$|G(f_c)| = \frac{R_2}{R_1}\frac{\sqrt{1+\left(\frac{f_z}{f_c}\right)^2}}{\sqrt{1+\left(\frac{f_c}{f_p}\right)^2}} \Rightarrow R_2 = G_{fc}R_1\frac{\sqrt{\left(\frac{f_c}{f_p}\right)^2+1}}{\sqrt{\left(\frac{f_z}{f_c}\right)^2+1}}$$

Type 2 – op amp

Type 2 with an op amp – design example

- You need to provide a 15-dB gain at 5 kHz with a 50°boo st
- How to calculate the component values?

$$f_p = \left[\tan(boost) + \sqrt{\tan^2(boost)+1}\right] f_c = 2.74 \times 5k = 13.7\ kHz$$

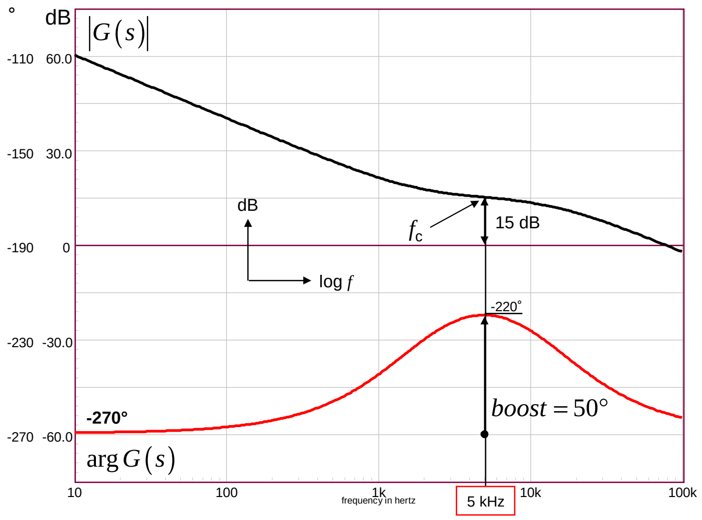
$$f_z = \frac{f_c^{\,2}}{f_p} = \frac{25k}{13.7k} \approx 1.8\ kHz \qquad R_2 = \frac{G_{f_c}R_1 f_p}{f_p - f_z}\frac{\sqrt{\left(\frac{f_c}{f_p}\right)^2+1}}{\sqrt{\left(\frac{f_z}{f_c}\right)^2+1}} = 64.8\ k\Omega$$

$$C_1 = \frac{1}{2\pi R_2 f_z} = 1.3\ nF \quad C_2 = \frac{C_1}{2\pi f_p C_1 R_2 - 1} = 206\ pF$$

- You can use the simplified formula in the general case
- For PFCs, C_2 is not small compared to C_1, use full formulas

Type 2 – op amp

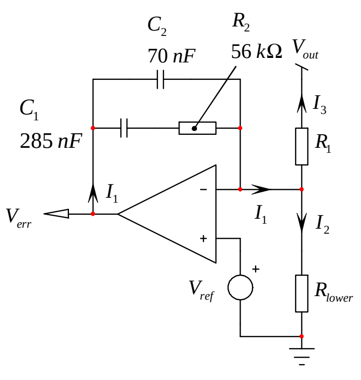
Type 2 with an op amp – Bode plot



Type 2 – op amp

Type 2 with an op amp – start-up issue

- For low bandwidth systems, capacitor values can be large
- For a PFC circuit, crossover can be as low as 20 Hz
- For a zero at 10 Hz and a pole at 40 Hz, we have:



- At power up, V_{out} = 0 and $R_1 >> R_{lower}$
- V_{err} should go to the op amp V_{cc}
- Big caps are in the compensation path
- I_1 is limited by R_1 and R_{lower}
  - op amp output slowly rises
  - delays the full power delivery

$$I_2 = I_1 - I_3 = \frac{V_{ref}}{R_{lower}} \qquad I_3 = \frac{V_{ref} - V_{out}}{R_1}$$

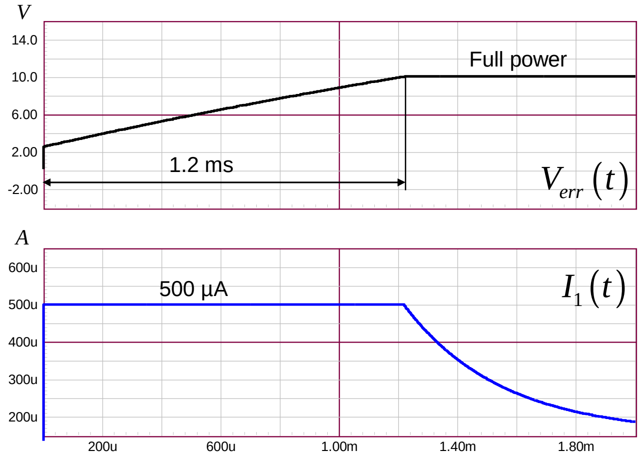
$$I_1(t) = \frac{V_{ref}}{R_{lower}} + \frac{V_{ref} - V_{out}(t)}{R_1} \approx \frac{V_{ref}}{R_{lower}}$$

$V_{out} = 0,\ R_{upper} >> R_{lower}$

Type 2 – op amp

Type 2 with an op amp – start-up issue

- The capacitor charging acts as an inexpensive soft-start
- If too small, V_{out} rises too quickly and an overshoot appears

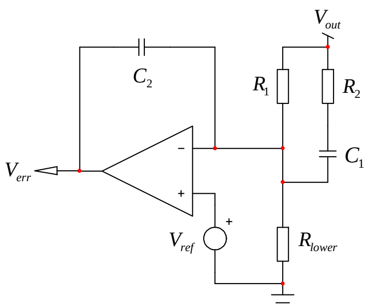


Type 2 – op amp

ON Semiconductor®

Type 2 with op amp – a different arrangement

- In the compensator, all the current flows in C_2, the smallest value
- Why not placing it differently then?



$$\frac{Z_f}{Z_i} = \frac{\frac{1}{sC_2}}{\left(\frac{1}{sC_1}+R_2\right)R_1 \Big/ \left(\frac{1}{sC_1}+R_2\right)+R_1}$$

$$G(s) = -\frac{(R_1+R_2)C_2}{R_1C_1}\,\frac{\frac{1}{s(R_1+R_2)C_2}+1}{1+sR_2C_2}$$

$$G_0 = \frac{R_1+R_2}{R_1}\frac{C_2}{C_1} \qquad C_2 = \frac{1}{\omega_z(R_1+R_2)} \qquad C_1 = \frac{1}{G_0R_1\omega_z} \qquad R_2 = \frac{R_1\omega_z}{\omega_p-\omega_z}$$

Type 2 – op amp

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Type 2 with op amp – automated calculation

A simple macro can be written to calculate all the elements around the compensator for both options.

parameters

R1=10k

fc=20
Gfc=-10
k=2

G=10^(-Gfc/20)
pi=3.14159

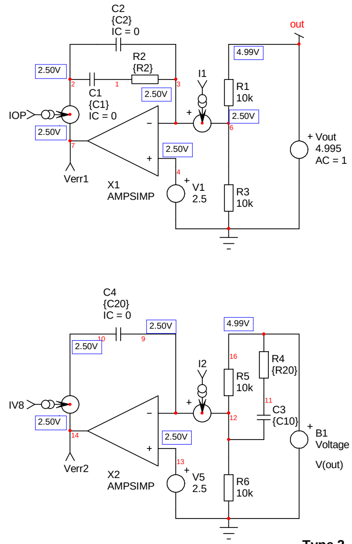
fp=fc*k
fz=fc/k

Wp=fp*2*pi
Wz=fz*2*pi

R20=R1*Wz/(Wp-Wz)
C10=1/(Wz*(R1+R20))
C20=1/(G*R1*Wz)

a=sqrt((fc^2/fp^2)+1)
b=sqrt((fz^2/fc^2)+1)

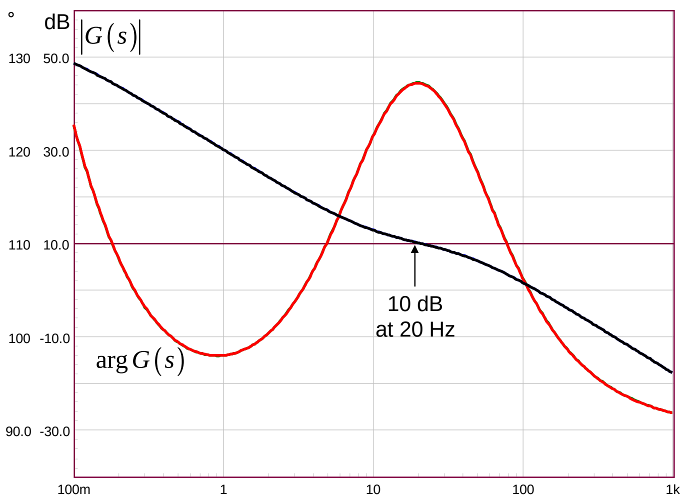
R2=((a/b)*G*R1*fp)/(fp-fz)
C2=C1/(C1*R2*2*pi*fp-1)
C1=1/(2*pi*R2*fz)



Type 2 – op amp

Type 2, Bode plot for both solutions

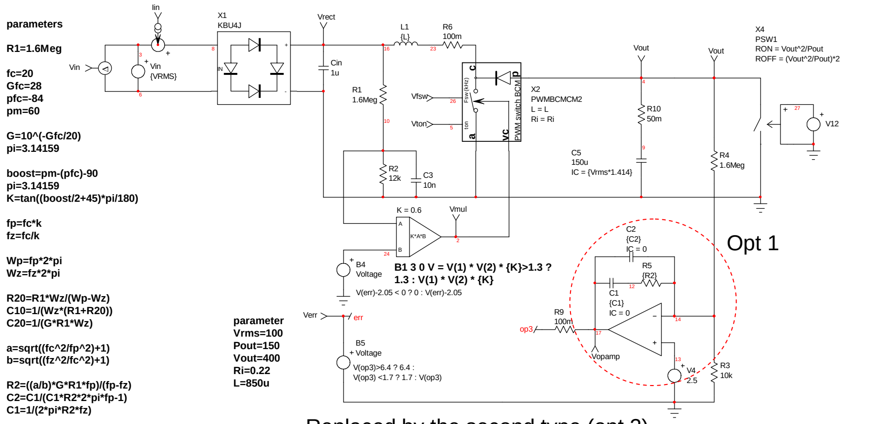
- ❑ Both curves perfectly superimpose on each other



Type 2 – op amp

Type 2 in a PFC circuit

- ❑ An average model is used to test both structures
- ❑ Start-up and transient response is studied in each case



Replaced by the second type (opt 2)

Type 2 – op amp

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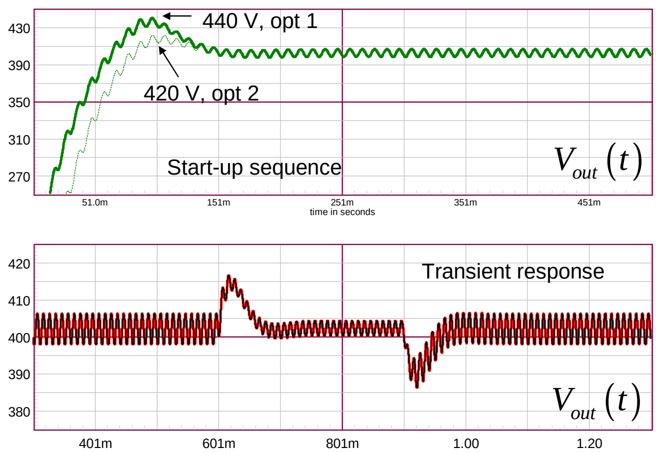
Type 2 in a PFC circuit – transient response

- ❑ The small-signal response is similar (f_c = 20 Hz, φ_m = 60°)
- ❑ Overshoot is reduced by 5% in the second option



Type 2 – op amp

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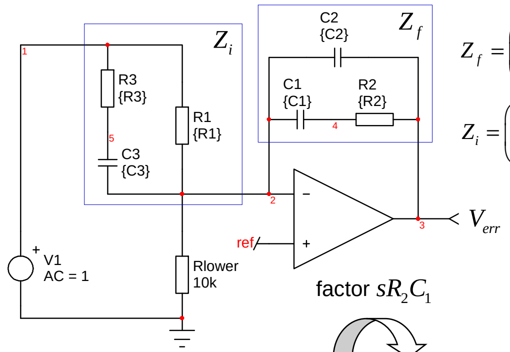
Type 3 with an op amp (full analysis)

❑ Type 3 keeps the origin pole but add a zero/pole pair



$$Z_f = \left(\frac{1}{sC_1}+R_2\right)\frac{1}{sC_2} \bigg/ \left(\left(\frac{1}{sC_1}+R_2\right)+\frac{1}{sC_2}\right)$$

$$Z_i = \left(\frac{1}{sC_3}+R_3\right)R_1 \bigg/ \left(\left(\frac{1}{sC_3}+R_3\right)+R_1\right)$$

$$G(s) = -\frac{\left(\frac{1}{sC_1}+R_2\right)\frac{1}{sC_2} \bigg/ \left(\left(\frac{1}{sC_1}+R_2\right)+\frac{1}{sC_2}\right)}{\left(\frac{1}{sC_3}+R_3\right)R_1 \bigg/ \left(\left(\frac{1}{sC_3}+R_3\right)+R_1\right)} = -\frac{R_2C_1}{R_1(C_1+C_2)}\,\frac{\frac{1}{sR_2C_1}+1}{1+sR_2\frac{C_1C_2}{C_1+C_2}}\,\frac{sC_3(R_1+R_3)+1}{sR_3C_3+1}$$

Type 3 – op amp

Type 3 with an op amp (full analysis)

❑ Re-write the expression in a more familiar form:

$$G(s) = -G_0\frac{\left(1+\frac{s_{z_1}}{s}\right)\left(1+\frac{s}{s_{z_2}}\right)}{\left(1+\frac{s}{s_{p_1}}\right)\left(1+\frac{s}{s_{p_2}}\right)} \qquad G_0 = \frac{R_2}{R_1}\frac{C_1}{C_1+C_2}$$

$$\omega_{z_1} = \frac{1}{R_2C_1} \qquad \omega_{p_1} = \frac{1}{R_2\frac{C_1C_2}{C_1+C_2}}$$

$$\omega_{z_2} = \frac{1}{(R_1+R_3)C_3} \qquad \omega_{p_2} = \frac{1}{R_3C_3}$$

❑ As ω_{z12}, ω_{p12}, G_0 and R_1 are given (boost, V_{out} etc.) how to get R_2?

$$|G(f_c)| = G_0\frac{\sqrt{1+\left(\frac{f_{z_1}}{f_c}\right)^2}\sqrt{1+\left(\frac{f_c}{f_{z_2}}\right)^2}}{\sqrt{1+\left(\frac{f_c}{f_{p_1}}\right)^2}\sqrt{1+\left(\frac{f_c}{f_{p_2}}\right)^2}} \Rightarrow R_2 = \frac{G_{f_c}R_1 f_{p_1}}{f_{p_1}-f_{z_1}}\,\frac{\sqrt{1+\left(\frac{f_c}{f_{p_1}}\right)^2}\sqrt{1+\left(\frac{f_c}{f_{p_2}}\right)^2}}{\sqrt{1+\left(\frac{f_{z_1}}{f_c}\right)^2}\sqrt{1+\left(\frac{f_c}{f_{z_2}}\right)^2}}$$

Type 3 – op amp

Type 3 with an op amp (simplified analysis)

- Extract the rest of the elements:

$$C_1 = \frac{1}{2\pi f_{z_1} R_2} \qquad C_2 = \frac{C_1}{2\pi f_{p_1} C_1 R_2 - 1} \qquad C_3 = \frac{f_{p_2} - f_{z_2}}{2\pi R_{upper} f_{p_2} f_{z_2}} \qquad R_3 = \frac{R_1 f_{z_2}}{f_{p_2} - f_{z_2}}$$

- In most cases, $C_2 << C_1$ and $R_3 << R_1$. Therefore:

$$G(s) \approx -\frac{R_2}{R_1} \frac{\frac{1}{sR_2C_1} + 1}{1 + sR_2C_2} \frac{sC_3R_1 + 1}{sR_3C_3 + 1} \quad C_1 = \frac{1}{2\pi f_{z_1} R_2} \quad C_2 = \frac{1}{2\pi f_{p_1} R_2} \quad C_3 = \frac{1}{2\pi f_{z_2} R_1} \quad R_3 = \frac{1}{2\pi f_{p_2} C_3}$$

$$|G(f_c)| = \frac{R_2}{R_1} \frac{\sqrt{1+\left(\frac{f_{z_1}}{f_c}\right)^2}\sqrt{1+\left(\frac{f_c}{f_{z_2}}\right)^2}}{\sqrt{1+\left(\frac{f_c}{f_{p_1}}\right)^2}\sqrt{1+\left(\frac{f_c}{f_{p_2}}\right)^2}} \Rightarrow R_2 = G_{fc} R_1 \frac{\sqrt{1+\left(\frac{f_c}{f_{p_1}}\right)^2}\sqrt{1+\left(\frac{f_c}{f_{p_2}}\right)^2}}{\sqrt{1+\left(\frac{f_{z_1}}{f_c}\right)^2}\sqrt{1+\left(\frac{f_c}{f_{z_2}}\right)^2}}$$

Type 3 – op amp

Type 3 with an op amp – design example

- You need to provide a -10-dB gain at 5 kHz with a 145°b oost
- How to calculate the component values?

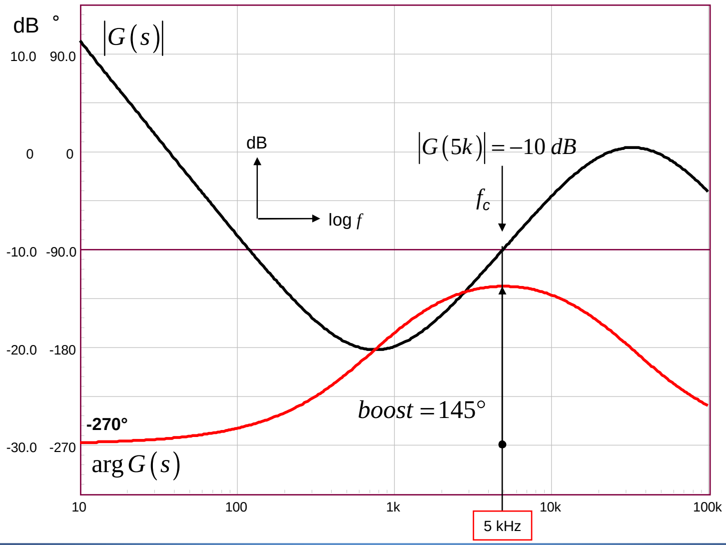
$$f_{p_{1,2}} = \frac{f_c}{\tan\left(45 - \frac{Boost}{4}\right)} = \frac{5k}{154m} \approx 32.5\ kHz \qquad f_{z_{1,2}} = \frac{f_c^{\ 2}}{f_{p_{1,2}}} = \frac{25k}{32.5k} \approx 769\ Hz$$

$$R_2 = \frac{G_{f_c} R_1 f_{p_1}}{f_{p_1} - f_{z_1}} \frac{\sqrt{1+\left(\frac{f_c}{f_{p_1}}\right)^2}\sqrt{1+\left(\frac{f_c}{f_{p_2}}\right)^2}}{\sqrt{1+\left(\frac{f_{z_1}}{f_c}\right)^2}\sqrt{1+\left(\frac{f_c}{f_{z_2}}\right)^2}} = 498\ \Omega$$

$$C_1 = 415\ pF \qquad C_2 = 10\ nF \qquad C_3 = 20\ nF \qquad R_3 = 242\ \Omega$$

Type 3 – op amp

Type 3 with an op amp – Bode plot



Type 3 – op amp

Agenda

- ❑ Feedback generalities
- ❑ The divider and the virtual ground
- ❑ Phase margin and crossover
- ❑ Poles and zeros
- ❑ Boosting the phase at crossover
- ❑ Compensator types
- ❑ Practical implementations: the op amp
- ❑ **Practical implementations: the OTA**
- ❑ Practical implementations: the TL431
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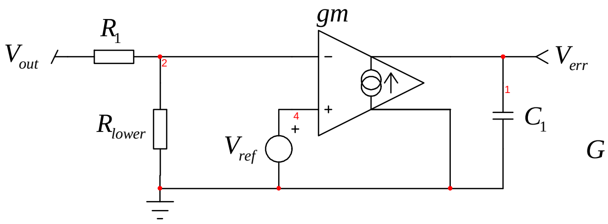
Type 1 with an OTA

- ❑ A type 1 with an OTA involves the transconductance value *gm*



$$I_{out} = -gmV_{in}\frac{R_{lower}}{R_{lower}+R_1}$$

$$G(s) = -gm\frac{R_{lower}}{R_{lower}+R_1}\frac{1}{sC_1}$$

$$G(s) = -\frac{1}{s\frac{R_{lower}+R_1}{gmR_{lower}}C_1} = -\frac{1}{sR_{eq}C_1} = -\frac{1}{\frac{s}{\omega_{po}}} \quad \text{with} \quad R_{eq} = \frac{R_{lower}+R_1}{gmR_{lower}}$$

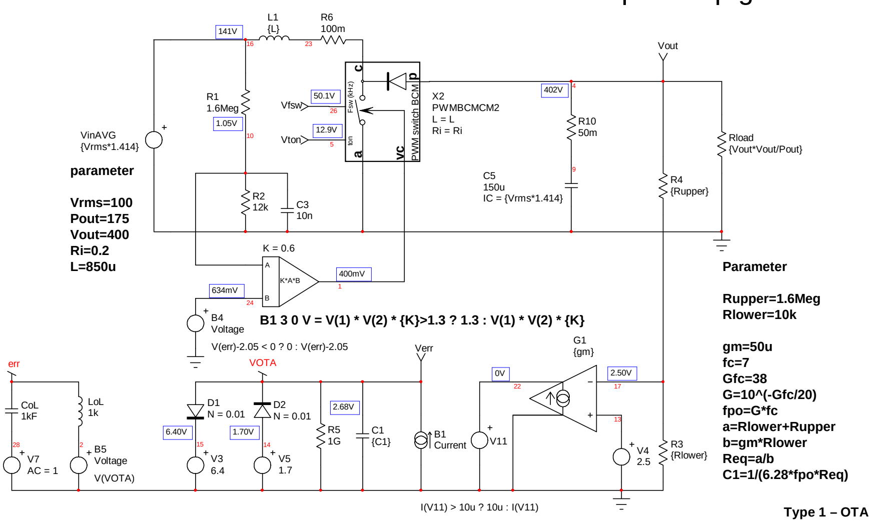
$$|G(s)| = \left|\frac{\omega_{po}}{j\omega}\right| = \frac{\omega_{po}}{\omega} \Rightarrow f_{po} = G_{fc}f_c \Rightarrow C_1 = \frac{1}{2\pi f_{po}\frac{R_{lower}+R_1}{gmR_{lower}}}$$

Type 1 – OTA

The divider network now enters the picture! →

Type 1 with an OTA – design example

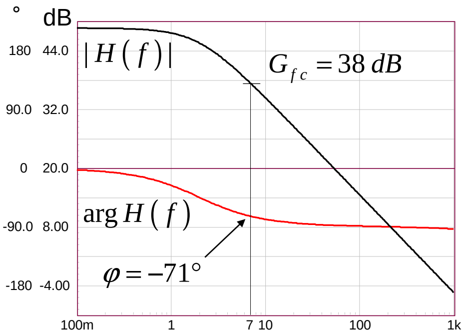
- ❑ A Borderline PFC with a MC33262 controller: open-loop gain test

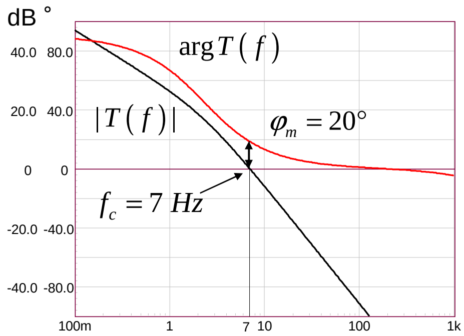


Type 1 – OTA

Type 1 with an OTA – design example

- A type 1 as examplified in the data-sheet give a weak φ_m!



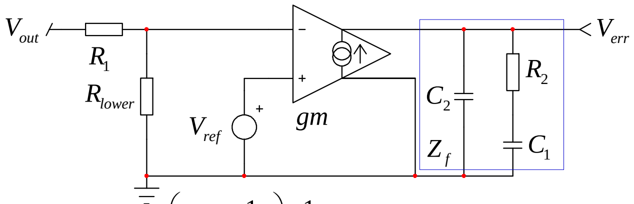


$$G_{fc} = 12.6m \qquad f_{po} = 12.6m \times 7 \approx 88\ mHz \longrightarrow C_1 = 560\ nF$$

Type 1 – OTA

Type 2 with an OTA

- A type 2 with an OTA requires the addition of a resistor and a cap.



$$G(s) = -\frac{R_{lower}}{R_1 + R_{lower}} gm \frac{\left(R_2 + \frac{1}{sC_1}\right)\frac{1}{sC_2}}{\left(R_2 + \frac{1}{sC_1}\right) + \frac{1}{sC_2}} \longrightarrow G(s) = -\frac{R_2 C_1}{C_1 + C_2} \frac{gmR_{lower}}{R_{lower} + R_1} \frac{\sqrt{1 + (f_z/f)^2}}{\sqrt{1 + (f/f_p)^2}}$$

Ratio dependent

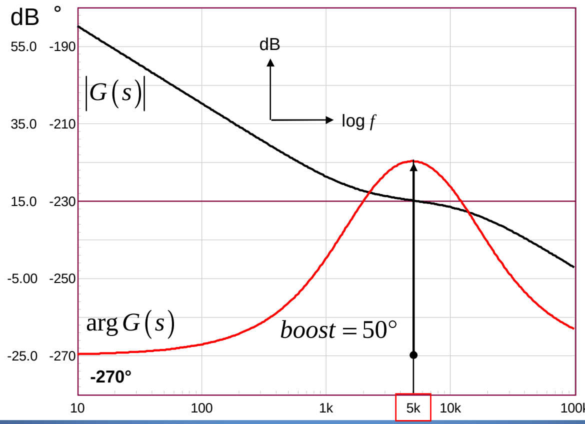
$$\omega_z = \frac{1}{R_2 C_1} \qquad \omega_p = \frac{1}{R_2\left(\frac{C_1 C_2}{C_1 + C_2}\right)} \qquad R_2 = \frac{G_{f_c} f_p}{f_p - f_z} \frac{R_{lower} + R_1}{gmR_1} \frac{\sqrt{1 + \left(\frac{f}{f_p}\right)^2}}{\sqrt{1 + \left(\frac{f_z}{f}\right)^2}}$$

Type 2 – OTA

Type 2 with an OTA – design example

- ❑ You need to provide a 15-dB gain at 5 kHz with a 50° boo st
- ❑ The poles and zero position are that of the op amp design

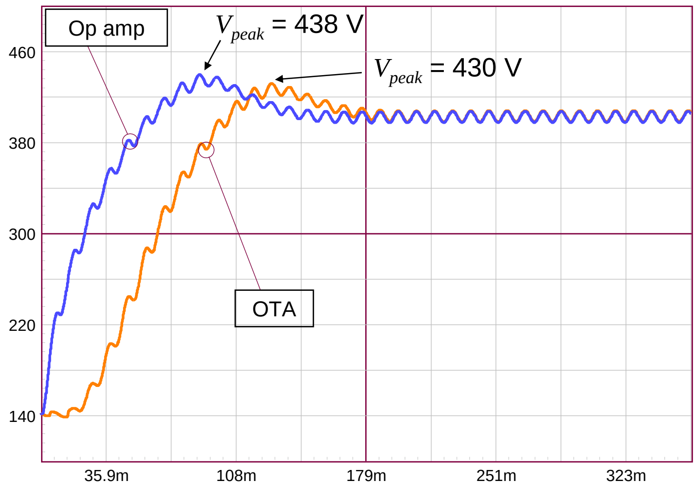
$R_2 = 260\ k\Omega \quad C_1 = 340\ pF \quad C_2 = 52\ pF \quad gm = 50\ \mu S \quad R_1 = R_{lower} = 10\ k\Omega$



Type 2 – OTA

PFC response: OTA versus op amp

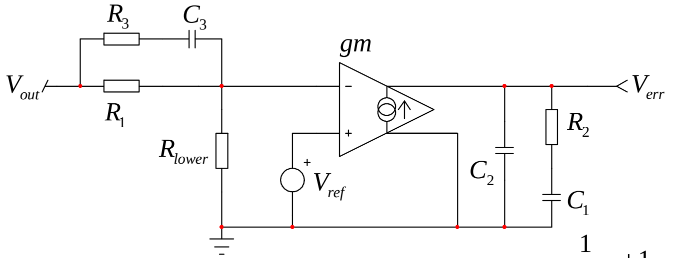
- ❑ The poles/zero are placed at the same location as in the op amp case



Type 2 – OTA

Type 3 with an OTA

- A type 3 with an OTA lets you boost the phase up to 180°. In theory…



$$G(s) = -\frac{R_{lower}\,\mathrm{gm}}{R_{lower}+R_1}\frac{R_2C_1}{C_1+C_2}\frac{sC_3(R_3+R_1)+1}{sC_3\left(\frac{R_{lower}R_1}{R_{lower}+R_1}+R_3\right)+1}\frac{\frac{1}{sR_2C_1}+1}{1+sR_2\frac{C_1C_2}{C_1+C_2}}$$

If $C_2 << C_1$

$$G(s) \approx -\frac{R_{lower}\,\mathrm{gm}R_2}{R_{lower}+R_1}\frac{sC_3(R_3+R_1)+1}{sC_3\left(\frac{R_{lower}R_1}{R_{lower}+R_1}+R_3\right)+1}\frac{\frac{1}{sR_2C_1}+1}{1+sR_2C_2}$$

Type 3 – OTA

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Type 3 with an OTA

- The extraction of the component values leads to complicated equations

$$\omega_{z_1} = \frac{1}{R_2C_1} \quad \omega_{z_2} = \frac{1}{C_3(R_3+R_1)} \quad \omega_{p_1} = \frac{1}{R_2C_2} \quad \omega_{p_2} = \frac{1}{C_3\left(\frac{R_{lower}R_1}{R_{lower}+R_1}+R_3\right)}$$

- First calculate R_3 but R_{lower} plays a role:
- No virtual ground as with the op amp!

$$R_3 = \frac{f_{z_2}V_{out} - f_{p_2}V_{ref}}{V_{ref}V_{out}\left(f_{p_2}-f_{z_2}\right)}R_{lower}\left(V_{out}-V_{ref}\right)$$

- Then calculate R_2 to crossover at the right frequency

$$R_2 = \frac{\sqrt{\left(f_{p_1}^2+f_c^2\right)\left(f_{z_1}^2+f_c^2\right)\left(f_{p_2}^2+f_c^2\right)\left(f_{z_2}^2+f_c^2\right)}}{f_{z_2}^2f_c^2+f_c^4++f_{z_1}^2f_c^2+f_{z_1}^2f_{z_2}^2}\frac{f_cf_{z_2}(R_{lower}+R_1)G_{f_c}}{gmf_{p_2}f_{p_1}R_{lower}}$$

↓

$$C_1 = \frac{1}{2\pi R_2 f_{z_1}} \quad C_2 = \frac{1}{2\pi R_2 f_{p_1}} \quad C_3 = \frac{1}{f_{z_2}(R_3+R_1)}$$

Type 3 – OTA

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Type 3 with an OTA

- If we look at R_3 definition, its numerator can be null:

$$f_{z_2} V_{out} - f_{p_2} V_{ref} = 0$$

$$\downarrow$$

$$f_{z_2} > \frac{V_{ref}}{V_{out}} f_{p_2}$$

If V_{out} = 12 V and V_{ref} = 2.5 V

$$\downarrow$$

$$f_{z_2} > \frac{f_{p_2}}{4} \qquad f_{p_2} = 10\ kHz \qquad f_{z_2} > 2.5\ kHz$$

If V_{out} = 400 V and V_{ref} = 2.5 V

$$\downarrow$$

$$f_{z_2} > \frac{f_{p_2}}{160} \qquad f_{p_2} = 10\ kHz \qquad f_{z_2} > 63\ Hz$$

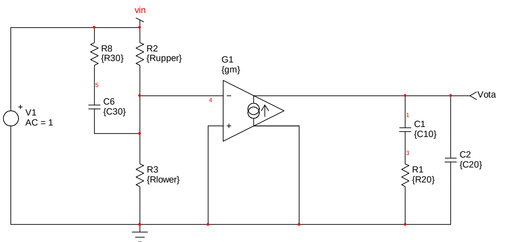
→ Less freedom to place the second pole and zero: limited boost!

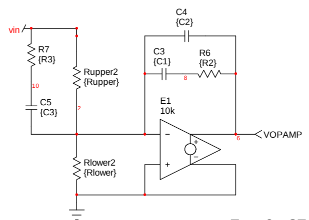
Type 3 – OTA

Type 3 with an OTA – a design example

parameters

Vout=12
Vref=2.5
Rlower=10k
Rupper=Rlower*(Vout-Vref)/Vref
fc=2k
Gfc=-20
G=10^(-Gfc/20)
pi=3.14159
fz1=300
fz2=900
fp1=26k
fp2=4.3k
C3=1/(2*pi*fz1*Rupper)
R3 =1/(2*pi*fp2*C3)
C1=1/(2*pi*fz2*R2)
C2=1/(2*pi*(fp1)*R2)
a=fc^4+fc^2*fz1^2+fc^2*fz2^2+fz1^2*fz2^2
c=fp2^2*fp1^2+fc^2*fp2^2+fc^2*fp1^2+fc^4
R2=sqrt(c/a)*G*fc*R3/fp1
gm=200u
d=(fp1^2+fc^2)*(fc^2+fz1^2) * (fp2^2+fc^2)*(fc^2+fz2^2)
e=fz1^2*fz2^2+fz1^2*fc^2+fc^2*fz2^2+fc^4
f=(Rlower+Rupper)*G*fz2*fc/(gm*Rlower*fp2*fp1)
R20=(sqrt(d)/e)*f
R30=((fz2*Vout-fp2*Vref)/(Vref*Vout*(fp2-fz2)))*Rlower*(Vout-Vref)
C30=1/(2*pi*(R30+Rupper))
C20=1/(2*pi*R20*fp1)
C10=1/(2*pi*R20*fz1)

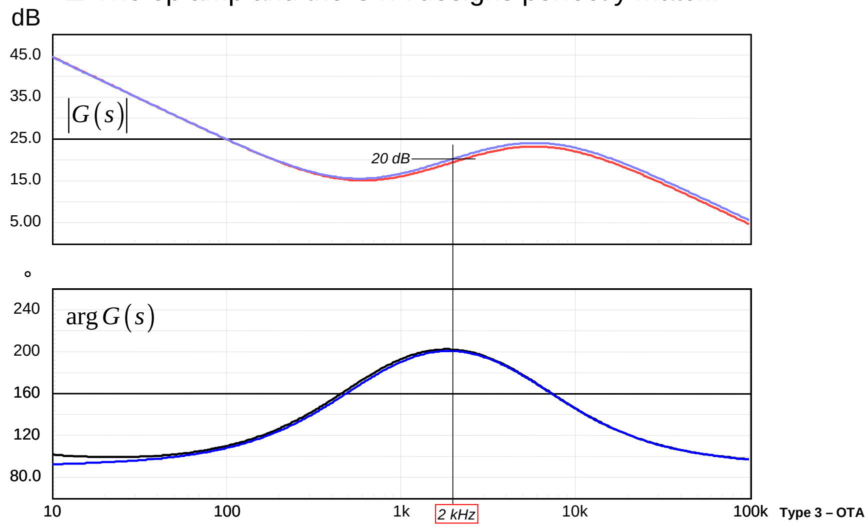




Type 3 – OTA

Type 3 with an OTA – a design example

- ❑ The op amp and the OTA designs perfectly match!



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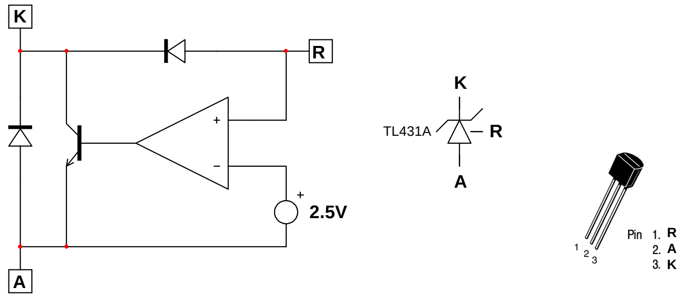
The TL431 programmable zener

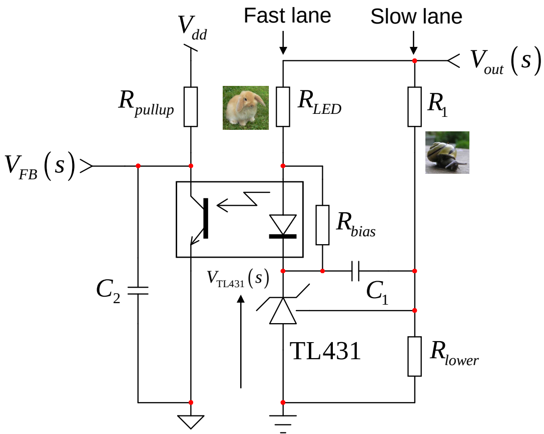
- ❑ The TL431 is the most popular choice in nowadays designs
- ❑ It associates an open-collector op amp and a reference voltage
- ❑ The internal circuitry is self-supplied from the cathode current
- ❑ When the R node exceeds 2.5 V, it sinks current from its cathode



- ❑ The TL431 is a shunt regulator

The TL431 programmable zener

- ❑ The TL431 lends itself very well to optocoupler control



$$I_{LED}(s) = \frac{V_{out}(s) - V_{TL431}(s)}{R_{LED}}$$

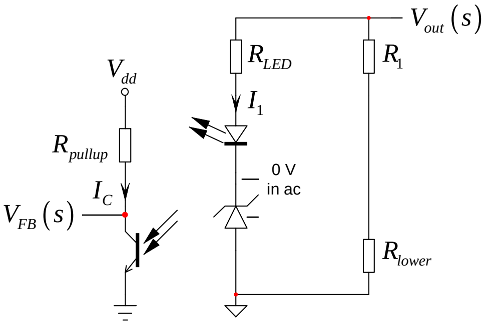
$$I_{LED}(s) = \boxed{\frac{V_{out}(s)}{R_{LED}}} - \boxed{\frac{V_{TL431}(s)}{R_{LED}}}$$

Fast lane   Slow lane

- ❑ R_{LED} connected to V_{out} offers a direct path to the LED: fast lane!

The TL431 programmable zener

- At high frequencies, the TL431 ac output is zero, C_1 is a short-circuit
- R_{LED} alone fixes the fast lane gain



$$V_{FB}(s) = -\mathrm{CTR} \cdot R_{pullup} \cdot I_1$$

$$I_1 = \frac{V_{out}(s)}{R_{LED}}$$

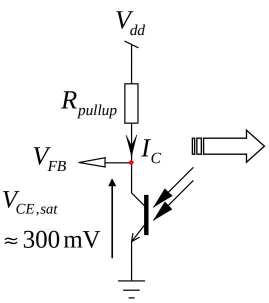
$$\frac{V_{FB}(s)}{V_{out}(s)} = -\mathrm{CTR}\frac{R_{pullup}}{R_{LED}}$$

- R_{LED} must also leave enough headroom to the TL431: upper limit!

TL431

The TL431 programmable zener

- R_{LED} cannot exceed a certain value because of bias limits
- V_{FB} must swing between $V_{CE,sat}$ and V_{cc}

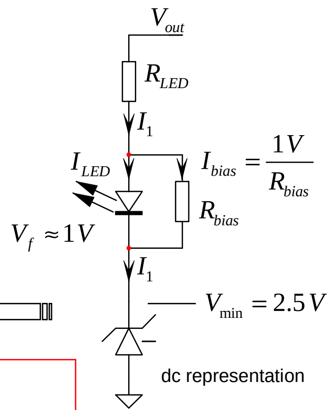


$$I_{C,\max} = \frac{V_{cc} - V_{CE,sat}}{R_{pullup}}$$

$$I_{1,\max} = \frac{V_{dd} - V_{CE,sat}}{R_{pullup}\mathrm{CTR}_{\min}} + I_{bias}$$

$$\Updownarrow$$

$$I_{1,\max} = \frac{V_{out} - V_f - V_{TL431,\min}}{R_{LED}}$$



$$R_{LED,\max} \le \frac{V_{out} - V_f - V_{TL431,\min}}{V_{dd} - V_{CE,sat} + I_{bias}\mathrm{CTR}_{\min}R_{pullup}}R_{pullup}\mathrm{CTR}_{\min}$$

The TL431 – the static gain limit

❑ Let us assume the following design:

$V_{out} = 5\,V$
$V_f = 1\,V$
$V_{TL431,\min} = 2.5\,V$
$V_{dd} = 4.8\,V$
$V_{CE,sat} = 300\,mV$
$I_{bias} = 1\,mA$
$\mathrm{CTR}_{\min} = 0.3$
$R_{pullup} = 20\,k\Omega$

$$R_{LED,\max} \le \frac{5-1-2.5}{4.8-0.3+1m\times0.3\times20k}\times20\mathrm{k}\times0.3$$

$$R_{LED,\max} \le 857\,\Omega$$

$$G_0 > \mathrm{CTR}\frac{R_{pullup}}{R_{LED}} > 0.3\frac{20}{0.857} > 7 \text{ or} \approx 17\,dB$$

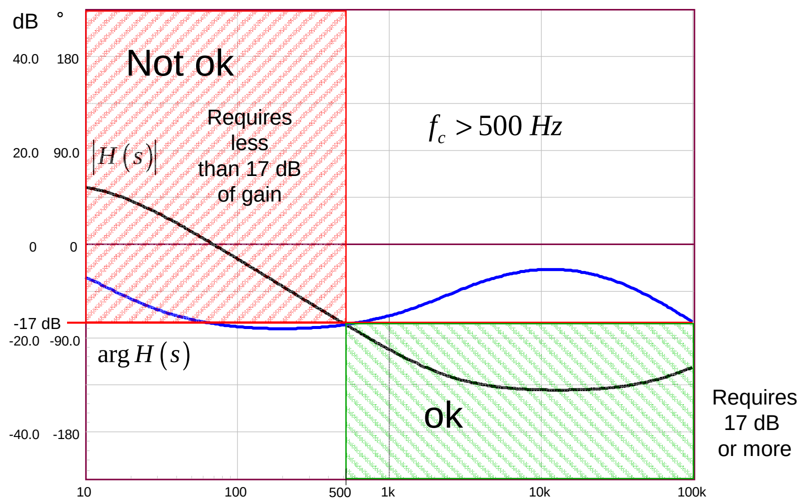
❑ In designs where R_{LED} fixes the gain, G_0 cannot be below 17 dB

⇨ You cannot "amplify" by less than 17 dB

TL431

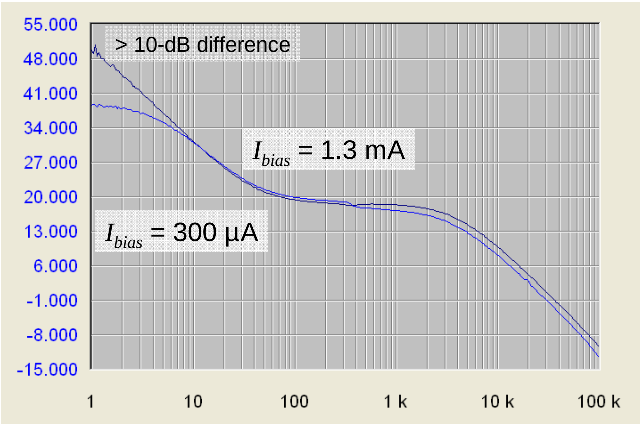
The TL431 – the static gain limit

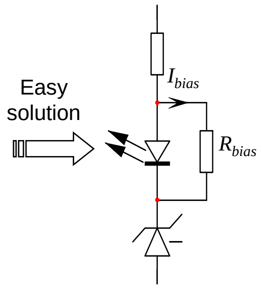
❑ You must identify the areas where compensation is possible



TL431 – injecting bias current

- ❑ Make sure enough current always biases the TL431: I_{bias} > 1 mA
- ❑ If not, its open-loop suffers – a 10-dB difference can be observed!



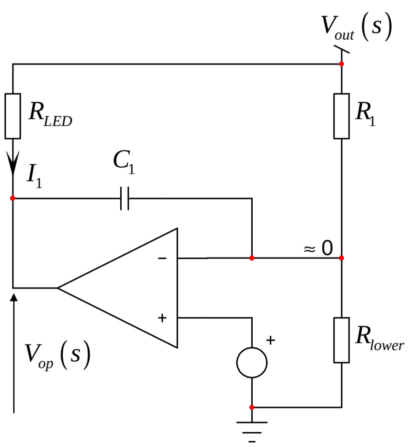


$$R_{bias} = \frac{1}{1m} = 1\,k\Omega$$

TL431

TL431 – small-signal analysis

- ❑ The TL431 is an open-collector op amp with a reference voltage
- ❑ Neglecting the LED dynamic resistance, we have:



$$I_1(s) = \frac{V_{out}(s) - V_{op}(s)}{R_{LED}}$$

$$V_{op}(s) = -V_{out}(s)\frac{\frac{1}{sC_1}}{R_1} = -V_{out}(s)\frac{1}{sR_1C_1}$$

$$I_1(s) = V_{out}(s)\frac{1}{R_{LED}}\left[1 + \frac{1}{sR_1C_1}\right]$$

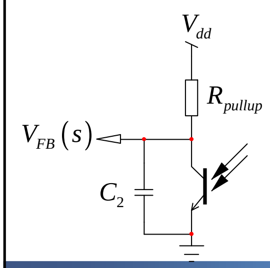
We know that: $V_{FB}(s) = -\mathrm{CTR} \cdot R_{pullup} \cdot I_1$

$$\frac{V_{FB}(s)}{V_{out}(s)} = -\frac{R_{pullup}\mathrm{CTR}}{R_{LED}}\left[\frac{1 + sR_1C_1}{sR_1C_1}\right]$$

TL431

TL431 – small-signal analysis

- ❑ In the previous equation we have:
- ✓ a static gain $G_0 = \mathrm{CTR}\frac{R_{pullup}}{R_{LED}}$
- ✓ a 0-dB crossover pole frequency $\omega_{po} = \frac{1}{R_1 C_1}$
- ✓ a zero $\omega_{z_1} = \frac{1}{R_1 C_1}$
- ❑ We are missing a pole for the type 2!



Add a cap. from collector to ground

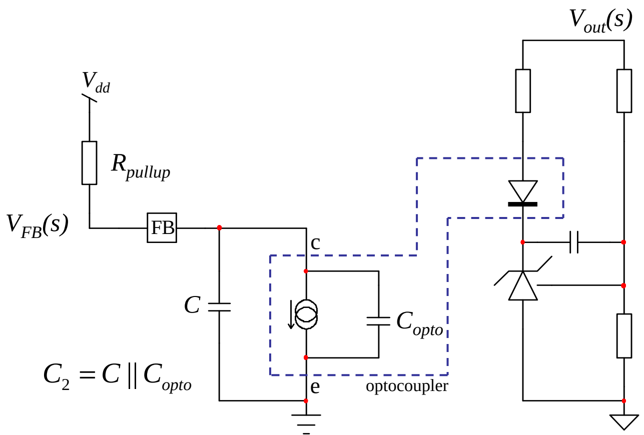
Type 2 transfer function

$$\frac{V_{FB}(s)}{V_{out}(s)} = -\frac{R_{pullup}\mathrm{CTR}}{R_{LED}}\left[\frac{1+sR_1C_1}{sR_1C_1\left(1+sR_{pullup}C_2\right)}\right]$$

TL431

TL431 – small-signal analysis

- ❑ The optocoupler also features a parasitic capacitor
- ➢ it comes in parallel with C_2 and must be accounted for

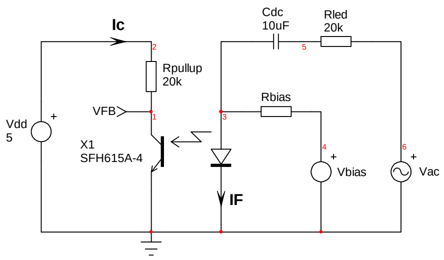


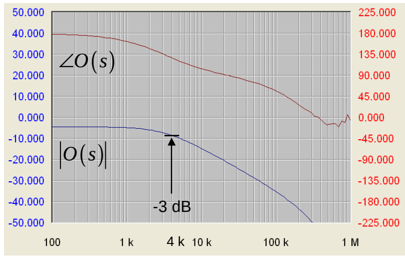
$C_2 = C \,||\, C_{opto}$

TL431

TL431 – small-signal analysis

- ❑ The optocoupler must be characterized to know where its pole is





- ❑ Adjust V_{bias} to have V_{FB} at 2-3 V to be in linear region, then ac sweep
- ❑ The pole in this example is found at 4 kHz

$$C_{opto} = \frac{1}{2\pi R_{pullup} f_{pole}} = \frac{1}{6.28 \times 20k \times 4k} \approx 2\ nF$$

⇨ Another design constraint!

TL431

The TL431 in a type 1 compensator

- ❑ To make a type 1 (origin pole only) neutralize the zero and the pole

$$\frac{V_{FB}(s)}{V_{out}(s)} = -\frac{R_{pullup}\text{CTR}}{R_{LED}}\left[\frac{1+sR_1C_1}{sR_1C_1\left(1+sR_{pullup}C_2\right)}\right] \qquad \omega_{po} = \frac{1}{\frac{R_1 R_{LED}}{R_{pullup}\text{CTR}}C_1}$$

$$\Rightarrow \quad C_1 = \frac{R_{pullup}\text{CTR}}{\omega_{po} R_1 R_{LED}}$$

$$\Rightarrow \quad sR_1C_1 = sR_{pullup}C_2 \quad \Rightarrow \quad C_2 = \frac{R_1}{R_{pullup}}C_1 \quad \Rightarrow \quad C_2 = \frac{\text{CTR}}{2\pi f_{po} R_{LED}}$$

- ❑ Once neutralized, you are left with an integrator

$$G(s) = -\frac{1}{\frac{s}{\omega_{po}}} \rightarrow |G(f_c)| = \frac{f_{po}}{f_c} \rightarrow f_{po} = G_{f_c} f_c \quad \Rightarrow \quad C_2 = \frac{\text{CTR}}{2\pi G_{f_c} f_c R_{LED}}$$

TL431

TL431 type 1 design example

- We want a 5-dB gain at 5 kHz to stabilize the 5-V converter

$V_{out} = 5\,V$
$V_f = 1\,V$
$V_{TL431,\min} = 2.5\,V$
$V_{dd} = 4.8\,V$
$V_{CE,sat} = 300\,mV$
$I_{bias} = 1\,mA$
$\mathrm{CTR}_{\min} = 0.3$
$R_{pullup} = 20\,k\Omega$

$$\Rightarrow R_{LED,\max} \leq 857\,\Omega \xrightarrow{\text{Apply 15\% margin}} R_{LED} = 728\,\Omega$$

$G_{fc} = 10^{\frac{5}{20}} = 1.77$
$f_c = 5\,kHz$

$$C_2 = \frac{\mathrm{CTR}}{2\pi G_{fc} f_c R_{LED}} = \frac{0.3}{6.28 \times 1.77 \times 5k \times 728} \approx 7.4\,nF$$

$R_1 = 10\,k\Omega$

C_{opto} = 2 nF

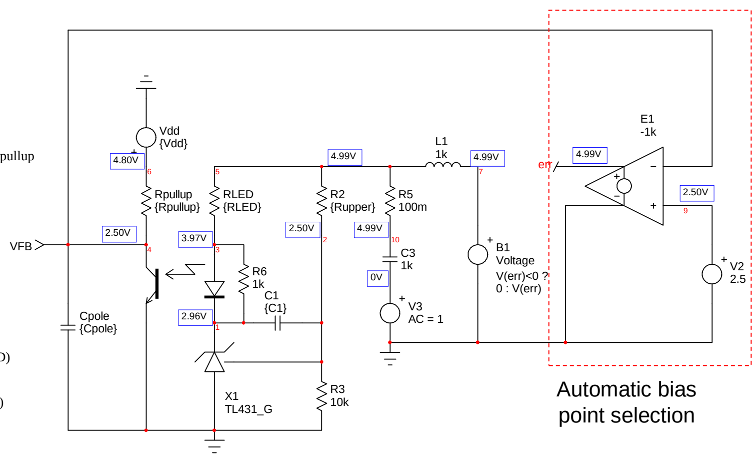
$$\Rightarrow C = 7.4n - 2n = 5.4\,nF \qquad C_1 = \frac{R_{pullup}}{R_1} C_2 \approx 14.7\,nF$$

TL431

TL431 type 1 design example

- SPICE can simulate the design – automate elements calculations…

parameters
Vout=5
Vf=1
Vref=2.5
VCEsat=300m
Vdd=4.8
Ibias=1m
A=Vout-Vf-Vref
B=Vdd-VCEsat+Ibias*CTR*Rpullup
Rmax=(A/B)*Rpullup*CTR
Rupper=(Vout-2.5)/250u
fc=5k
Gfc=-5
G=10^(-Gfc/20)
pi=3.14159
Fpo=G*fc
Rpullup=20k
RLED=Rmax*0.85
C1=Cpole1*Rpullup/Rupper
Cpole1=CTR/(2*pi*Fpo*RLED)
Cpole=Cpole1-Copto
Fopto=4k
Copto=1/(2*pi*Fopto*Rpullup)
CTR = 0.3



Automatic bias point selection

TL431

TL431 type 1 design example

- We have a type 1 but 1.3 dB of gain is missing?

TL431

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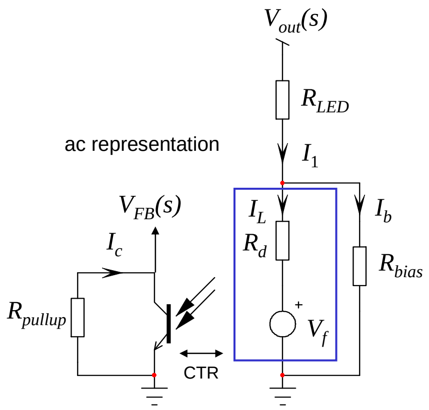
TL431 type 1 design example

- The 1-kΩ resistor in parallel with the LED is an easy bias
- However, as it appears in the loop, does it affect the gain?



$$V_{FB} = I_c R_{pullup} = I_L R_{pullup} \mathrm{CTR}$$

$$I_L = I_1 \frac{R_{bias}}{R_{bias}+R_d}$$

$$I_L = \frac{V_{out}}{R_{LED}+R_{bias} \| R_d} \frac{R_{bias}}{R_{bias}+R_d}$$

$$\left.\frac{V_{FB}}{V_{out}}\right|_{s=0} = \frac{R_{pullup}\mathrm{CTR}}{R_{LED}+R_{bias} \| R_d} \frac{R_{bias}}{R_{bias}+R_d}$$

- Both bias and dynamic resistances have a role in the gain expression

TL431

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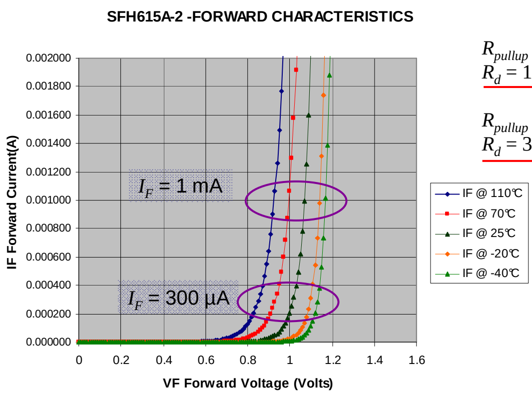
TL431 type 1 design example

- A low operating current increases the dynamic resistor R_d



R_{pullup} = 20 kΩ, I_F = 300 µA (CTR = 0.3)
R_d = 158 Ω

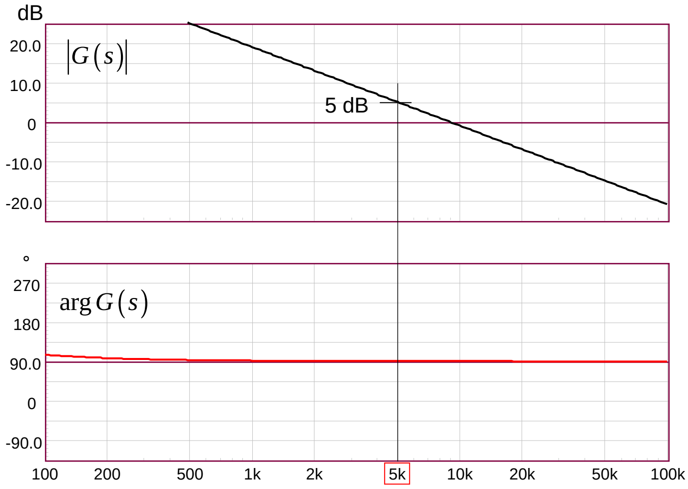
R_{pullup} = 1 kΩ, I_F = 1 mA (CTR = 1)
R_d = 38 Ω

- Make sure you have enough LED current to keep R_d small

TL431

TL431 type 1 design example

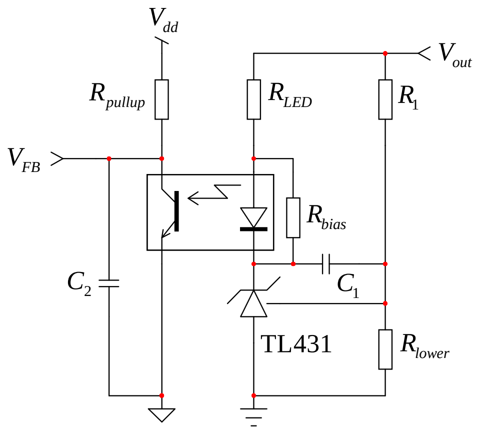
- The pullup resistor is 1 kΩ and the target now reaches 5 dB



TL431

The TL431 in a type 2 compensator

- ❑ Our first equation was already a type 2 definition, we are all set!



$$G_0 = \mathrm{CTR}\frac{R_{pullup}}{R_{LED}}$$

$$\omega_{po} = \frac{1}{R_1 C_1}$$

$$\omega_{z_1} = \frac{1}{R_1 C_1}$$

$$\omega_{p_1} = \frac{1}{R_{pullup} C_2}$$

TL431

TL431 type 2 design example

- ❑ You need to provide a 15-dB gain at 5 kHz with a 50° boo st
- ❑ The output voltage is 12 V
- ❑ The poles and zero position are that of the op amp design

$$G_0 = \mathrm{CTR}\frac{R_{pullup}}{R_{LED}} = 10^{15/20} = 5.62 \qquad f_p = 13.7\,kHz \;\; f_z = 1.8\,kHz$$

- ❑ With a 250-µA bridge current, the divider resistor is made of:

$$R_{lower} = 2.5/250u = 10\,k\Omega \qquad R_1 = (12-2.5)/250u = 38\,k\Omega$$

- ❑ The pole and zero respectively depend on R_{pullup} and R_1:

$$C_2 = 1/2\pi f_p R_{pullup} = 581\,pF \qquad C_1 = 1/2\pi f_z R_1 = 2.3\,nF$$

- ❑ The LED resistor depends on the needed mid-band gain:

$$R_{LED} = \frac{R_{pullup}\mathrm{CTR}}{G_0} = 1.06\,k\Omega \xrightarrow{\text{ok}} R_{LED,\max} \le 4.85\,k\Omega$$

TL431

TL431 type 2 design example

- The optocoupler is still at a 4-kHz frequency:

$$C_{pole} \approx 2\,nF$$

Already above!

- Type 2 pole capacitor calculation requires a 581-pF cap.!

⇨ The bandwidth cannot be reached, reduce f_c!

- For noise purposes, we want a minimum of 100 pF for C
- With a total capacitance of 2.1 nF, the highest pole can be:

$$f_{pole} = \frac{1}{2\pi R_{pullup} C} = \frac{1}{6.28 \times 20k \times 2.1n} = 3.8\,kHz$$

- For a 50° phase boost and a 3.8-kHz pole, the crossov er must be:

$$f_c = \frac{f_p}{\tan(boost) + \sqrt{\tan^2(boost) + 1}} \approx 1.4\,kHz$$

TL431

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TL431 type 2 design example

- The zero is then simply obtained:

$$f_z = \frac{f_c^{\,2}}{f_p} = 516\,Hz$$

- We can re-derive the component values and check they are ok

$$C_2 = 1/2\pi f_p R_{pullup} = 2.1\,nF \qquad C_1 = 1/2\pi f_z R_1 = 8.1\,nF$$

- Given the 2-nF optocoupler capacitor, we just add 100 pF
- In this example, $R_{LED,max}$ is 4.85 kΩ

$$G_0 > \text{CTR}\frac{R_{pullup}}{R_{LED}} > 0.3\frac{20}{4.85} > 1.2 \text{ or } \approx 1.8\,dB$$

- You <u>cannot</u> use this type 2 if an attenuation is required at f_c!

TL431

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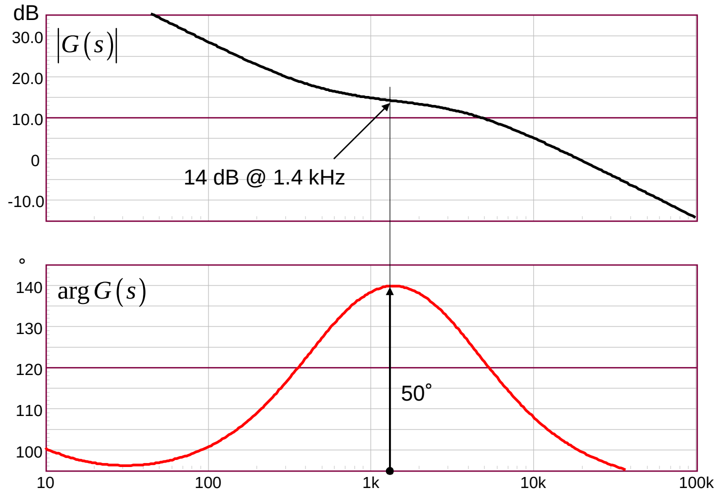
TL431 type 2 design example

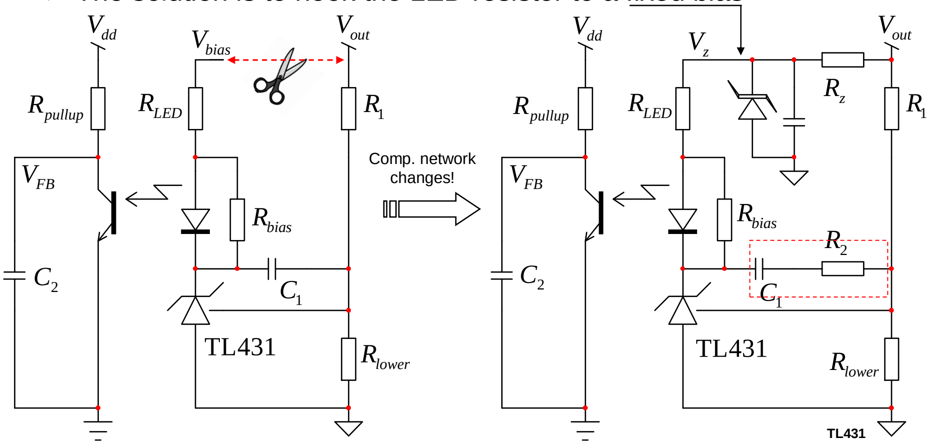
- The 1-dB gain difference is linked to R_d and the bias current



TL431

TL431 – suppressing the fast lane

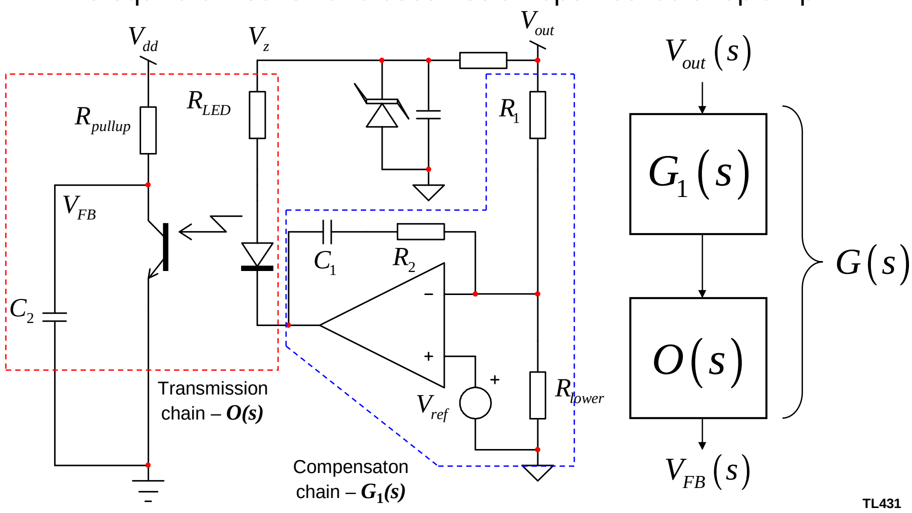
- The gain problem comes from the fast lane presence
- Its connection to V_{out} creates a parallel input
- ➢ The solution is to hook the LED resistor to a fixed bias



TL431

TL431 – suppressing the fast lane

❑ The equivalent schematic becomes an open-collector op amp

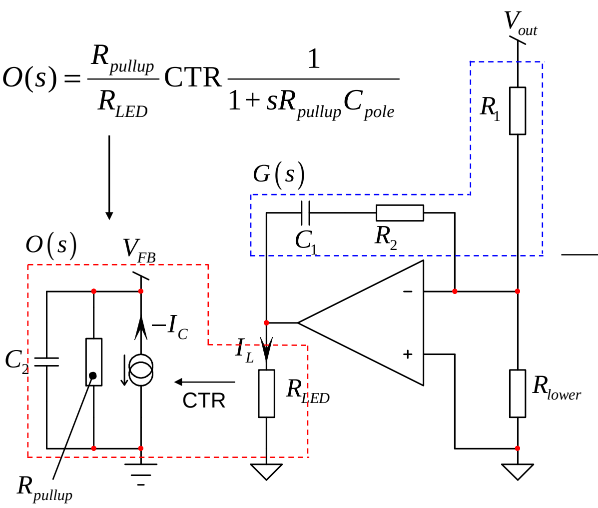


TL431

TL431 – suppressing the fast lane

❑ The small-signal ac representation puts all sources to 0

$$O(s) = \frac{R_{pullup}}{R_{LED}} CTR \frac{1}{1 + sR_{pullup}C_{pole}}$$



$$G_1(s) = \frac{1 + R_2C_1}{sR_1C_1}$$

TL431

TL431 – suppressing the fast lane

- The op amp can now be wired in any configuration!
- Just keep in mind the optocoupler transmission chain

$$O(s) = \frac{R_{pullup}}{R_{LED}} \mathrm{CTR} \frac{1}{1 + sR_{pullup}C_{pole}}$$

- Wire the op amp in type 2A version (no high frequency pole)

$$G_1(s) = \frac{1 + R_2C_1}{sR_1C_1}$$

- When cascaded, you obtain a type 2 with an extra gain term

$$G(s) = \underbrace{-\frac{R_{pullup}}{R_{LED}} \mathrm{CTR}}_{G_2} \frac{1 + R_2C_1}{sR_1C_1\left(1 + sR_{pullup}C_{pole}\right)}$$

TL431

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TL431 type 2 design example – no fast lane

- We still have a constraint on R_{LED} but only for dc bias purposes

$$R_{LED,\max} \le \frac{V_z - V_f - V_{TL431,\min}}{V_{dd} - V_{CE,sat} + I_{bias}\mathrm{CTR}_{\min} R_{pullup}} R_{pullup}\mathrm{CTR}_{\min}$$

- You need to attenuate by -10-dB at 1.4 kHz with a 50° boost
- The poles and zero position are that of the previous design

$$\left.\begin{aligned} V_z &= 6.2\,V \\ V_f &= 1\,V \\ V_{TL431,\min} &= 2.5\,V \\ V_{dd} &= 4.8\,V \\ V_{CE,sat} &= 300\,mV \\ I_{bias} &= 1\,mA \\ \mathrm{CTR}_{\min} &= 0.3 \\ R_{pullup} &= 20\,k\Omega \end{aligned}\right\} R_{LED,\max} \le 1.5\,k\Omega \xrightarrow{\text{Apply 15\% margin}} R_{LED} = 1.27\,k\Omega$$

$$f_z = 516\,Hz \quad f_p = 3.8\,kHz$$

TL431

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TL431 type 2 design example – no fast lane

- We need to account for the extra gain term:

$$G_2 = \frac{R_{pullup}}{R_{LED}} \mathrm{CTR} = \frac{20\mathrm{k}}{1.27\mathrm{k}} 0.3 = 4.72$$

- The required total mid-band <u>attenuation</u> at 1.4 kHz is -10 dB

$$G_{f_c} = 10^{-10/20} = 0.316$$

- The mid-band gain from the type 2A is therefore:

$$G_1 = \frac{G_0}{G_2} = \frac{0.316}{4.72} = 0.067 \text{ or } -23.5\ dB$$

- Calculate R_2 for this attenuation:

$$R_2 = G_1 R_1 \frac{\sqrt{\left(\frac{f_c}{f_p}\right)^2 + 1}}{\sqrt{\left(\frac{f_z}{f_c}\right)^2 + 1}} = 2.6\ k\Omega$$

TL431

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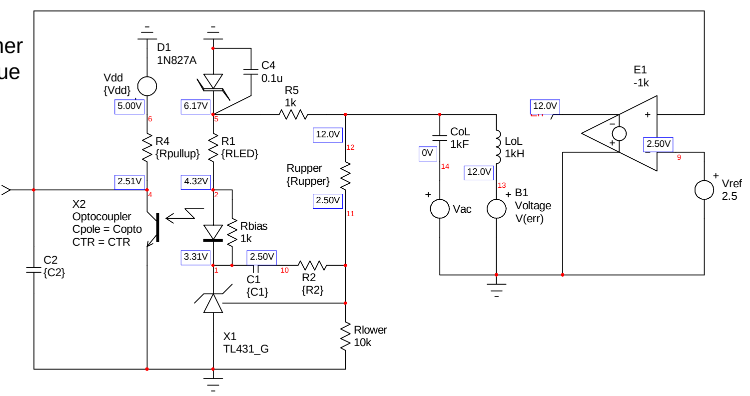
TL431 type 2 design example – no fast lane

- An automated simulation helps to test the calculation results

parameters

Vout=12
Rupper=(Vout-2.5)/250u
fc=1.4k
Gfc=10
Vf=1
Ibias=1m
Vref=2.5
VCEsat=300m
Vdd=5
Vz=6.2
Rpullup=20k
Fopto=4k
Copto=1/(2*pi*Rpullup*Fopto)
CTR=0.3
G1=Rpullup*CTR/RLED
G2=10^(-Gfc/20)
G2=G2/G1
pi=3.14159
fz=516
fp=3.8k
C1=1/(2*pi*fz*R2)
Cpole2=1/(2*pi*fp*Rpullup)
C2=Cpole2-Copto
a=(fz^2+fc^2)*(fp^2+fc^2)
c=(fz^2+fc^2)
R2=(sqrt(a)/c)*G*fc*Rupper/fp
Rmax1=(Vz-Vf-Vref)
Rmax2=(Vdd-VCEsat+Ibias*(Rpullup*CTR))
RLED=(Rmax1/Rmax2)*Rpullup*CTR*0.85

Zener value



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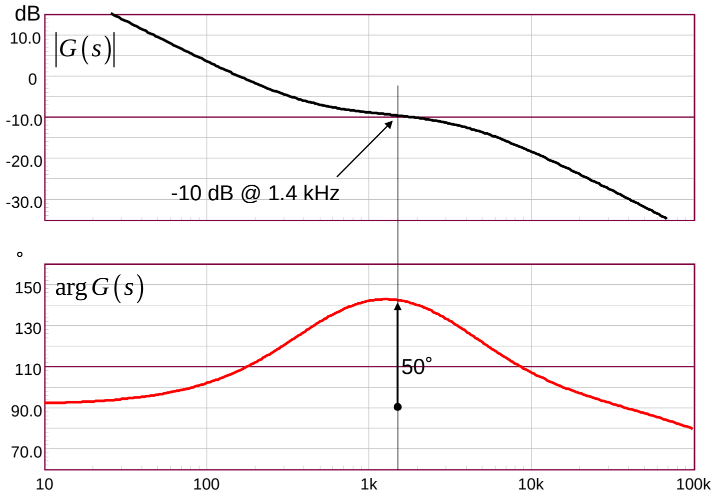
TL431 type 2 design example – no fast lane

- The simulation results confirm the calculations are ok



TL431

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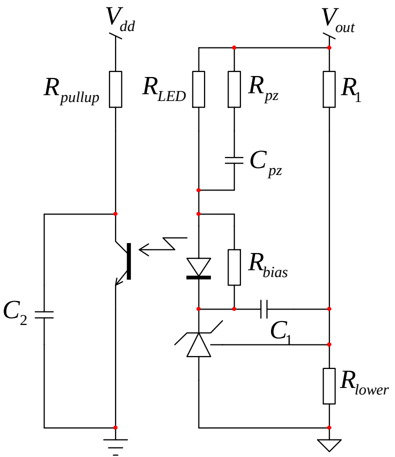
The TL431 in a type 3 compensator

- The type 3 with a TL431 is difficult to put in practice



$$f_{z_1} = \frac{1}{2\pi R_1 C_1} \qquad f_{z_2} = \frac{1}{2\pi\left(R_{LED} + R_{pz}\right)C_{pz}}$$

$$f_{p_1} = \frac{1}{2\pi R_{pz} C_{pz}} \qquad f_{p_2} = \frac{1}{2\pi R_{pullup}\left(C_2 \,||\, C_{opto}\right)}$$

$$G = \frac{R_{pullup}}{R_{LED}} \mathrm{CTR}$$

R_{LED} fixes the gain and a zero position

- Suppress the fast lane for an easier implementation!

TL431

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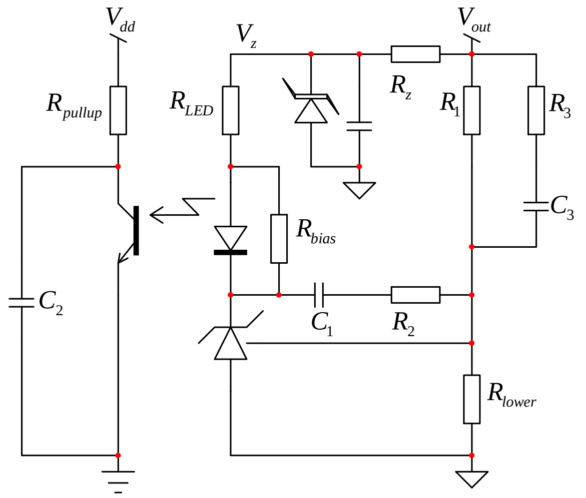
The TL431 in a type 3 compensator

- ❑ Once the fast lane is removed, you have a classical configuration



$$f_{z_1} = \frac{1}{2\pi R_2 C_1}$$

$$f_{z_2} = \frac{1}{2\pi R_1 C_3}$$

$$f_{p_1} = \frac{1}{2\pi R_3 C_3}$$

$$f_{p_2} = \frac{1}{2\pi R_{pullup}\left(C_2 \parallel C_{opto}\right)}$$

$$G = \frac{R_{pullup}}{R_{LED}}\text{CTR}$$

TL431

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TL431 type 3 design example – no fast lane

- ❑ We want to provide a 10-dB attenuation at 1 kHz
- ❑ The phase boost needs to be of 120°
- ➢ place the double pole at 3.7 kHz and the double zero at 268 Hz
- ❑ Calculate the maximum LED resistor you can accept, apply margin

$$R_{LED,\max} \le \frac{V_z - V_f - V_{TL431,\min}}{V_{dd} - V_{CE,sat} + I_{bias}\text{CTR}_{\min} R_{pullup}} R_{pullup}\text{CTR}_{\min} \le 1.5\,k\Omega \xrightarrow{\text{X 0.85}} 1.3\,k\Omega$$

- ❑ We need to account for the extra gain term:

$$G_2 = \frac{R_{pullup}}{R_{LED}}\text{CTR} = \frac{20\text{k}}{1.3\text{k}}0.3 = 4.6$$

- ❑ The required total mid-band <u>attenuation</u> at 1 kHz is -10 dB

$$G_{f_c} = 10^{-10/20} = 0.316$$

TL431

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TL431 type 3 design example – no fast lane

❑ The mid-band gain from the type 3 is therefore:

$$G_1 = \frac{G_0}{G_2} = \frac{0.316}{4.6} = 0.068 \text{ or } -23.3\ dB$$

❑ Calculate R_2 for this attenuation:

$$R_2 = \frac{G_1 R_1 f_{p_1}}{f_{p_1} - f_{z_1}} \frac{\sqrt{1+\left(\frac{f_c}{f_{p_1}}\right)^2}\sqrt{1+\left(\frac{f_c}{f_{p_2}}\right)^2}}{\sqrt{1+\left(\frac{f_{z_1}}{f_c}\right)^2}\sqrt{1+\left(\frac{f_c}{f_{z_2}}\right)^2}} = 744\ \Omega$$

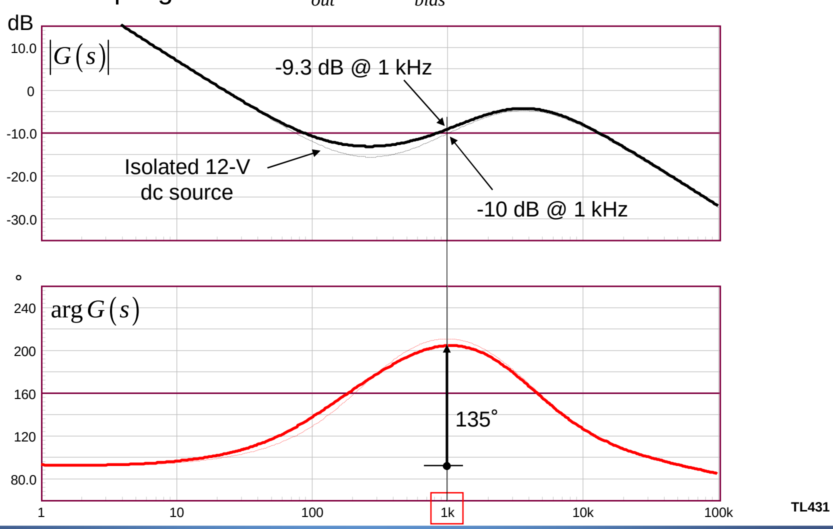
$$C_1 = 800\ nF\ \ C_2 = 148\ pF\ \ C_3 = 14.5\ nF\ \ C_{opto} = 2\ nF$$

❑ The optocoupler pole limits the upper double pole position

❑ The maximum boost therefore depends on the crossover frequency

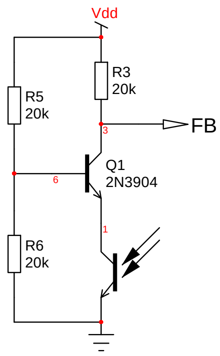
TL431 type 3 design example – no fast lane

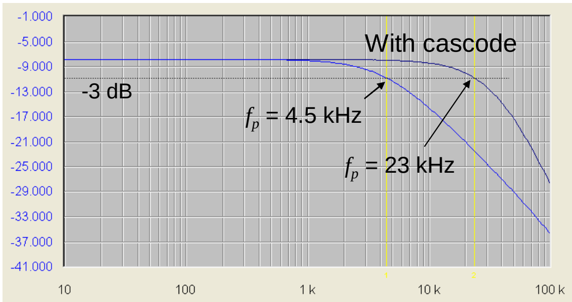
❑ The decoupling between V_{out} and V_{bias} affects the curves



Pushing the opto pole with the cascode

- ❑ The optocoupler pole is clearly a limiting factor
- ❑ A possibility exists to push its position to a higher region
  - ➢The cascode fixes the optocoupler collector potential
  - ➢It neutralizes the Miller capacitance of the optocoupler

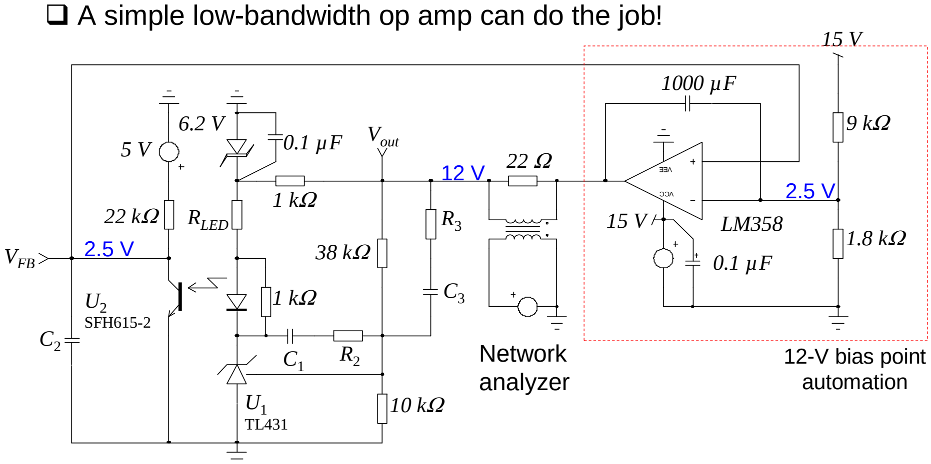




SFH615A-2

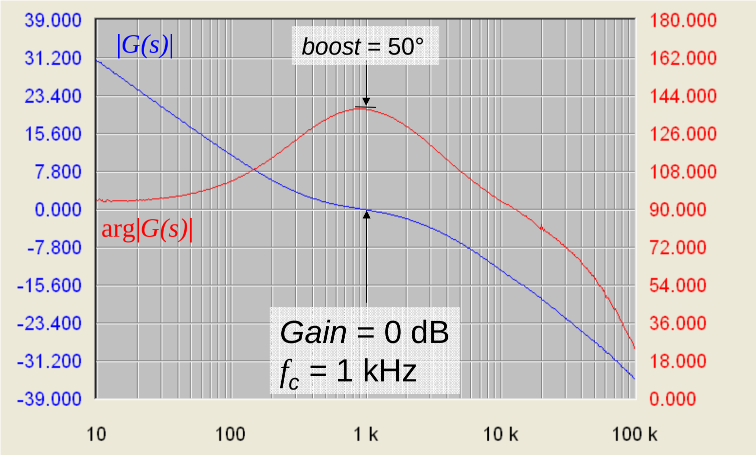
Testing the TL431 fast lane structures

- ❑ Simulations are a good indication, but lab. results are better →
- ❑ The TL431 needs to be exactly biased at V_{out} to ac sweep it
- ❑ A simple low-bandwidth op amp can do the job!



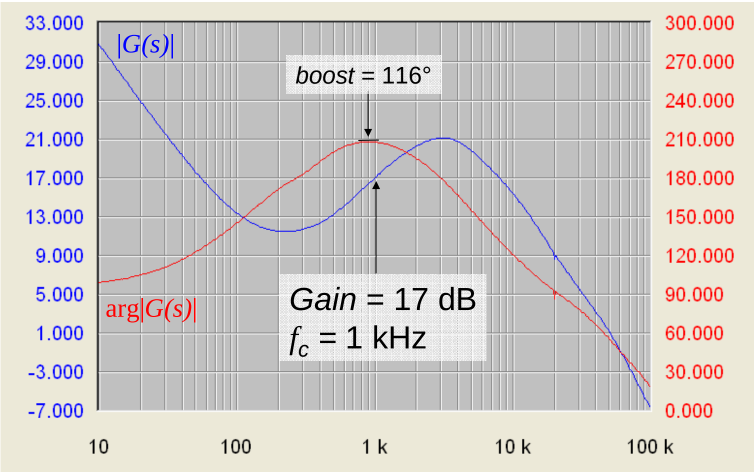
Testing the TL431 fast lane structures

❑ The results confirm the calculation procedures: a type 2



Testing the TL431 fast lane structures

❑ The results confirm the calculation procedures: a type 3

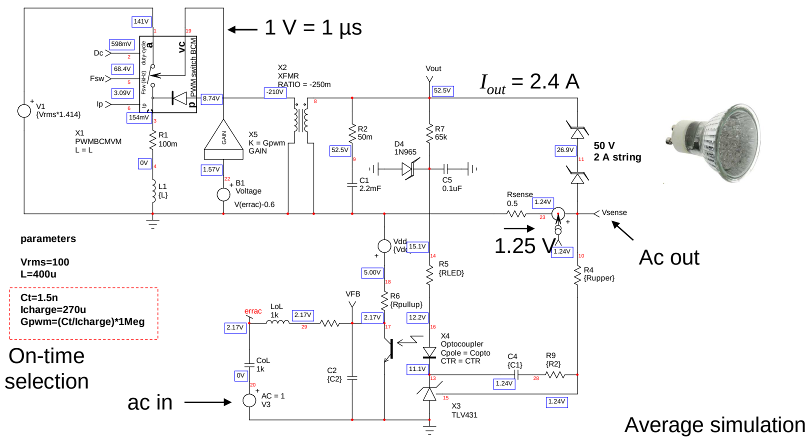


Agenda

- ❑ Feedback generalities
- ❑ The divider and the virtual ground
- ❑ Phase margin and crossover
- ❑ Poles and zeros
- ❑ Boosting the phase at crossover
- ❑ Compensator types
- ❑ Practical implementations: the op amp
- ❑ Practical implementations: the OTA
- ❑ Practical implementations: the TL431
- ❑ **Design examples**
- ❑ A real case study
- ❑ Conclusion

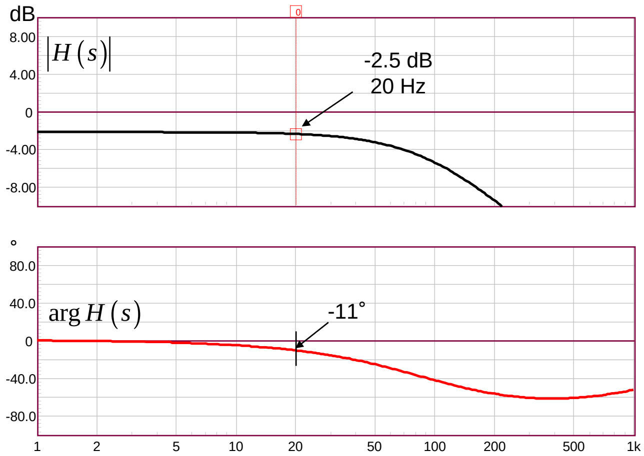
Design Example 1 – a single-stage PFC

- ❑ The single-stage PFC is often used in LED applications
- ❑ It combines isolation, current-regulation and power factor correction
- ❑ Here, a constant on-time BCM controler, the ***NCP1608***, is used



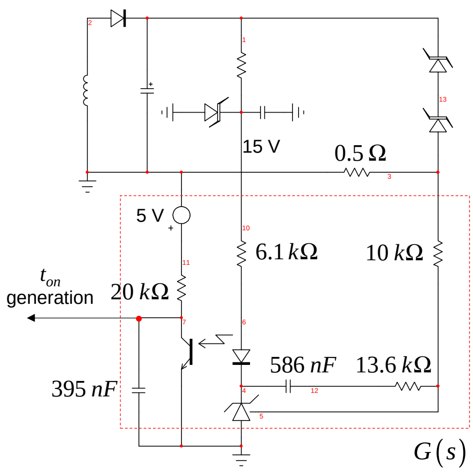
Design Example 1 – a single-stage PFC

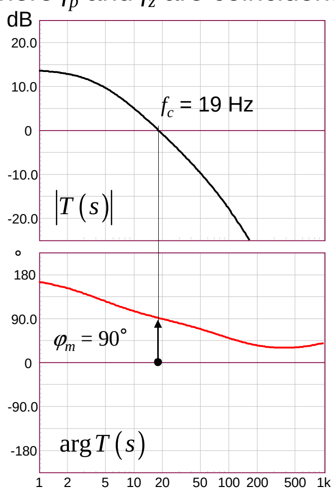
- ❑ Once the converter elements are known, ac-sweep the circuit
- ❑ Select a crossover low enough to reject the ripple, e.g. 20 Hz



Design Example 1 – a single-stage PFC

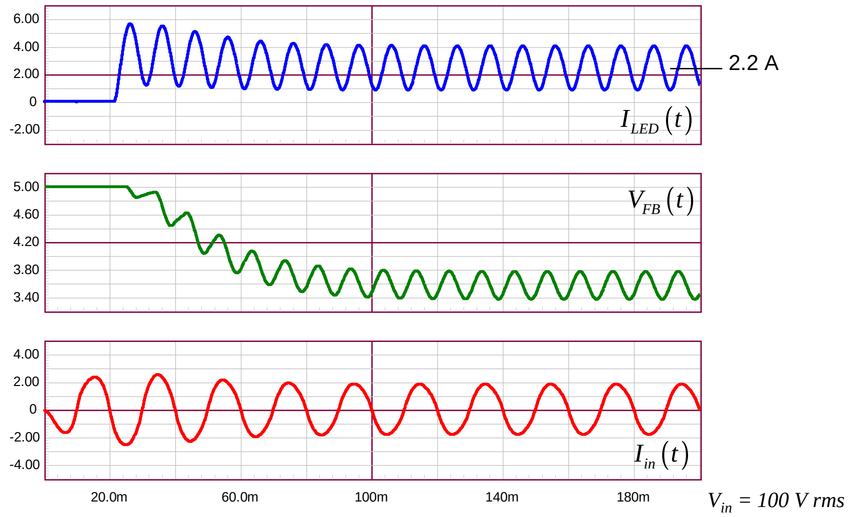
- ❑ Given the low phase lag, a type 1 can be chosen
- ➢ Use the type 2 with fast lane removal where f_p and f_z are coincident





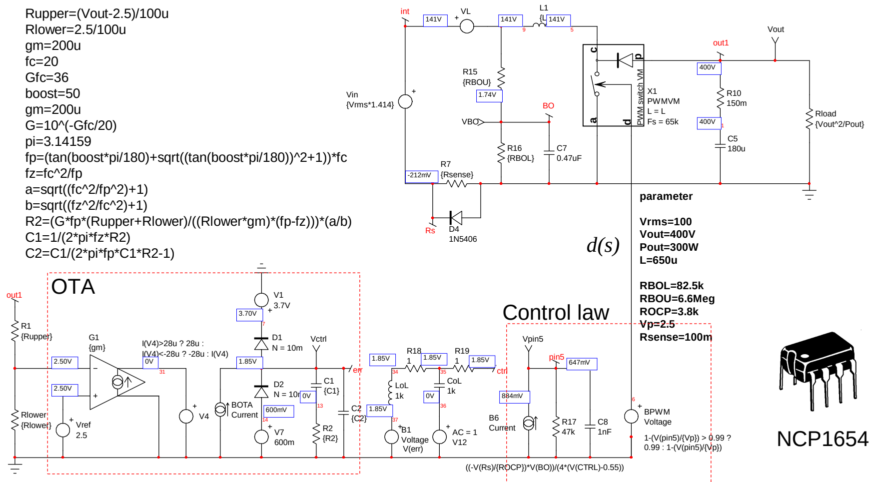
Design Example 1 – a single-stage PFC

- ❑ A transient simulation helps to test the system stability



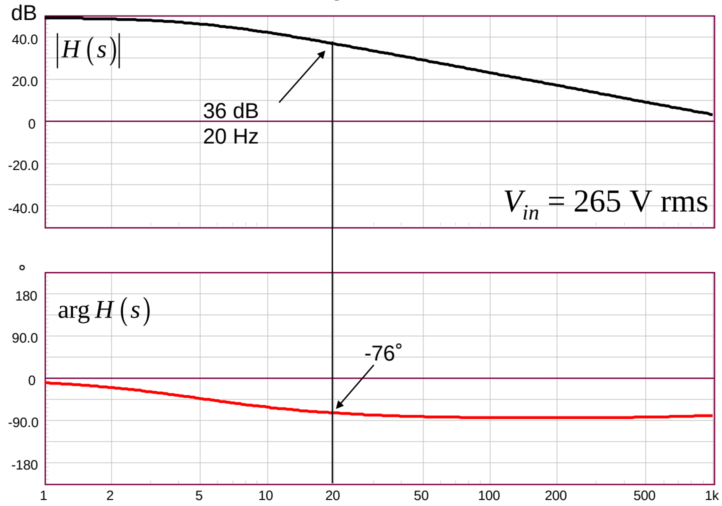
Design Example 2 – a 300-W PFC

- ❑ A CCM PFC is delivering 300 W in universal mains
- ❑ Use an average model to plot it transfer function



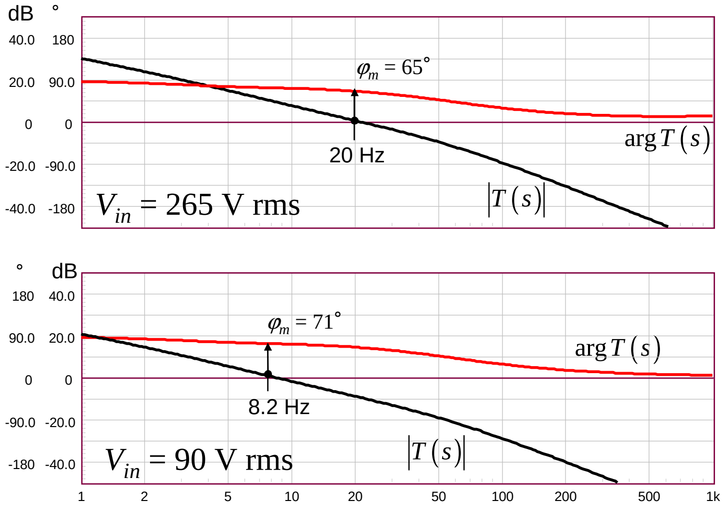
Design Example 2 – a 300-W PFC

❑ Select the bandwidth at high line: 20 Hz



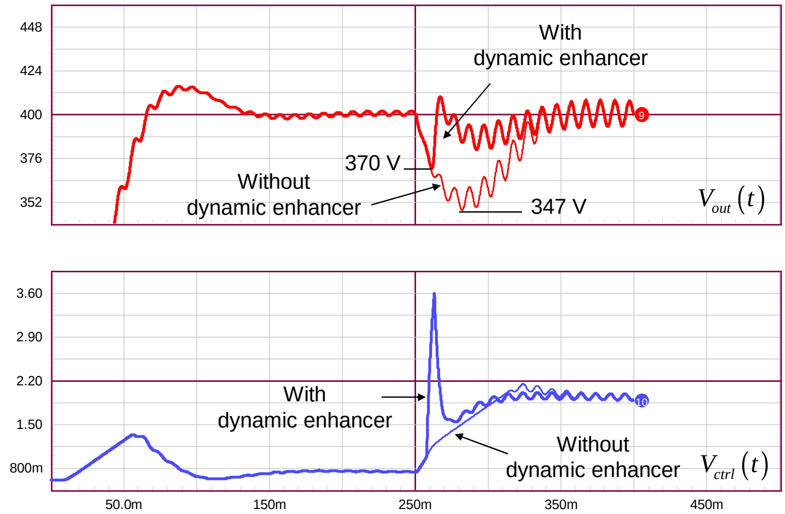
Design Example 2 – a 300-W PFC

❑ Use a type 2 configuration and boost by 50°



Design Example 2 – a 300-W PFC

- ❑ Test the transient response and see dynamic enhancer effects



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Design example 3: a DCM flyback converter

- ❑ We want to stabilize a 20-W DCM adapter
- ❑ V_{in} = 85 to 265 V rms, V_{out} = 12 V/1.7 A
- ❑ F_{sw} = 65 kHz, R_{pullup} = 20 kΩ
- ❑ Optocoupler is SFH-615A, pole is at 6 kHz
- ❑ Cross over target is 1 kHz
- ❑ Selected controller: NCP1216

1. Obtain a power stage open-loop Bode plot, *H(s)*
2. Look for gain and phase values at cross over
3. Compensate gain and build phase at crossover, *G(s)*
4. Run a loop gain analysis to check for margins, *T(s)*
5. Test transient responses in various conditions

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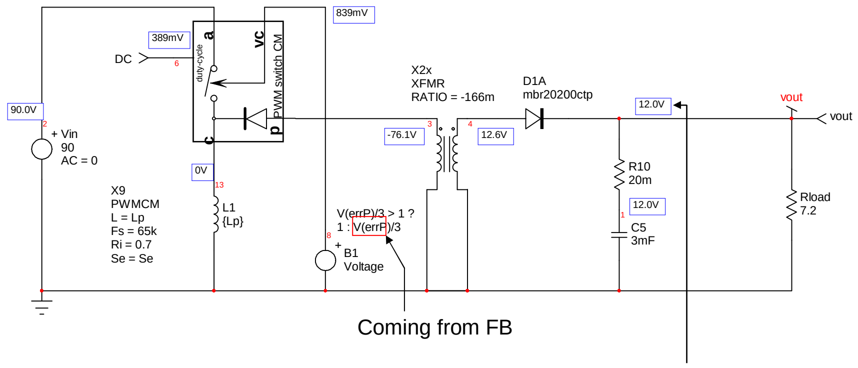
Design example 3: a DCM flyback converter

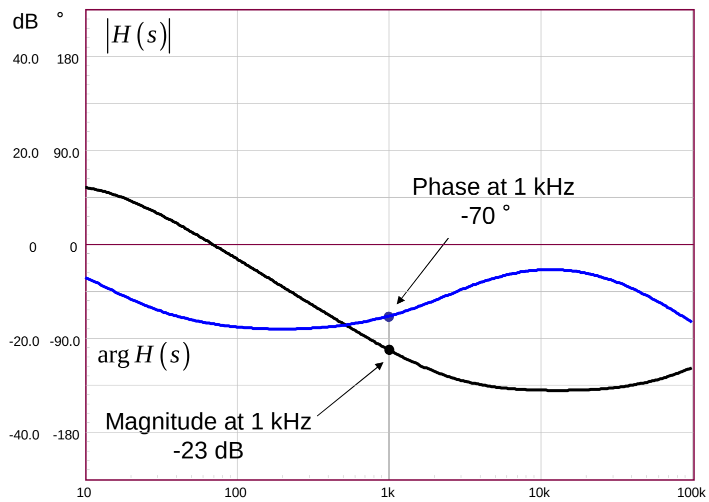
- ❑ Capture a SPICE schematic with an averaged model



- ❑ Look for the bias points values: V_{out} = 12 V, ok

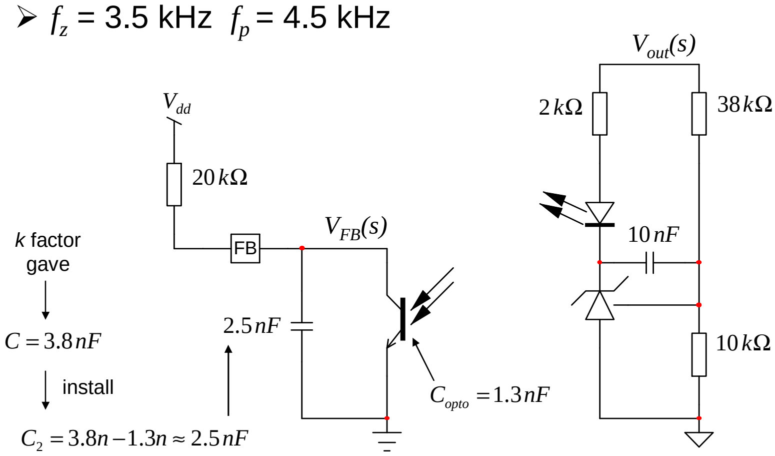
Design example 3: a DCM flyback converter

- ❑ Observe the open-loop Bode plot and select f_c: 1 kHz



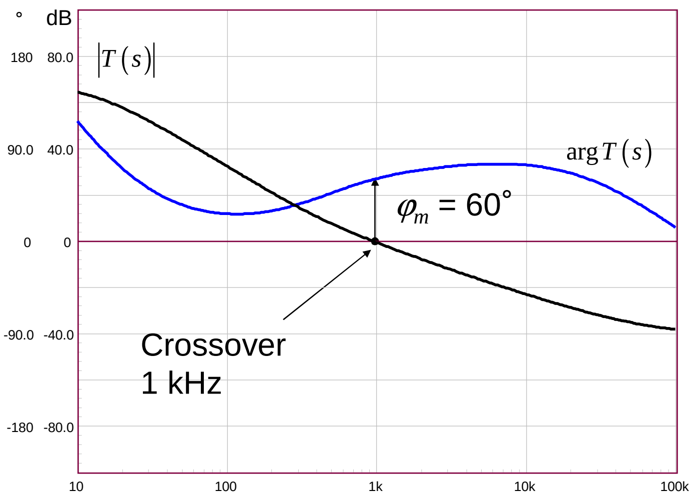
Design example 3: a DCM flyback converter

- ❑ Apply *k* factor or other method, get f_z and f_p
- ➢ f_z = 3.5 kHz f_p = 4.5 kHz



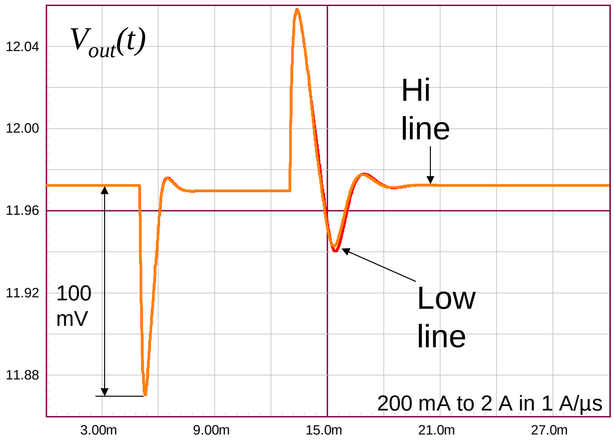
Design example 3: a DCM flyback converter

- ❑ Check loop gain and watch phase margin at f_c



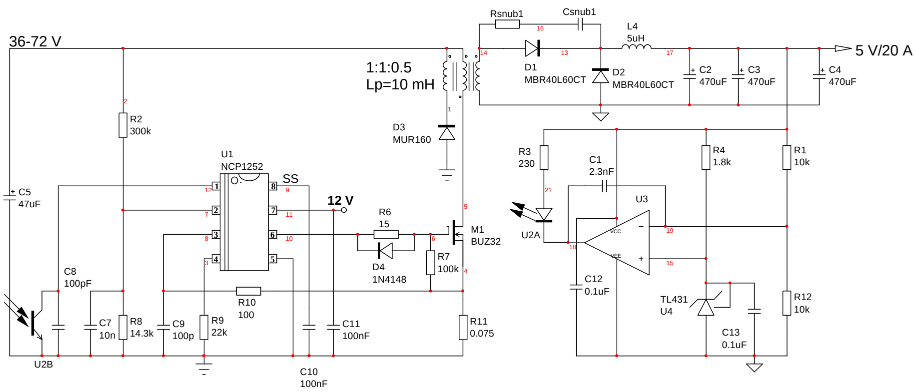
Design example 3: a DCM flyback converter

- ❑ Sweep ESR values and check margins again



Design example 4: a CCM forward converter

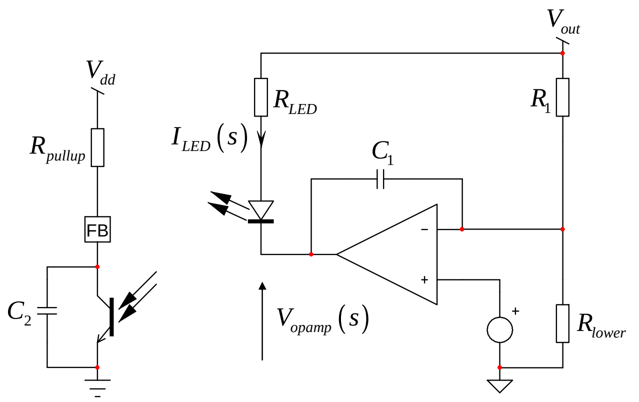
- ❑ We have designed a 5-V/20-A telecom input converter
- ❑ We use the NCP1252, fixed-frequency current-mode



- ❑ We need high dc gain, an op amp is adopted, f_c = 10 kHz

Design example 4: a CCM forward converter

- Despite the op amp, we still have a fast lane issue!



Same transfer function as with a TL431!

$$I_{LED}(s) = \frac{V_{out}(s) - V_{opamp}(s)}{R_{LED}} \longrightarrow \frac{V_{FB}(s)}{V_{out}(s)} = -\frac{R_{pullup}\mathrm{CTR}}{R_{LED}}\left[\frac{1+sR_1C_1}{sR_1C_1\left(1+sR_{pullup}C_2\right)}\right]$$

Design example 4: a CCM forward converter

- The LED resistor still limits the gain you can get:

$$R_{LED,\max} \le \frac{V_{out} - V_f - V_{opamp,\min}}{V_{dd} - V_{CE,sat}} R_{pullup}\mathrm{CTR}_{\min}$$

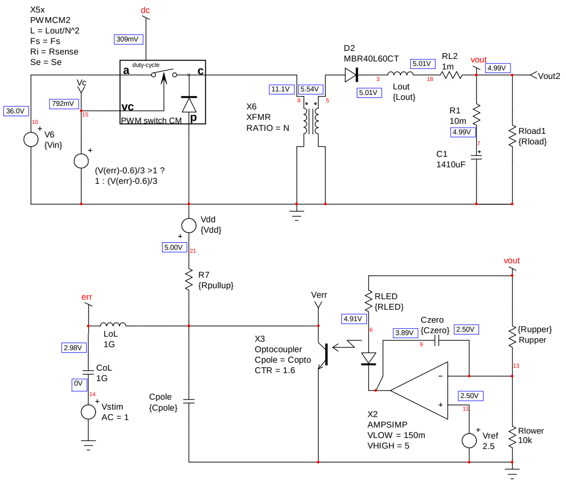
$$\left.\begin{aligned} V_{out} &= 5\,V \\ V_f &= 1\,V \\ V_{opamp,\min} &= 150\,mV \\ V_{dd} &= 4.8\,V \\ V_{CE,sat} &= 300\,mV \\ \mathrm{CTR}_{\min} &= 0.5 \\ R_{pullup} &= 3.3\,k\Omega \end{aligned}\right\} R_{LED,\max} \le 1.4\,k\Omega$$

- In this case, we cannot provide less than:

$$G_0 = \frac{R_{pullup}\mathrm{CTR}}{R_{LED}} = \frac{3.3k \times 0.5}{1.4k} = 1.17 = 1.4\,dB$$

Design example 4: a CCM forward converter

- ❑ Build an average model to extract $H(s)$



parameters

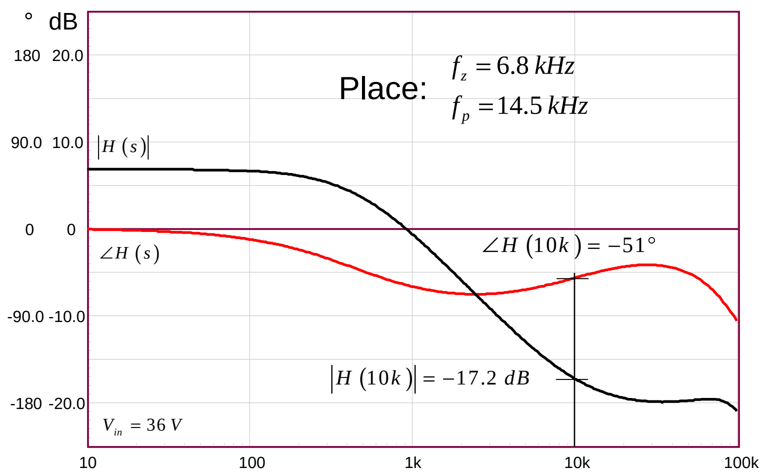
Vout=5
Lout=5u
Lmag=10mH
Fs=200k
N=0.5
Rsense=70m
Rload=250m
Vin=36

Vinmin=36
D=0.31
Smag=(Vinmin/Lmag)*Rsense
Sn=(((N*Vinmin-Vout)/Lout)*N)*Rsense
Q=1/(pi*((1-D)-0.5))
mc=((1/pi)+0.5)/(1-D)
Se=(mc-1)*Sn-Smag

Ramp compensation calculations

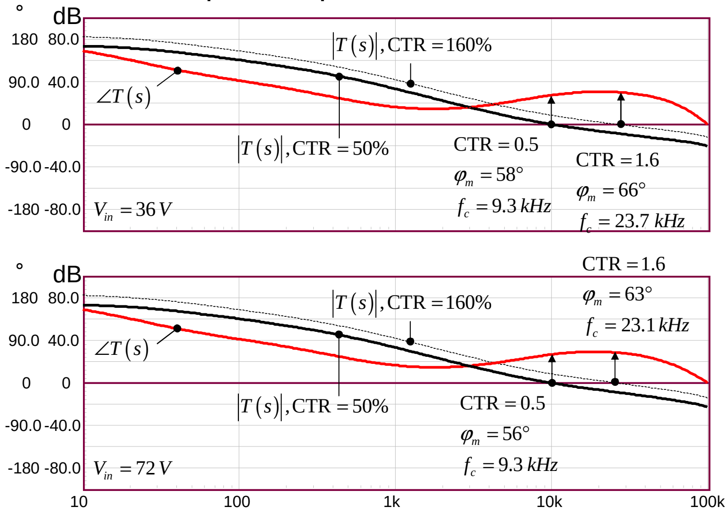
Design example 4: a CCM forward converter

- ❑ From the power stage Bode plot, extract the data at f_c



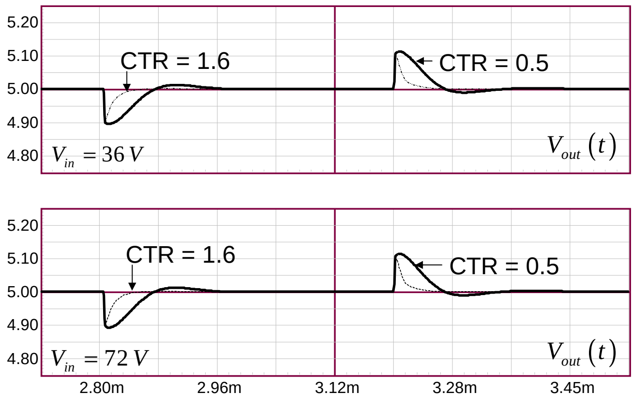
Design example 4: a CCM forward converter

- Check the impact on parameters such CTR, ESR etc.



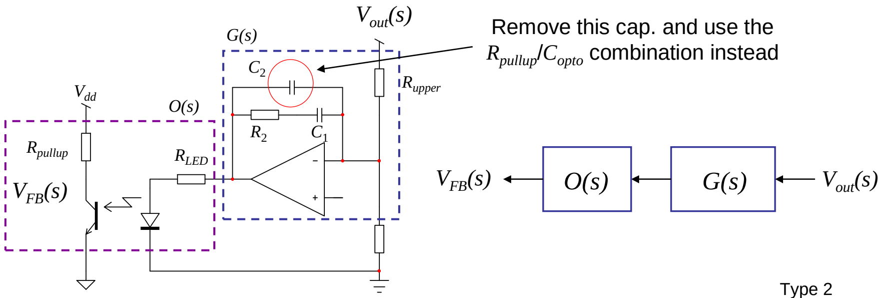
Design example 4: a CCM forward converter

- The CTR variation induces an upper crossover of 23 kHz
- This is an aggressive target, prone to collecting noise
- Better reduce the initial crossover to limit f_c to ≈15 kHz



Design example 4: a CCM forward converter

❑ The opto wired to the ground, the fast lane goes away



Type 2

$$\frac{V_{FB}(s)}{V_{out}(s)} = \frac{R_{pullup}\mathrm{CTR}}{R_{LED}}\,\frac{1}{1+sR_{pullup}C_{pole}}\,\frac{1+sR_2C_1}{sR_{upper}(C_1+C_2)\left(1+sR_2\left[\frac{C_1C_2}{C_1+C_2}\right]\right)}$$

Optocoupler contribution

Opto equivalent capacitor

Suppress C_2

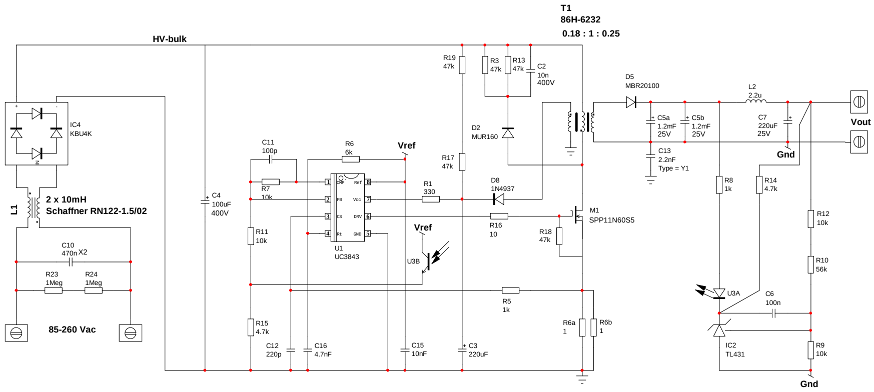
❑ The control phase is reversed, watch for the right polarity!

Agenda

❑ Feedback generalities
❑ The divider and the virtual ground
❑ Phase margin and crossover
❑ Poles and zeros
❑ Boosting the phase at crossover
❑ Various compensator types
❑ Practical implementations: the op amp
❑ Practical implementations: the OTA
❑ Practical implementations: the TL431
❑ Design examples
❑ **A real case study**
❑ Conclusion

A real-case example with a UC384X

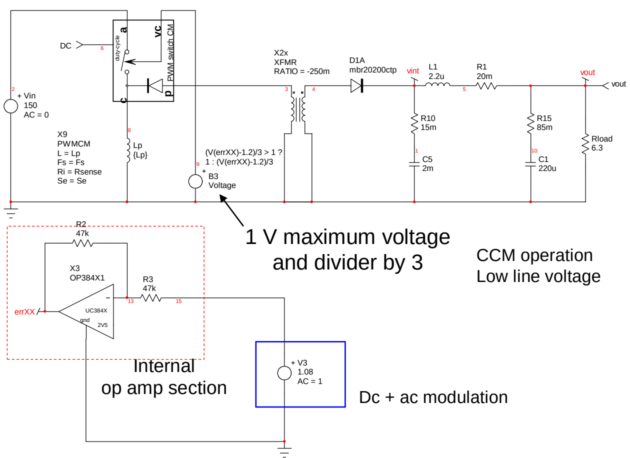
- ❑ A 19-V/3-A converter is built around an UC3843



- ❑ The converter operates in CCM at full load low line

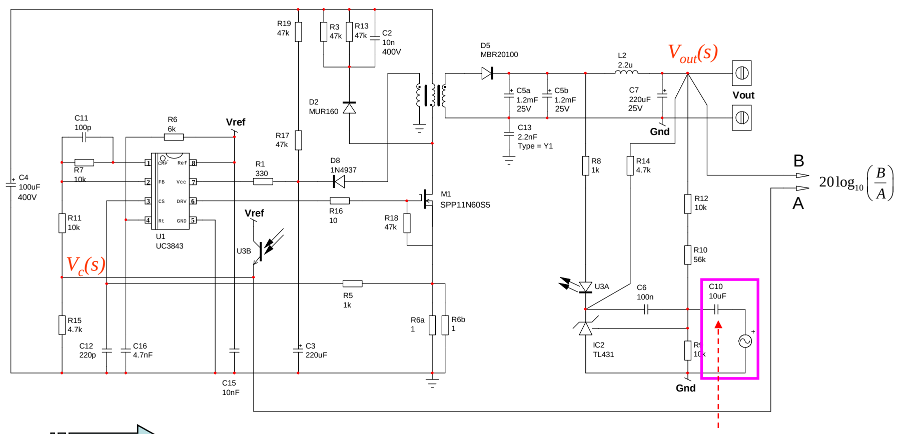
A real-case example with a UC384X

- ❑ Use an auto-toggling current-mode average model



A real-case example with a UC384X

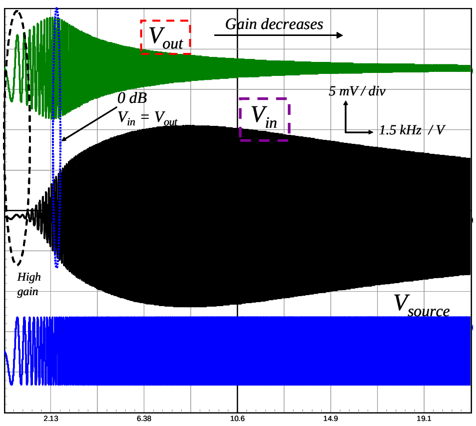
- ❑ $H(s)$ alone can be measured without loop opening

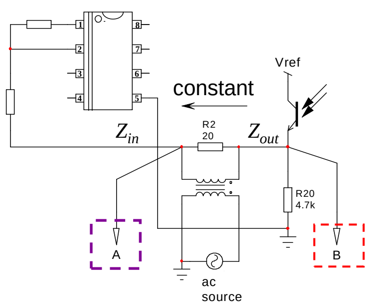


Watch out for capacitor connection (short-circuit to GND when discharged)

A real-case example with a UC384X

- ❑ For closed-loop measurements, a transformer is the solution

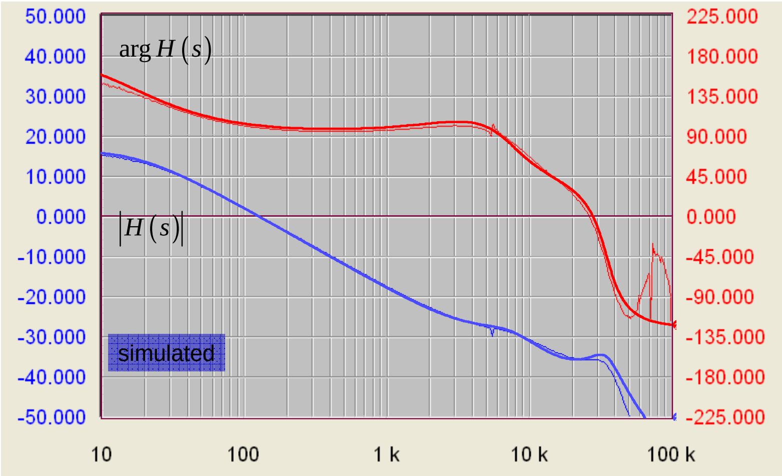




$$\left|T(s)\right| = \left|20\log_{10}\frac{V_B}{V_A}\right.$$

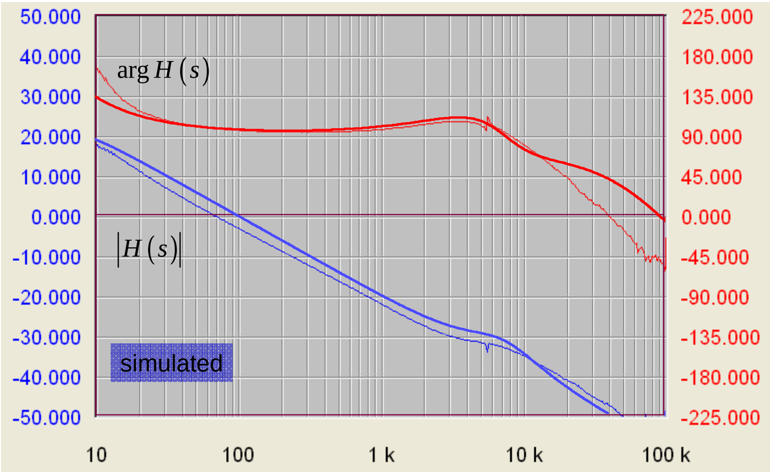
- ❑ Make sure $Z_{out} << Z_{in}$ to avoid gain errors

A real-case example with a UC384X



CCM operation, R_{load} = 6.3 Ω

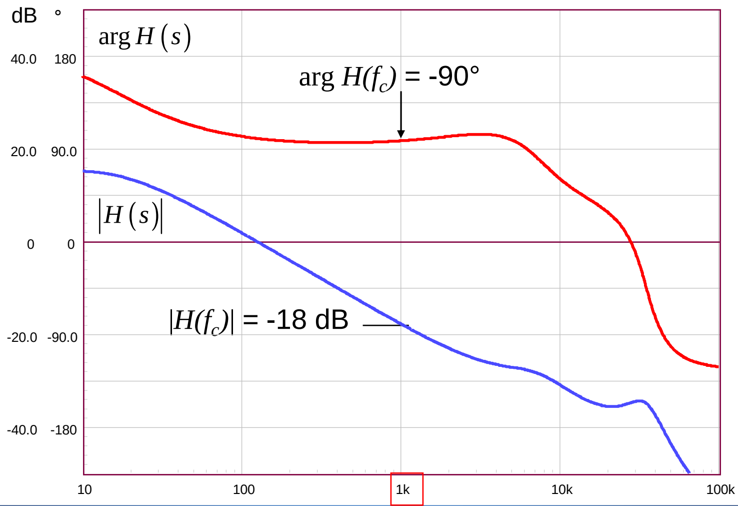
A real-case example with a UC384X



DCM operation, R_{load} = 20 Ω

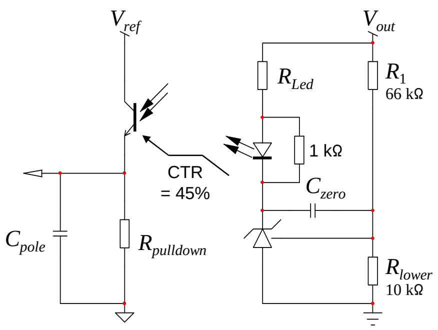
A real-case example with a UC384X

❑ Select the crossover point on the open-loop Bode plot



A real-case example with a UC384X

❑ The TL431 is tailored to pass a 1-kHz bandwidth



Calculate mid-band gain: +18 dB

$$R_{LED} = \frac{R_{pullup}\text{CTR}}{10^{\frac{18}{20}}} = \frac{4.7k \times 0.45}{7.94} = 266\ \Omega$$

We place a zero at 300 Hz:

$$C_{zero} = \frac{1}{2\pi f_{zero} R_1} = \frac{1}{6.28 \times 300 \times 66k} = 8\ nF$$

We place a pole at 3.3 kHz:

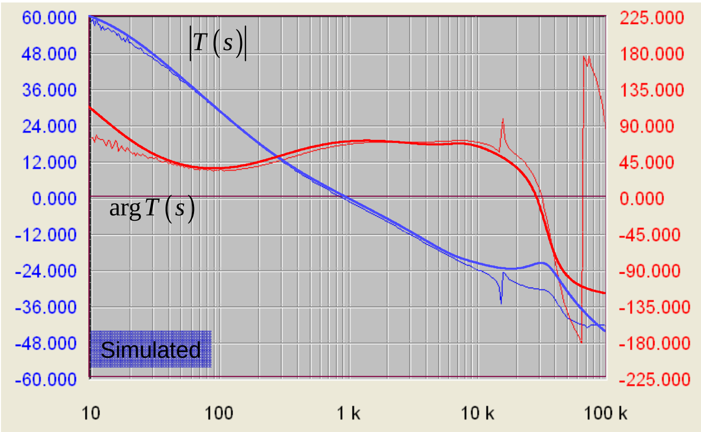
$$C_{pole} = \frac{1}{2\pi f_{pole} R_{pulldown}} = \frac{1}{6.28 \times 3.3k \times 4.7k} = 10\ nF$$

k factor method

"Switch-Mode Power Supplies: SPICE Simulations and Practical Designs", McGraw-Hill

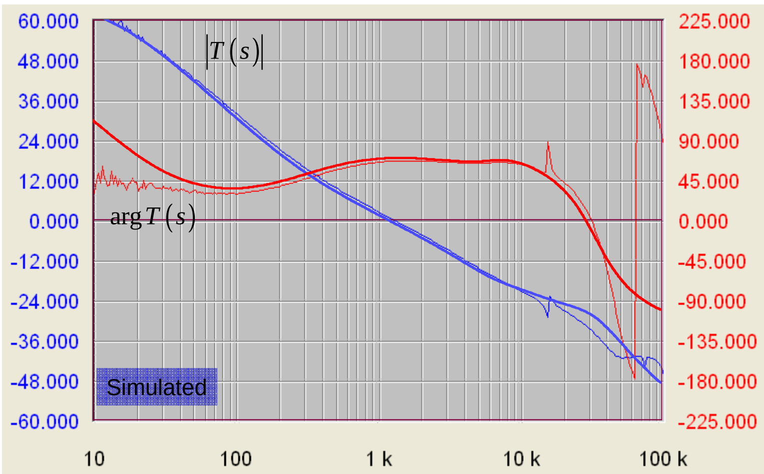
A real-case example with a UC384X

- ❑ Sweep extreme voltages and loads as well!



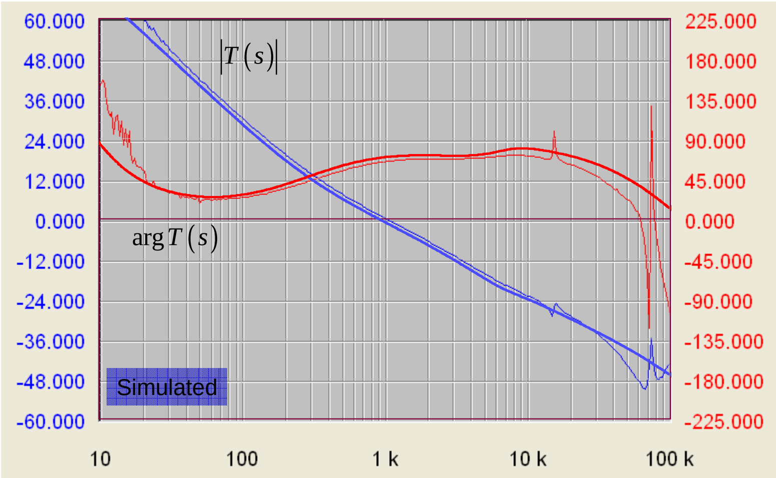
CCM operation, R_{load} = 6.3 Ω, V_{in} = 150 Vdc

A real-case example with a UC384X



CCM operation, R_{load} = 6.3 Ω, V_{in} = 330 Vdc

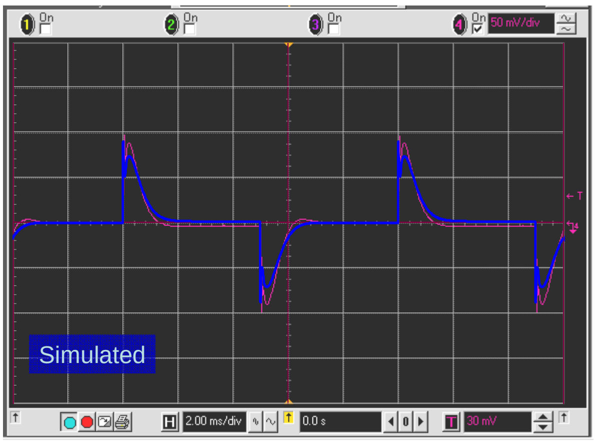
A real-case example with a UC384X



DCM operation, R_{load} = 20 Ω, V_{in} = 330 Vdc

A real-case example with a UC384X

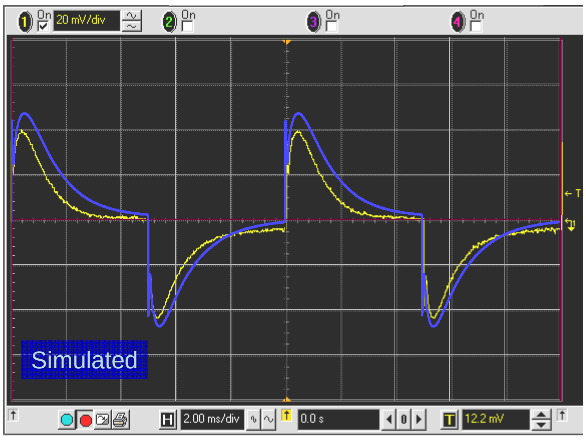
- ❑ Good agreement between curves!



V_{in} = 150 V
CCM
2 to 3 A
1 A/µs

A real-case example with a UC384X

❑ DCM operation at high line is also stable



V_{in} = 330 V
DCM
0.5 to 1 A
1 A/µs

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Conclusion

❑ We have seen how to apply loop theory to a switching converter

❑ Classical type 1, 2 and 3 compensators have been covered

❑ Their implementation with op amps, OTAs and TL431 studied

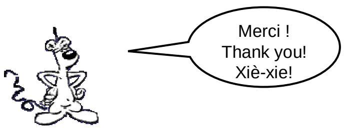
❑ Op amps are the most flexible, OTAs and TL431 have limits

❑ In isolated supplies, the optocoupler affects the transmission chain

❑ Design examples showed the power of averaged models

❑ Use them to extensively test the loop stability (sweep ESRs etc.)

❑ Applying these recipes is key to design success!



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