K_{gfe} Background

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K_{gfe} Derivation

Constituent equations:

$$P_{fe} = K_{fe} \cdot A_e \cdot l_c \cdot {B_{PK}}^{\beta}$$

$$P_{cu} = \frac{{I_{tot}}^2 \cdot MLT \cdot n_1^2 \cdot \rho}{K_u \cdot W_a}$$

$$P_{total} = P_{fe} + P_{cu}$$

$$n_1 = \frac{\lambda_1}{2 \cdot A_e \cdot B_{PK}}$$

$$I_{tot} = \sum_{j=1}^{k} \frac{i_j \cdot n_j}{n_1}$$

Solving $\frac{\partial P_{total}}{\partial B_{PK}} = 0$ for B_{PK} yields:

$$B_{PK} = 2^{-\frac{1}{\beta+2}} \cdot \left(\frac{{I_{tot}}^2 \cdot {\lambda_1}^2 \cdot MLT \cdot \rho}{\beta \cdot K_{fe} \cdot W_a \cdot A_e^3 \cdot l_c \cdot K_u} \right)^{\frac{1}{\beta+2}}$$

Substituting B_{PK} (et. al.) into P_{total} and rearranging yields K_{gfe}.

$$K_{gfe} = \left(\left(\frac{\beta}{2}\right)^{\frac{2}{\beta+2}} + \left(\frac{\beta}{2}\right)^{-\frac{\beta}{\beta+2}} \right)^{-\frac{\beta+2}{\beta}} \cdot \frac{W_a \cdot A_e^{\frac{2\cdot(\beta-1)}{\beta}}}{MLT \cdot l_c^{\frac{2}{\beta}}} = \frac{{I_{tot}}^2 \cdot {\lambda_1}^2 \cdot \rho \cdot \left(\frac{K_{fe}}{P_{total}}\right)^{2/\beta}}{4 \cdot P_{total} \cdot K_u}$$

I_{tot} is derived by using the Lagrangian multiplier ξ.

$$P_{cu} = \frac{MLT \cdot \rho \cdot \sum_{j=1}^{k} \frac{i_j^2 \cdot n_j^2}{\alpha_j}}{W_a \cdot K_u}$$

Λ is the optimization equation.

$$\Lambda = \frac{MLT \cdot \rho \cdot \sum_{j=1}^{k} \frac{i_j^2 \cdot n_j^2}{\alpha_j}}{W_a \cdot K_u} + \xi \cdot \left(1 - \sum_{j=1}^{k} \alpha_j\right)$$

Solving $\frac{\partial \Lambda}{\partial \xi} = 0$ and $\frac{\partial \Lambda}{\partial \alpha_j} = 0$ yeilds:

$$\xi = -\frac{MLT \cdot \rho \cdot \left(\sum_{j=1}^{k} i_j \cdot n_j\right)^2}{W_a \cdot K_u}$$

$$\alpha_i = \frac{i_i \cdot n_i}{\sum_{j=1}^{k} i_j \cdot n_j}$$

Scale K_{gfe} to be proportional to volume

- $K_{gfe} = \left(\left(\frac{\beta}{2}\right)^{\frac{2}{\beta+2}} + \left(\frac{\beta}{2}\right)^{-\frac{\beta}{\beta+2}}\right)^{-\frac{\beta+2}{\beta}} \cdot \frac{W_a \cdot A_e^{\frac{2\cdot(\beta-1)}{\beta}}}{MLT \cdot l_c^{\frac{2}{\beta}}} = \frac{I_{tot}{}^2 \cdot \lambda_1{}^2 \cdot \rho \cdot \left(\frac{K_{fe}}{P_{total}}\right)^{2/\beta}}{4 \cdot P_{total} \cdot K_u}$
- The units for K_{gfe} as defined above are $m^{5-\frac{6}{\beta}}$
- $Core\ Volume \sim K_{gfe}^{\frac{3}{5-\frac{6}{\beta}}}$