A PRIMER ON ELECTRIC POWER

Power in general is defined as the rate of energy change with time. **Energy** is the capacity for doing work. **Work** is done when an object is moved by a force. Numerically work done by any force $\vec{\mathbf{F}}$ when an object moves from a point A to a point B is specified as linear (or path) integral of the force component in the direction of motion times distance moved:

$$W = \int_{A}^{B} \vec{F} * d\vec{s} \qquad (1)$$

Note that $\vec{\mathbf{X}} * \vec{\mathbf{Y}}$ is a scalar product of two vectors $\vec{\mathbf{X}}$ and $\vec{\mathbf{Y}}$ which is equal to $|\vec{\mathbf{X}}| * |\vec{\mathbf{Y}}| * \mathbf{cos}\,\varphi$, where φ is an angle between these vectors.

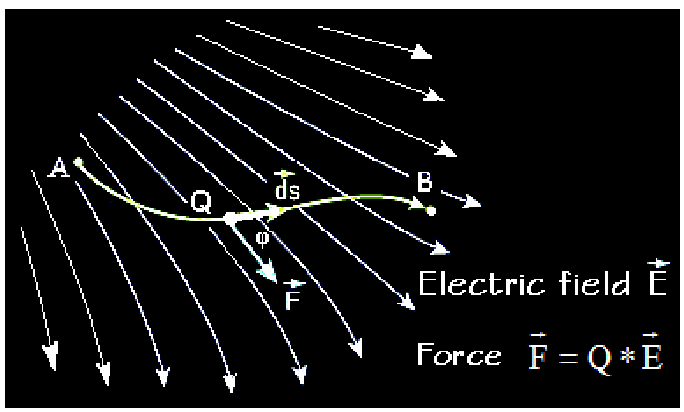
If a point electric charge Q is placed in electric field it will experience an electric force, which is proportional to the amount of Q. The force per unit charge is called **electric field**:

$$\vec{E} = \frac{\vec{F}}{Q} \qquad (2)$$

We presume here that Q has a negligible size and does not distort the field.

Combining (1) and (2) yields the expression for **work in electric field** to move a charge Q from a point A to a point B:

$$W = \int_{A}^{B} \vec{E} * Q * d\vec{s} \qquad (3)$$



From (3) the power required to move charge Q in electric field from a point A to a point B is:

$$P = \frac{dW}{dt} = \int_{A}^{B} \frac{dQ}{dt} * \vec{E} * d\vec{s} = i * \int_{A}^{B} \vec{E} * d\vec{s} \quad , \qquad (4)$$

where $i = \dfrac{dQ}{dt}$ is the rate of charge flow past a given area called **electric current**.

The linear integral in (4) is called **voltage** **(V)** or potential difference between points A and B

V= $\int_{A}^{B} \vec{E} * d\vec{s}$ (5)

Substituting (5) into (4) yields familiar expression for the instantaneous **electric power**:

$$P(t) = V(t) * i(t) \ , \qquad (6)$$

where $V(t)$ and $i(t)$ are instantaneous values of voltage and current.

Note that generally, any linear integral is a function of the path from A to B. In electrostatic fields however, this integral does not depend on the path and is the function only of the electric field and coordinates of points A and B. Likewise, work in electrostatic field does not depend on the path. Particularly, the work to move a charge around any closed loop (A=B) is zero. Fields in which work does not depend on path are called **conservative (or potential) fields**.

Electrostatic fields are generated by electric charges that remain in rest or are moving with a constant speed. Therefore, such fields present mainly academic interest since no electronic circuit would work without accelerating electric charges. In changing electric fields integral (5) along a closed loop is no longer zero:

$$\oint \vec{E} * d\vec{s} = -\frac{d\psi}{dt} = -\frac{d}{dt} * \int_{A} \vec{B} * d\vec{A} \qquad (7)$$

where Ψ- magnetic flux through the closed loop, $\vec{B}$ - magnetic flux density through the loop, A – area of the loop.

As the result, the linear integral in (5) generally depends on the pass from a point A to a point B. Therefore the voltage between any two points A and B is no longer defined. Strictly speaking, in variable electric fields we may only talk about voltage between points A and B **along a given path**. For different paths between the same points A and B the voltage (5) will be different.

If magnetic flux $\vec{B}$ is the same through entire surface A formed by the loop, then (7) can be simplified:

$$\oint \vec{E} * d\vec{s} = -\frac{dB}{dt} * A \qquad (8)$$

where A is the area of the loop.

We see from (8) that in order to minimize the effects of changing magnetic field we need to reduce the rate of change of magnetic field dB/dt and the area A of the loop A. If their product is small enough we may neglect it and consider the electric field quasi-potential with the familiar definition of voltage.

In AC circuits all quantities in (4), (5) and (6) are continuously varying and are functions of time **t**. The average value of the power over certain period of time **T** is given by

$$Pavg = \frac{1}{T} * \int_0^T V(t) * i(t) * dt \tag{9}$$

Average power (9) is called **active power** or **real power**, or simply watts.
Note that AC values are often stated as root-mean-square (RMS). The **RMS** value of any variable X(t) is generally defined by

$$Xrms = \sqrt{\frac{1}{T} * \int_0^T X(t)^2 * dt} \tag{10}$$

Product **Vrms*Irms** is called **apparent power** (or **volt-amps**).

The ratio between active power and apparent power is called **power factor**:

$$PF = \frac{Pavg}{Vrms * Irms} = \frac{\int_0^T V(t) * i(t) * dt}{\sqrt{\int_0^T V(t)^2 * dt} * \sqrt{\int_0^T i(t)^2 * dt}} \tag{11}$$

Any periodic non-sinusoidal current can be presented by Fourier transform:

$$i(t) = Io + i_1(t) + i_2(t) + ... + i_n(t) + ... \tag{12}$$

For a sinusoidal voltage V(t), substituting (12) into (9) gives active power as:

$$Pavg = \frac{1}{T}\left[\int_0^T V(t) * i_1(t) * dt + \sum_{n=2}^{\infty} \int_0^T V(t) * i_n(t) * dt\right] \tag{13}$$

It can be shown that for $n \geq 2$: $\int_0^T V(t) * i_n(t) * dt = 0$, that is active power is provided only by first (fundamental) harmonic of the current:

$$Pavg = \frac{1}{T} * \int_0^T V(t) * il(t) * dt \qquad (14)$$

If voltage and first harmonic of the current have the same frequency, we derive from (14):

$$Pavg = \frac{1}{T} * \int_0^T V_{pk} * \sin(\omega t) * I_{1pk} * \sin(\omega t + \varphi) * dt = \frac{V_{pk} * I_{1pk}}{2} * \cos\varphi \qquad (15)$$

where V_{pk} and I_{1pk}- peak (maximum) values of the voltage and fundamental harmonic of the current respectively,

ω - angular frequency (in radian/sec).

φ - is the phase angle (in radians) between the fundamental harmonic of the current and the voltage.

For sinusoidal signals, it can be derived from (8) that $X_{rms} = X_{pk}/\sqrt{2}$.

This yields the following expression for **<u>active power</u>**:

$$P_{avg} = V_{rms} * I_{1rms} * \cos\varphi \qquad (16)$$

where V_{rms}- RMS voltage, I_1rms – RMS value of fundamental harmonic of the current.

Comparing (11) and (16) we see that power factor is

$$PF = \frac{I_{1rms}}{I_{rms}} \cos\varphi \qquad (17)$$

where φ - is the phase angle between the fundamental harmonic of the current and the voltage.

The ratio between apparent power associated with higher order harmonics and apparent power associated with fundamental harmonic is called **<u>Total Harmonic Distortion (THD):</u>**

$$THD = \frac{\sqrt{I^2{}_{2rms} + I^2{}_{3rms}... + I^2{}_{nrms} + ...}}{I_{1rms}} \qquad (18)$$

where **Inrms**- RMS value of the **n-th** harmonic of the current.

For a periodic current from (12):

$$Irms = \sqrt{I^2 o + I^2{}_{1rms} + I^2{}_{2rms} + ... + I^2{}_{nrms} + ...} \quad , \qquad (19)$$

where **Io** – DC component of the current.

In AC lines Io=0. Then from (18) and (19) THD can be expressed as

$$THD = \frac{\sqrt{Irms^2 - I_{1rms}{}^2}}{I_{1rms}} \qquad (20)$$

From (17) and (20) we can also derive the relationship between PF and THD:

$$PF = \frac{\cos\varphi}{\sqrt{1 + THD^2}} \qquad (21)$$