TEXAS INSTRUMENTS

Implementing PR controllers as mapped to a 2p2z structure using C2000 Microcontrollers

v 1.0, 4/3/2018

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PR controllers in analog domain can be written as:

$$G_i = \frac{K_i 2\omega_{rc} s}{s^2 + 2\omega_{rc} s + \omega_o^2}$$

For digital implementation Tustin transform is used $s = 2F_s \frac{(z-1)}{(z+1)}$ and the coefficients of the digital compensator derived as follows:

$$G_{id} = \frac{K_i 2\omega_{rc} 2F_s \frac{(z-1)}{(z+1)}}{4F_s^2 \frac{(z-1)^2}{(z+1)^2} + 2\omega_{rc} 2F_s \frac{(z-1)}{(z+1)} + \omega_o^2}$$

$$= \frac{4K_i \omega_{rc} F_s (z^2 - 1)}{z^2(4F_s^2 + \omega_o^2 + 4F_s\omega_{rc}) + z(-8F_s^2 + 2\omega_o^2) + (\omega_o^2 + 4F_s^2 - 4F_s\omega_{rc})}$$

Notice the drift in the resonance frequency when using Tustin approximation

Now assuming the PR controller without damping

$G_i = \frac{K_i s}{s^2 + \omega_o^2}$ & using First Order Hold i.e. ramp invariant method the z-transform is given as

$$G_i = \frac{K_i}{T\omega_o^2} \frac{(z^2 - 1)(1 - \cos(\omega_o T))}{z^2 - 2z\cos(\omega_o T) + 1}$$

Tustin discretization of the above yields

$$G_{id} = \frac{K_i 2F_s \dfrac{(z-1)}{(z+1)}}{4F_s^2 \dfrac{(z-1)^2}{(z+1)^2} + \omega_o^2}$$

$$= \frac{4K_i F_s (z^2 - 1)}{z^2(4F_s^2 + \omega_o^2) + z(-8F_s^2 + 2\omega_o^2) + (\omega_o^2 + 4F_s^2)}$$

To correct for the drift in the resonance point placement, the resonant frequency can be adjusted as below

$$-\cos(\omega_o T) = \frac{(\omega_{o1}^2 - 4F_s^2)}{(\omega_{o1}^2 + 4F_s^2)}$$

$$\Rightarrow \omega_{o1} = 2F_s \sqrt{\frac{(1 + \cos(\omega_o T))}{(1 - \cos(\omega_o T))}}$$

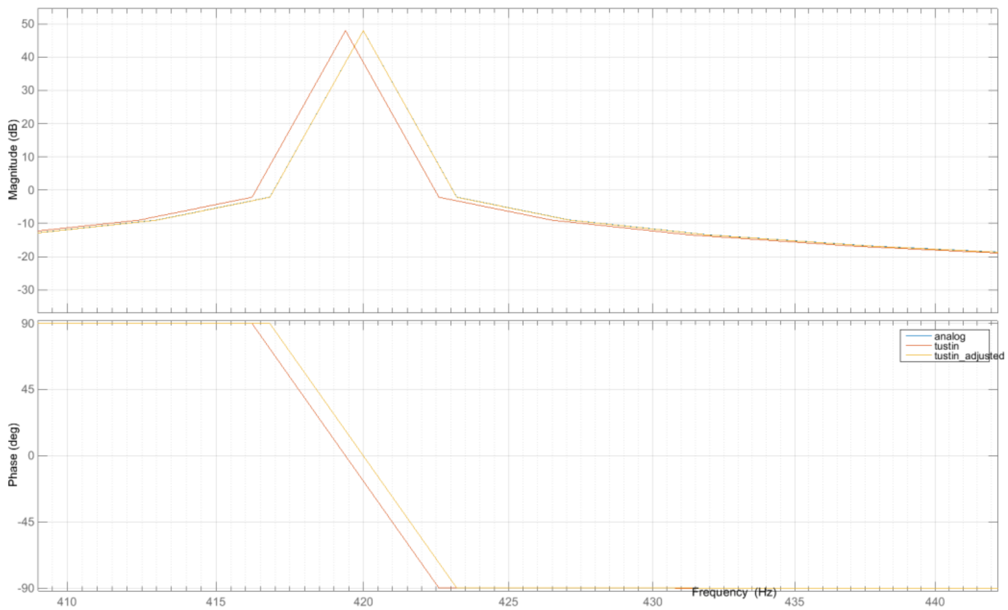
Using Trig Identity $\sin^2\theta = \dfrac{1 - \cos 2\theta}{2}$ & $\cos^2\theta = \dfrac{1 + \cos 2\theta}{2}$

$$\Rightarrow \omega_{o1} = 2F_s \tan(\omega_o T / 2)$$

And the discrete version of the Tustin transform adjusted as

$$G_{id} = \frac{4K_i \omega_{rc} F_s (z^2 - 1)}{z^2(4F_s^2 + \omega_{o1}^2 + 4F_s\omega_{rc}) + z(-8F_s^2 + 2\omega_{o1}^2) + (\omega_{o1}^2 + 4F_s^2 - 4F_s\omega_{rc})}$$

With the adjusted frequency there is no drift that is observed in the resonance point when using tustin transform for discretizing the controller.



This two pole two zero structure can be mapped to a 2p2z structure available as part of the DCL library easily.