Analyzing the Sepic Converter

In the last issue, we talked about the simplest of all converters, the buck converter, and showed how its control transfer functions could be extraordinarily complex. In this issue, we'll go to the other end of the spectrum, and look at a converter that is far more complex, yet is often used by engineers who are unaware of the difficulties that follow.

By Dr. Ray Ridley, Ridley Engineering

The Sepic Converter

The most basic converter that we looked at last month is the buck converter. It is so named because it always steps down, or bucks, the input voltage. The output of the converter is given by:

$$V_o = DV_g$$

Interchange the input and the output of the buck converter, and you get the second basic converter – the boost. The boost always steps up, hence its name. The output voltage is always higher than the input voltage, and is given by:

$$V_o = \frac{1}{D'}V_g$$

What if you have an application where you need to both step up and step down, depending on the input and output voltage? You could use two cascaded converters – a buck and a boost. Unfortunately, this requires two separate controllers and switches. It is, however, a good solution in many cases.

The buck-boost converter has the desired step up and step down functions:

$$V_o = \frac{-D}{D'}V_g$$

The output is inverted. A flyback converter (isolated buck-boost) requires a transformer instead of just an inductor, adding to the complexity of the development.

One converter that provides the needed input-to-output gain is the Sepic (single-ended primary inductor converter) converter. A Sepic converter is shown in Fig. 1. It has become popular in recent

years in battery-powered systems that must step up or down depending upon the charge level of the battery.

Fig. 2 shows the circuit when the power switch is turned on. The first inductor, L1, is charged from the input voltage source during this time. The second inductor takes energy from the first capacitor, and the output capacitor is left to provide the load current. The fact that both L1 and L2 are disconnected from the load when the switch is on leads to complex control characteristics, as we will see later.

When the power switch is turned off, the first inductor charges the capacitor C1 and also provides current to the load, as shown in Fig. 3. The second inductor is also connected to the load during this time.

The output capacitor sees a pulse of current during the off time, making it inherently noisier than a buck converter.

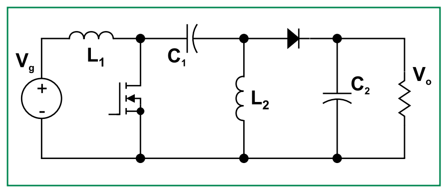


Figure 1. The Sepic converter can both step up and step down the input voltage, while maintaining the same polarity and the same ground reference for the input and output.

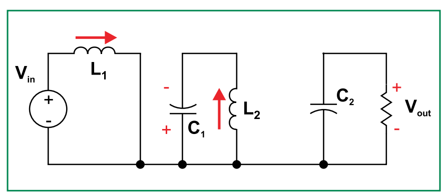


Figure 2. When the switch is turned on, the input inductor is charged from the source, and the second inductor is charged from the first capacitor. No energy is supplied to the load capacitor during this time. Inductor current and capacitor voltage polarities are marked in this figure.

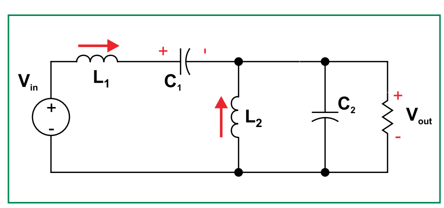


Figure 3. With the switch off, both inductors provide current to the load capacitor.

The input current is non-pulsating, a distinct advantage in running from a battery supply.

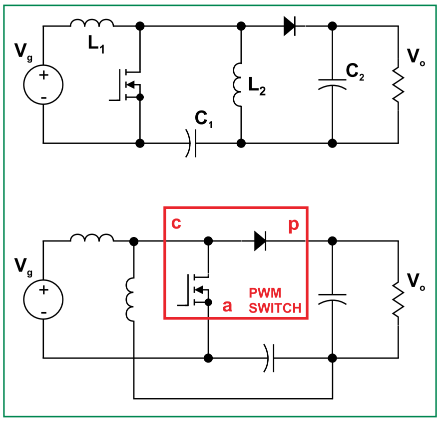


Figure 4. In order to take advantage of Vorpérian's PWM switch model, the circuit elements must first be rearranged. The function of the original topology is retained when the capacitor is moved, and the second inductor is redrawn.

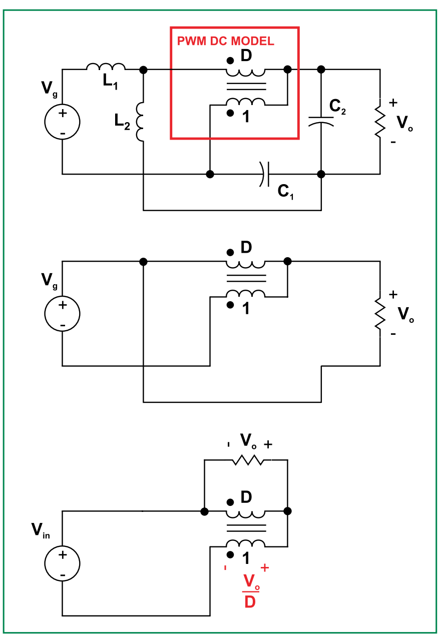


Figure 5. For DC analysis, the small signal sources are set to zero, inductors become short circuits, and capacitors become open circuits. After the circuit is redrawn, it is a trivial matter to write KVL around the outer loop of the circuit to solve for the conversion gain of the converter.

The PWM Switch Model in the Sepic Converter

The best way to analyze both the AC and DC characteristics of the Sepic converter is by using the PWM switch model, developed by Dr. Vatché Vorpérian in 1986. Some minor circuit manipulations are first needed to reveal the location of the switch model, and this is shown in Fig. 4.

First, capacitor C1 is moved to the bottom branch of the converter. Then, inductor L2 is pulled over to the left, keeping its ends connected to the same nodes of the circuit. This reveals the PWM switch model of the converter, with its active, passive, and common ports, allowing us to use well-established analysis results for this converter.

For more background on the PWM switch model, the text book "Fast Analytical Techniques for Electrical and Electronic Circuits" [1] is highly recommended.

DC Analysis of the Sepic Converter

Fig. 5 shows the equivalent circuit of the Sepic converter with the DC portion of the PWM switch model in place. The DC model is just a 1:D transformer. We replace the inductors with short circuits, and the capacitors with open circuits for the DC analysis. You can, if you like, include any parasitic resistances in the model [2], but that's beyond the scope of this article.

After the circuit is manipulated as shown in the figure, we can write the KVL equation around the outer loop of the converter:

$$V_g + V_o - \frac{1}{D} V_o = 0$$

Rearranging gives:

$$V_g = \left(\frac{1}{D} - 1 \right) V_o = \frac{D'}{D}$$

And the DC gain is given by:

$$V_o = \frac{D}{D'} V_g$$

Here we see the ability of the converter to step up or down, with a gain of 1 when D=0.5. Unlike the buck-boost and Cuk converters, the output is not inverted.

AC Analysis of the Sepic Converter

You won't find a complete analysis of the Sepic converter anywhere in printed literature. What you will find are application notes with comments like, "the Sepic is not well-understood." Despite the lack of documentation for the converter, engineers continue to use it when applicable.

Proper small-signal analysis of the Sepic converter is a difficult analytical task, only made practical by advanced circuit analysis techniques originally developed by Dr. David Middlebrook and continued by Vorpérian. [1]

If you're going to build a Sepic, as a minimum, you need to understand the control characteristics. Fortunately, Vorpérian's work is now available for this converter, and you can download the complete analysis notes .[2]

The simplified analysis of the Sepic converter, derived in detail in [2], ignores parasitic resistances of the inductors and capacitors, and yields the following result for the control-to-output transfer function:

$$\frac{v_o(s)}{d(s)} \approx \frac{1}{D'^2} \frac{\left(1 - s\frac{L_1}{R}\frac{D^2}{D'^2}\right)\left(1 - s\frac{C_1(L_1+L_2)R}{L_1}\frac{D'^2}{D^2} + s^2\frac{L_2C_1}{D}\right)}{\left(1 + \frac{s}{\omega_{o1}Q_1} + \frac{s^2}{(\omega_{o1})^2}\right)\left(1 + \frac{s}{\omega_{o2}Q_2} + \frac{s^2}{(\omega_{o2})^2}\right)}$$

Where

$$\omega_{o1} \approx \frac{1}{\sqrt{L_1\left(C_2\frac{D^2}{D'^2} + C_1\right) + L_2(C_1 + C_2)}}$$

$$Q_1 \approx \frac{R}{\omega_{o1}\left(L_1\frac{D^2}{D'^2} + L_2\right)}$$

$$\omega_{o2} \approx \sqrt{\frac{1}{L_2\frac{C_1}{D^2}\left\|\frac{C_2}{D'^2}\right.} + \frac{1}{L_1C_1\|C_2}}$$

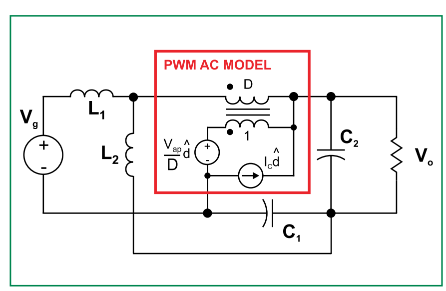


Figure 6. The small-signal AC sources are included in the switch model, and we can either solve the analysis by hand, or use PSpice to plot desired transfer functions. The hand analysis is crucial for symbolic expressions and design equations.

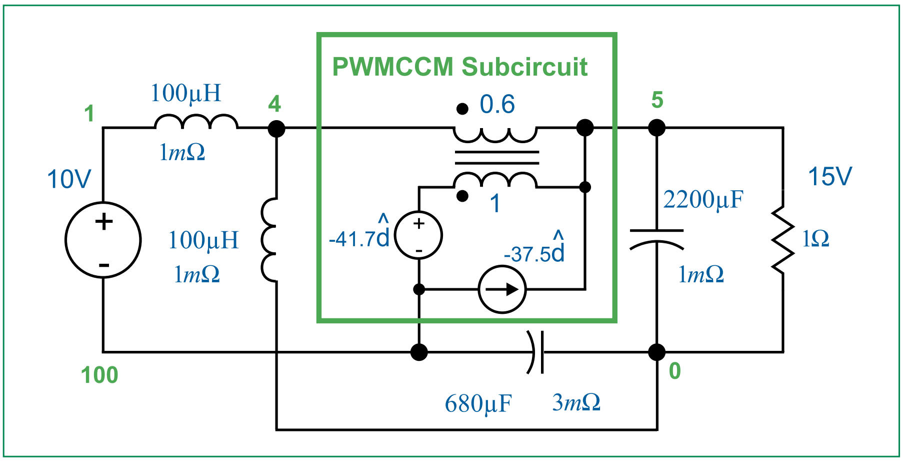


Figure 7. Analysis can also be done with PSpice. This figure shows a specific design example for a 15 W converter. Parasitic resistances are included in the PSpice model.

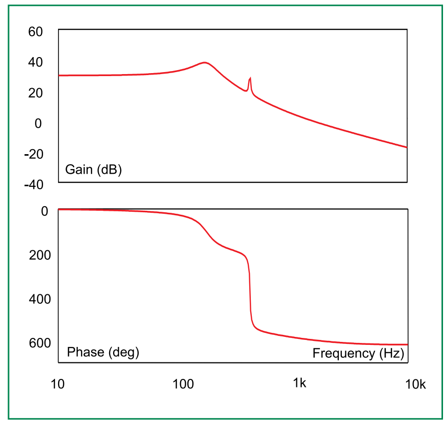


Figure 8. This shows the control-to-output transfer function for the Sepic converter. With low values of damping resistors, the converter has four poles, and three right-half-plane zeros. This results in an extreme phase delay of 630 degrees!

$$Q_2 \approx \frac{R}{\omega_{o2}\left(L_1 + L_2\right)\dfrac{C_1}{C_2}\dfrac{\omega_{o1}^2}{\omega_{o2}^2}}$$

As you can see from these expressions, the "simplified" analysis is anything but simple. Including the parasitic resistances greatly complicates the analysis, but may be necessary for worst-case analysis of the Sepic converter. The analysis of this converter involves the use of the powerful extra element theorem, and Vorpérian's book on circuit analysis techniques. [1]

In addition to the inevitable fourth-order denominator of the Sepic, the most important features to note in the control transfer function are the terms in the numerator. The first term is a single right-half-plane (RHP) zero. Right-half-plane zeros are a result of converters where the response to an increased duty cycle is to initially decrease the output voltage.

When the power switch is turned on, the first inductor is disconnected from the load, and this directly gives rise to the first-order RHP zero. Notice that the expression only depends on the input inductor, L1, the load resistor, R, and the duty cycle.

The complex RHP zeros arise from the fact that turning on the switch disconnects the second inductor from the load. These zeros will actually move with the values of parasitic resistors in the circuit, so careful analysis of your converter is needed to ensure stability under all conditions.

PSpice Modeling of the Sepic Converter

The analytical solution above does not include all of the parasitic circuit elements. As you will see from [2], there is a prodigious amount of work to be done even without the resistances.

We can also use PSpice to help understand the Sepic better. Fig. 7 shows the circuit model for a specific numerical application of the Sepic, and it includes resistances which will affect the stability of the converter, sometimes in dramatic ways.

The PSpice file listing can be downloaded from [2] so you can reproduce these results to analyze your own Sepic converter.

Fig. 8 shows the result of the PSpice analysis. The two resonant frequencies predicted by the hand analysis can clearly be seen in the transfer function plot. What is remarkable is the extreme amount of phase shift after the second resonance. This is caused by the delay of the second pair of poles, and the additional delay of the complex RHP zeros. The total phase delay through the converter is an astonishing 630 degrees. Controlling this converter at a frequency beyond the second resonance is impossible.

Summary

The Sepic converter definitely has some select applications where it is the topology of choice. How do designers get away with building such a converter? There are several possibilities. First, the dynamic and step load requirements on the system may be very benign, with no reason to design a loop with high bandwidth. This allows the loop gain to be reduced below 0 dB before the extreme phase delay of the second resonance.

Secondly, in many practical cases, the parasitic resistances of the circuit move the RHP zeros to the left half plane, greatly reducing the phase delay. This can also be done with the addition of damping networks to the power stage, a topic beyond the scope of this article.

Thirdly, some engineers do not build a proper Sepic. In some application notes, the two inductors are wound on a single toroidal core, which provides almost unity coupling between the two. In this case, the circuit no longer works as a proper Sepic. Don't fall into this design trap - the circuit will be far from optimum.

Additional Reading

[1] "Fast Analytical Techniques for Electrical and Electronic Circuits", Vatché Vorpérian, Cambridge University Press 2002. ISBN 0 521 62442 8.

[2] http://www.switchingpowermagazine.com. Click on Articles and Sepic Analysis Notes.

www.ridleyengineering.com

Analysis of the Sepic Converter

by

Dr. Vatché Vorpérian

Notes on the small-signal analysis of the

The isolated sepic converter with synchronous rectifiers

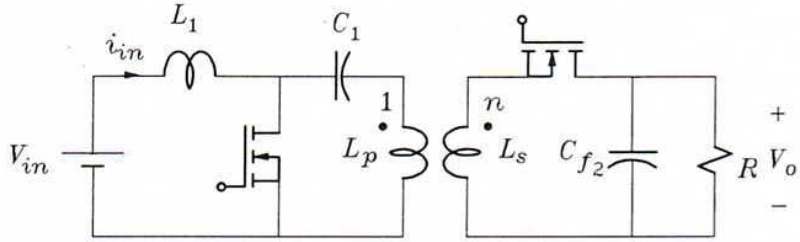


Fig. 1

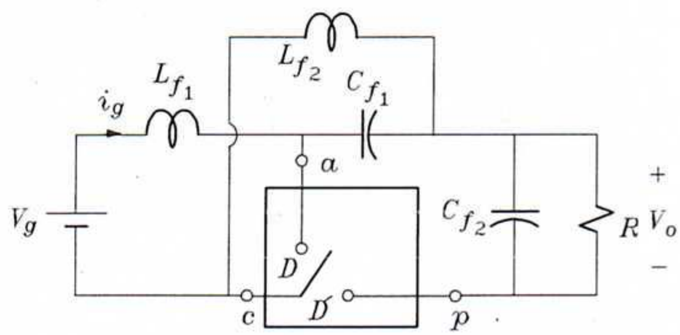
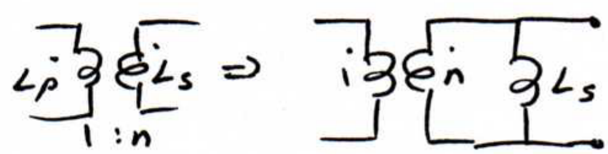


Fig. 2

- To get from fig 1 to fig 2 replace the transformer with it equivalent circuit model:

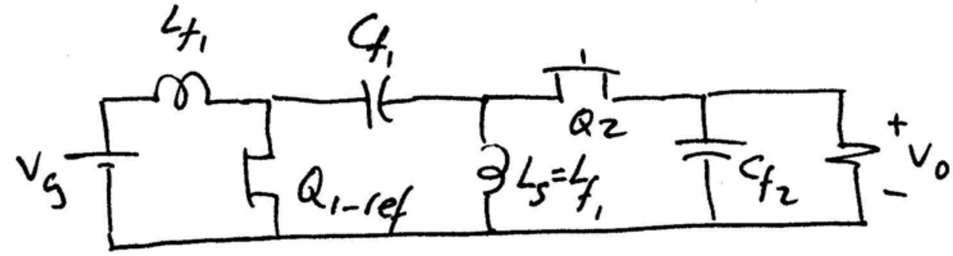


- reflect V_{in}, L_1, C_1 & Q_1 as follows to the secondary side

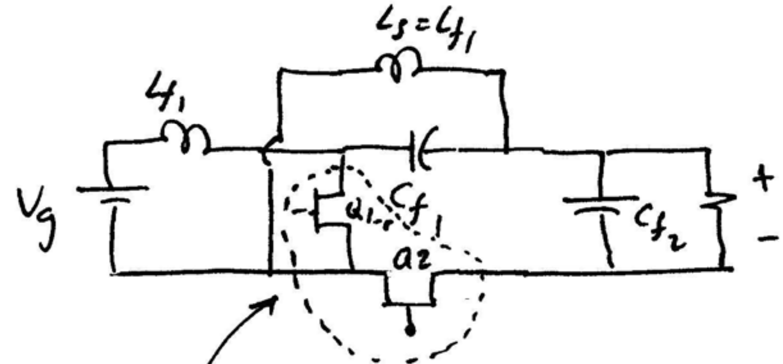
$$L_{f_1} = L_1 n^2 \qquad Q_1 \rightarrow Q_{1\text{-}ref} \Rightarrow \begin{cases} V_{DSref} = n V_{DS} \\ I_{Dref} = \dfrac{I_D}{n} \end{cases}$$

$$C_{f_1} = \frac{C_1}{n^2}$$

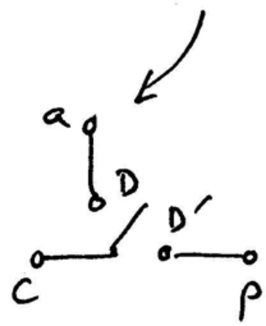
$$V_g = V_{in} n$$



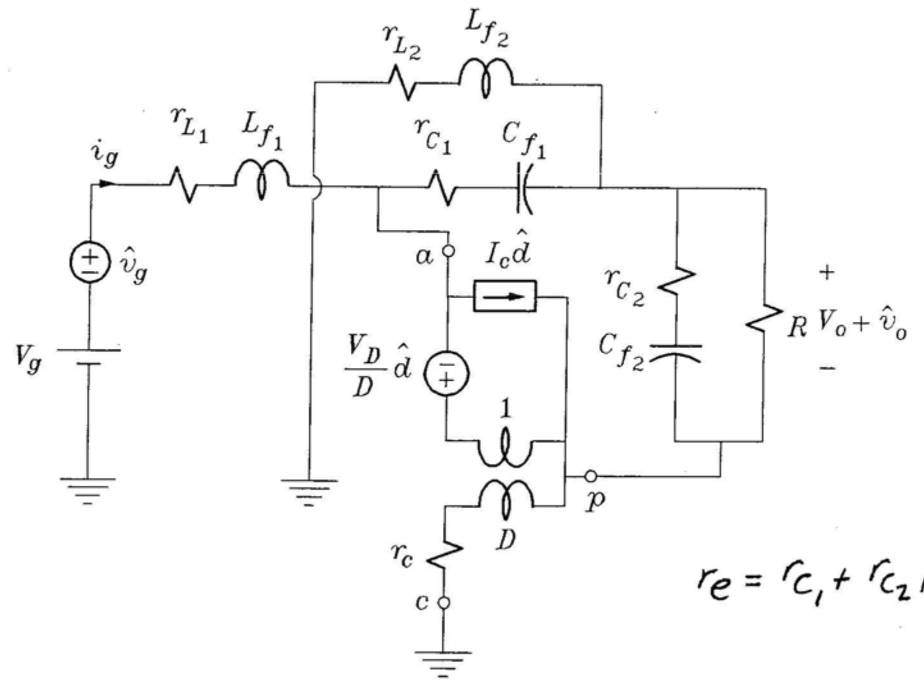
- now redraw L_{f_1} around C_{f_1} and slide Q_2 next to Q_{1-ref} in the return line like this



This is the PWM switch

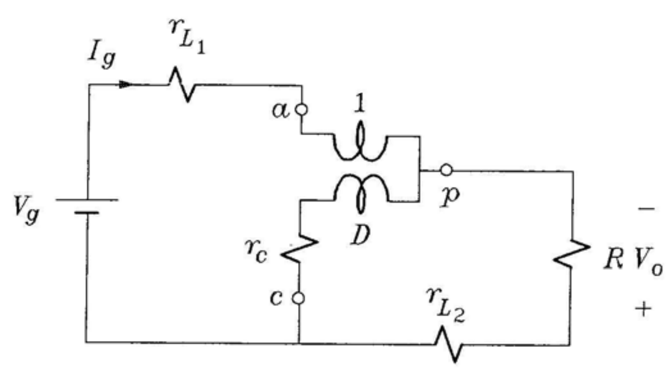


Replace the PWM switch with its equivalent circuit model

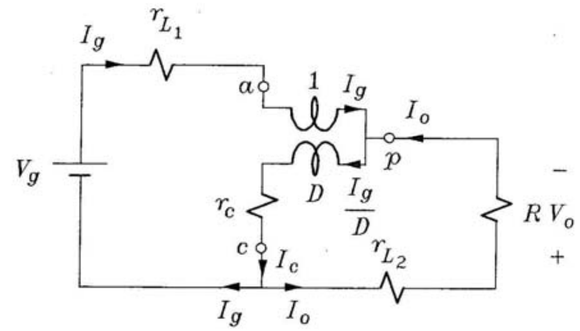


$r_e = r_{c_1} + r_{c_2} // R$

Set the small-signal sources to zero to obtain the dc model



1. Determination of the conversion ratio and the dc operating point of the PWM switch



As in the buck-boost converter, the current conversion is not effected by the parasitic elements and is given by KCL at point "p."

$$I_o = \frac{I_g}{D} - I_g \Rightarrow \frac{I_g}{I_o} = \frac{D}{D'} \qquad \text{also} \quad \frac{I_c}{I_o} = \frac{1}{D'}$$

We use now the efficiency to determine the conversion ratio

$$M = \frac{V_o}{V_g} = \frac{I_g}{I_o} \cdot \eta \qquad \eta = \frac{P_{out}}{P_{out} + P_{Lost}}$$

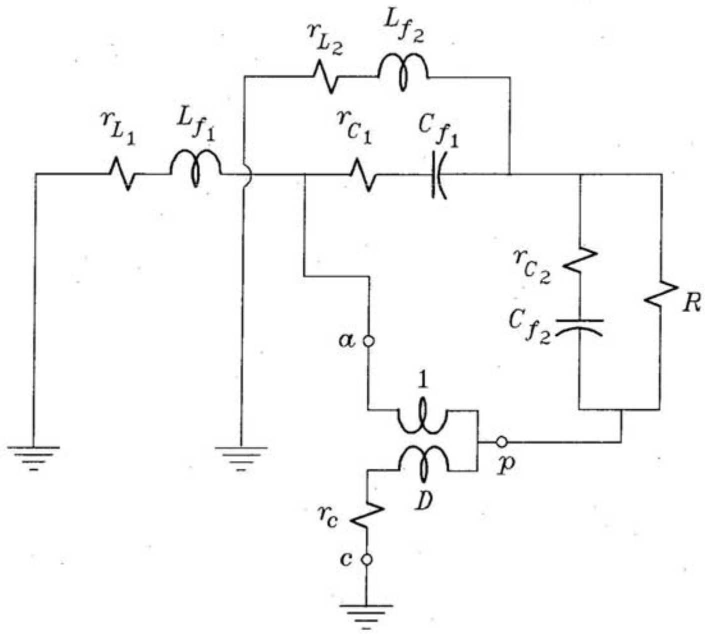
$$\left.\begin{array}{l} P_{out} = I_o^2 R \\ P_{Lost} = I_o^2 r_{L_2} + I_g^2 r_{L_1} + I_c^2 r_c \end{array}\right\} \Rightarrow \eta = \frac{1}{1 + \frac{r_{L_2}}{R} + \frac{r_{L_1}}{R}\left(\frac{I_g}{I_o}\right)^2 + \frac{r_c}{R}\left(\frac{I_c}{I_o}\right)^2}$$

$$\therefore M = \frac{D}{D'} \frac{1}{1 + \frac{r_{L_2}}{R} + \frac{r_{L_1}}{R}\left(\frac{D}{D'}\right)^2 + \frac{r_c}{R}\frac{1}{D'^2}}$$

Operating point: $I_c = \frac{I_o}{D'}$, $V_{ap} = V_g + V_o + I_o r_{L_2} - I_g r_{L_1}$

$$= V_g + V_o\left(1 + \frac{r_{L_2}}{R} - \frac{D}{D'}\frac{r_{L_1}}{R}\right)$$

2. Determination of D(s)



With $\hat{v}_g$ and $\hat{d}$ set to zero, we obtain the circuit above from we will determine $D(s)$ for the small-signal transfer functions:

$$\frac{\hat{v}_o}{\hat{v}_g} = M\,\frac{N_l(s)}{D(s)}$$

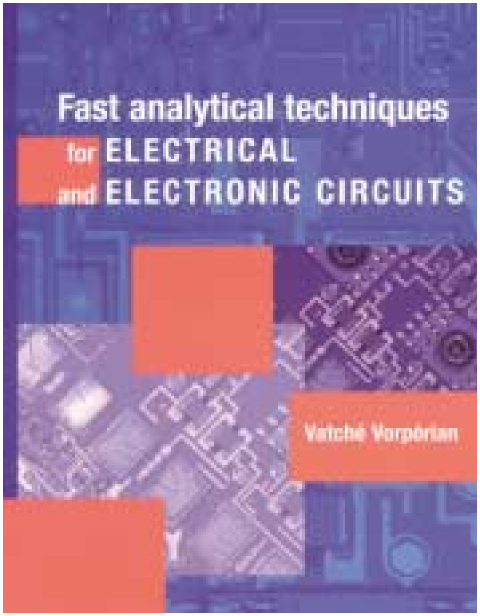
$$\frac{\hat{v}_o}{\hat{d}} = K_D\,\frac{N_d(s)}{D(s)}$$

$$Y_{in}(s) = \frac{\hat{i}_g}{\hat{v}_g} = G_{in}\,\frac{N_i(s)}{D(s)}$$

$$Z_o(s) = \frac{\hat{v}_T}{\hat{i}_T} = R_o\,\frac{N_o(s)}{D(s)}$$; $\hat{i}_T$ is test current source connected at the output

Fast analytical techniques for ELECTRICAL and ELECTRONIC CIRCUITS

by Vatché Vorpérian
Jet Propulsion Laboratory
California Institute of Technology
Cambridge University Press
www.cambridge.org



Today, the only method of circuit analysis known to most engineers and students is nodal or loop analysis. Although this works well for obtaining numerical solutions, it is almost useless for obtaining analytical solutions in all but the simplest cases.

In this unique book, Vorpérian describes remarkable alternative techniques to solve, almost by inspection, complicated linear circuits in symbolic form and obtains meaningful analytical answers for any transfer function or impedance. Although not intended to replace traditional computer based methods, these techniques provide engineers with a powerful set of tools for tackling circuit design problems. They also have great value in enhancing students understanding of circuit operation. The numerous problems and worked examples in this book make it an ideal textbook for senior/graduate courses, or a reference book.

This book will show you how to:

- Use less algebra and do most of it directly on the circuit diagram.
- Obtain meaningful analytical solutions to complex circuits with reactive elements and dependent sources by reducing them to a set of simple and purely resistive circuits which can be analyzed by inspection.
- Analyze feedback amplifiers easily using the simplest and most natural formulation.
- Analyze PWM converters easily using the model of the PWM switch.

Originally developed and taught at institutions and companies around the world by Professor David Middlebrook at Caltech, the extended and new techniques described in this book are an indispensable set of tools for linear electronic circuit analysis and design.

Publisher's note: Dr. Vatché Vorpérian is one of the rare few researchers who delight in the process of analysis of analog circuits, and in finding simple and elegant solutions to seemingly insurmountable problems. I have observed, on numerous occasions, his ability to derive models and equations overnight. In this latest book for the electrical engineer, he reveals many of his techniques. Much of it is applied to power conversion circuits-which is one of the few remaining disciplines where hand analysis is crucial to the development of circuit topologies and new technologies. It's a 'must have' text for anyone serious about the field of power electronics.

Vatché's major contributions to our field include the PWM switch model, the ZCS and ZVS quasi-resonant switch model, and the analysis of the series and parallel resonant converter.

Since the converter is of fourth order, we have:

$$D(s) = 1 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 \tag{1}$$

For a properly designed fourth-order converter, the denominator consists of two quadratic factors whose resonances are well seperated and almost entirely damped by the load. The parasitic resistances have almost no effect on the two resonant frequencies of $D(s)$ and contribute very little to the damping of the resonances under normal loading conditions. Therefore we can write:

$$D(s) = \left(1 + \frac{s}{\omega_{o1}Q_1} + \frac{s^2}{\omega_{o1}^2}\right)\left(1 + \frac{s}{\omega_{o2}Q_2} + \frac{s^2}{\omega_{o2}^2}\right) \tag{2}$$

Expanding Eq. (2) and comparing to Eq. (1) under the assumption of moderate to high Q and well-separated resonances, we get

$$a_1 \simeq \frac{1}{\omega_{o1}Q_1} \tag{3a}$$

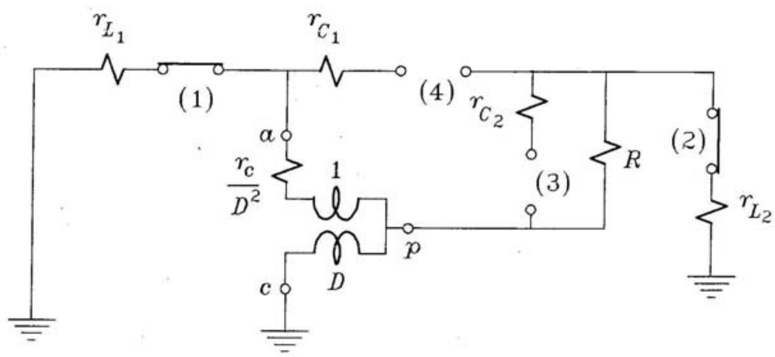
$$a_2 \simeq \frac{1}{\omega_{o1}^2} \tag{3b}$$

$$a_3 \simeq \frac{1}{\omega_{o1}Q_1\omega_{o2}^2} + \frac{1}{\omega_{o2}Q_2\omega_{o1}^2} \tag{3c}$$

$$a_4 \simeq \frac{1}{\omega_{o1}^2\omega_{o2}^2} \tag{3d}$$

We will determine these coefficients according to Eqs. (4-7) given below and perform all necessary approximations as we go along. The reference circuit is shown on the next page.

$$a_1 = \frac{L_{f_1}}{R^{(1)}} + \frac{L_{f_2}}{R^{(2)}} + C_{f_2}R^{(3)} + C_{f_1}R^{(4)} \tag{4}$$



reference circuit indicating normal port conditions

$$a_2 = \frac{L_{f_1}}{R^{(1)}}\frac{L_{f_2}}{R^{(2)}_{(1)}} + \frac{L_{f_1}}{R^{(1)}}C_{f_2}R^{(3)}_{(1)} + \frac{L_{f_1}}{R^{(1)}}C_{f_1}R^{(4)}_{(1)}$$

$$+ \frac{L_{f_2}}{R^{(2)}}C_{f_2}R^{(3)}_{(2)} + \frac{L_{f_2}}{R^{(2)}}C_{f_1}R^{(4)}_{(2)} + C_{f_2}R^{(3)}C_{f_1}R^{(4)}_{(3)} \tag{5}$$

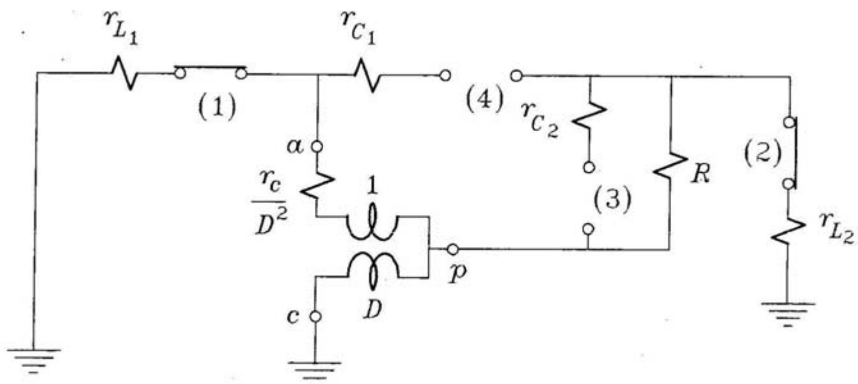
$$a_3 = \frac{L_{f_1}}{R^{(1)}}\frac{L_{f_2}}{R^{(2)}_{(1)}}C_{f_2}R^{(3)}_{(21)} + \frac{L_{f_1}}{R^{(1)}}\frac{L_{f_2}}{R^{(2)}_{(1)}}C_{f_1}R^{(4)}_{(21)}$$

$$+ \frac{L_{f_1}}{R^{(1)}}C_{f_2}R^{(3)}_{(1)}C_{f_1}R^{(4)}_{(31)} + \frac{L_{f_2}}{R^{(2)}}C_{f_2}R^{(3)}_{(2)}C_{f_1}R^{(4)}_{(32)} \tag{6}$$

$$a_4 = \frac{L_{f_1}}{R^{(1)}}\frac{L_{f_2}}{R^{(2)}_{(1)}}C_{f_2}R^{(3)}_{(21)}C_{f_1}R^{(4)}_{(123)} \tag{7}$$

The order in which the elements, or the ports, are taken in any one term is immeterial. For example:

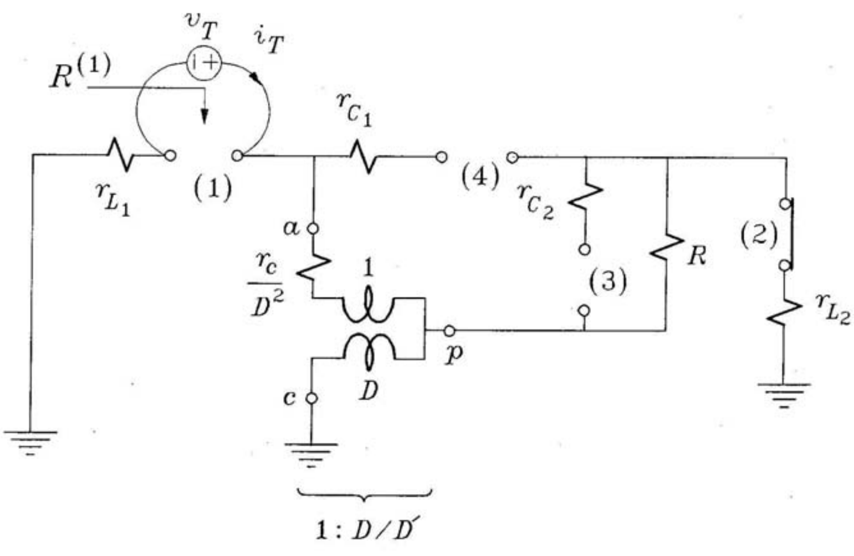
$$\frac{L_{f_1}}{R^{(1)}}\frac{L_{f_2}}{R^{(2)}_{(1)}}C_{f_1}R^{(4)}_{(21)} = \frac{L_{f_1}}{R^{(1)}}C_{f_1}R^{(4)}_{(1)}\frac{L_{f_2}}{R^{(2)}_{(41)}} \tag{8}$$

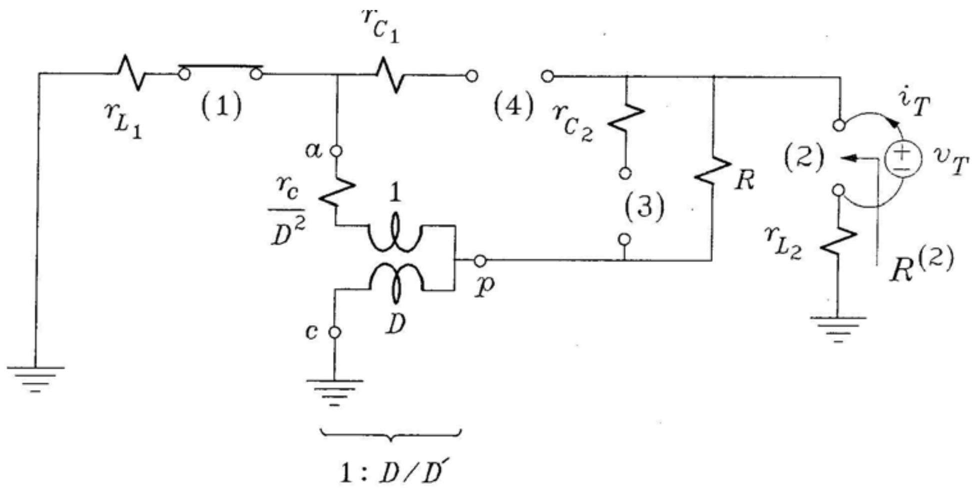


reference circuit indicating normal port conditions

i) Determination of a_1

$$R^{(1)} = r_{L_1} + \frac{r_c}{D^2} + \left(\frac{D'}{D}\right)^2 (R + r_{L_2}) \simeq \left(\frac{D'}{D}\right)^2 R$$

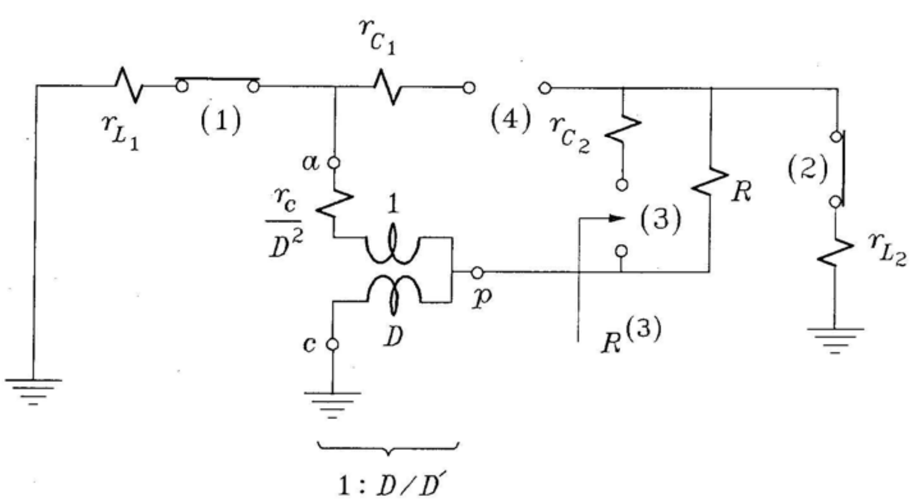


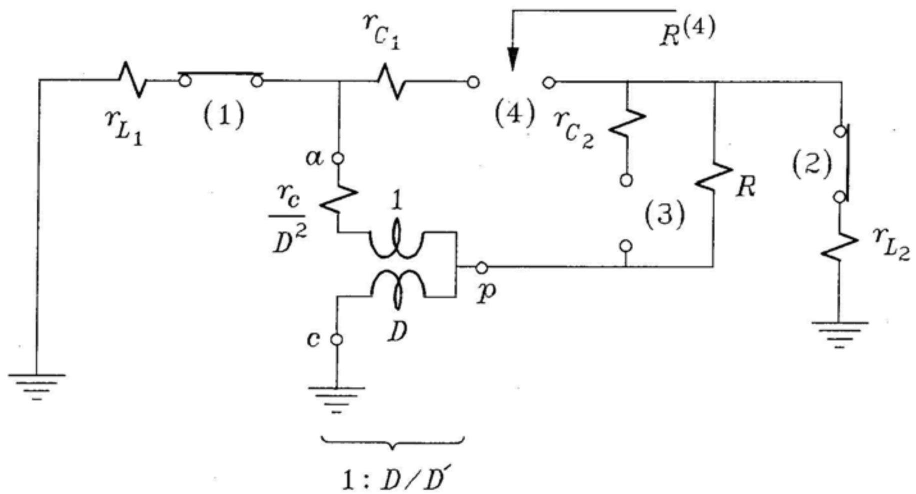


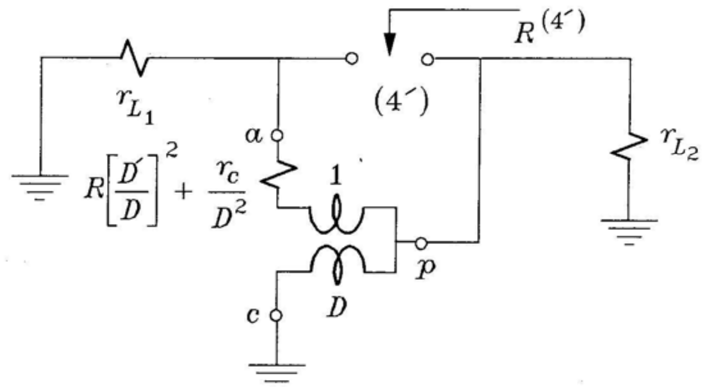
$$R^{(2)} = r_{L_2} + R + \left(\frac{D}{D'}\right)^2\left(\frac{r_c}{D^2} + r_{L_1}\right) \simeq R$$

$$R^{(3)} = r_{C_2} + R \,\|\, \left(r_{L_2} + \left(\frac{D}{D'}\right)^2\left(\frac{r_c}{D^2} + r_{L_1}\right)\right)$$

$$\simeq r_{C_2} + r_{L_2} + \frac{r_c}{D'^2} + r_{L_1}\left(\frac{D}{D'}\right)^2$$







$$R^{(4)} = r_{C_1} + R^{(4')}$$

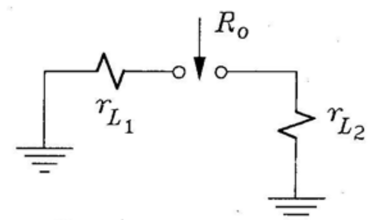
According to the EET

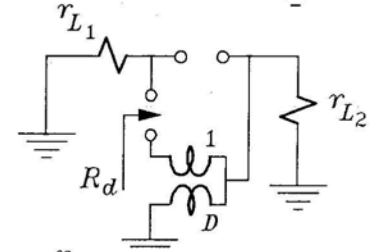
$$R^{(4')} = R_o \frac{1 + \dfrac{R_n}{r_c/D^2 + R(D'/D)^2}}{1 + \dfrac{R_d}{r_c/D^2 + R(D'/D)^2}}$$

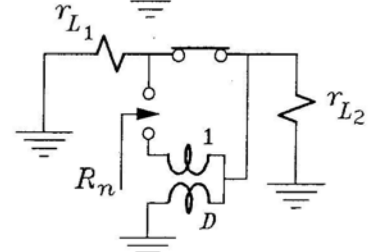
$$R_o = r_{L_1} + r_{L_2}, \quad R_d = r_{L_1} + \left(\frac{D'}{D}\right)^2 r_{L_2}$$

$$R_n = \frac{r_{L_1} \,//\, r_{L_2}}{D^2}$$

$$\therefore \; R^{(4)} \simeq r_{C_1} + r_{L_1} + r_{L_2}$$







Determination of a_2

In order to determine a_2, we need to determine the following:

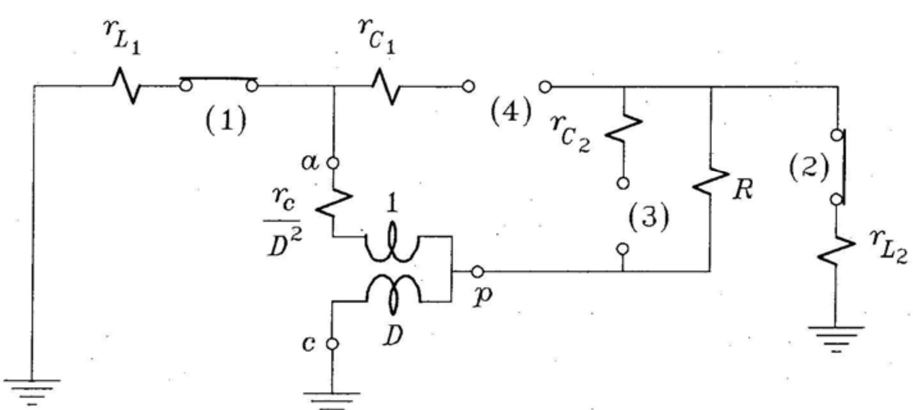
$$R^{(2)}_{(1)},\ R^{(4)}_{(1)},\ R^{(3)}_{(1)},\ R^{(4)}_{(2)},\ R^{(3)}_{(2)},\ R^{(4)}_{(3)}$$

According to the reference circuit below, all of the above can be deduced from $R^{(1)}$, $R^{(2)}$, $R^{(3)}$ and $R^{(4)}$ as follows:

$$R^{(n)}_{(1)} = \lim_{r_{L_1}\to\infty} R^{(n)} \quad n = 2, 3, 4$$

$$R^{(n)}_{(2)} = \lim_{r_{L_2}\to\infty} R^{(n)} \quad n = 3, 4$$

$$R^{(4)}_{(3)} = R^{(4)}\Big|_{R\to r_{C_2}\parallel R}$$



reference circuit indicating normal port conditions

we now have

$$R^{(2)}_{(1)} \to \infty$$

$$R^{(3)}_{(1)} = r_{C_2} + R \simeq R$$

$$R^{(3)}_{(2)} = r_{C_2} + R \simeq R$$

$$R^{(4)}_{(1)} = r_{C_1} + \frac{r_c}{D^2} + R\left(\frac{D'}{D}\right)^2 + \frac{r_{L_2}}{D^2} \simeq R\left(\frac{D'}{D}\right)^2$$

$$R^{(4)}_{(2)} = r_{C_1} + \frac{r_c}{D'^2} + \frac{r_{L_1}}{D'^2} + R \simeq R$$

$$R^{(4)}_{(3)} = r_{C_1} + R_0 \frac{1 + \dfrac{R_n}{r_c/D^2 + r_{C_2} \| R\,(D'/D)^2}}{1 + \dfrac{R_d}{r_c/D^2 + r_{C_2} \| R\,.(D'/D)^2}}$$

where R_n, R_d and R_0 are the same as before given for $R^{(4)}$!

Hence, ignoring parasitic elements, we have

$$a_2 = \underbrace{\cancel{\frac{L_{f_1}}{R(D'/D)^2} \cdot \frac{L_{f_2}}{\infty}}}_{\to 0} + \frac{L_{f_1}}{R(D'/D)^2} \cdot C_{f_2} R + \frac{L_{f_1}}{R(D'/D)^2} C_{f_1} R\left(\frac{D'}{D}\right)^2$$

$$+ \frac{L_{f_2}}{R} C_{f_1} R + \frac{L_{f_2}}{R} C_{f_2} R + \underbrace{\cancel{C_{f_2} R^{(3)} C_{f_1} R^{(4)}_{(3)}}}_{\to 0}$$

$$a_2 \simeq L_{f_1}\left(C_{f_2}\left(\frac{D}{D'}\right)^2 + C_{f_1}\right) + L_{f_2}\left(C_{f_1} + C_{f_2}\right)$$

$$\omega_{01} = \frac{1}{\sqrt{a_2}} = \frac{1}{\sqrt{L_{f_1}\left(C_{f_2}\left(\frac{D}{D'}\right)^2 + C_{f_1}\right) + L_{f_2}\left(C_{f_1} + C_{f_2}\right)}}$$

Determination of a_3

In order to determine a_3, we need to determine the following

$$R^{(3)}_{(21)} = \lim_{r_{L_1}\to\infty} R^{(3)}_{(2)} = r_{C_2} + R \simeq R$$

$$R^{(4)}_{(21)} = \lim_{r_{L_1}\to\infty} R^{(4)}_{(2)} \to \infty$$

$$R^{(4)}_{(31)} = \lim_{r_{L_1}\to\infty} R^{(4)}_{(3)} = r_{C_1} + \frac{r_{L_2} + r_{C_1}\|R + r_{C_2}\|R\, D'^2}{D^2}$$

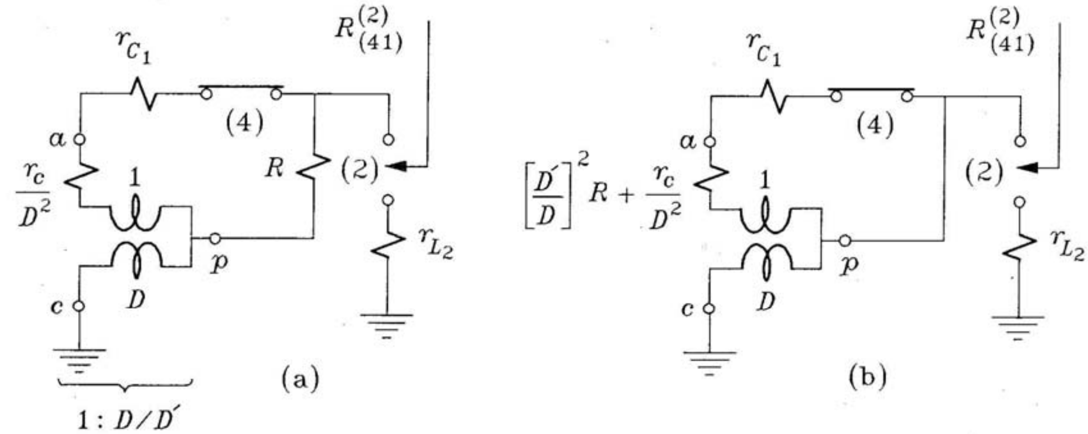
$$R^{(4)}_{(32)} = \lim_{r_{L_2}\to\infty} R^{(4)}_{(3)} = r_{C_1} + r_{C_2}\|R + \frac{r_{L_1} + r_{C_1}\|R}{D'^2}$$

$$\therefore\quad a_3 = \underbrace{\cancel{\frac{L_{f_1}}{\left(\frac{D'}{D}\right)^2 R}\cdot\frac{L_{f_2}}{\infty}\cdot C_{f_2} R}}_{0} + \overbrace{\frac{L_{f_1}}{\left(\frac{D'}{D}\right)^2 R}\cdot\frac{L_{f_2}}{\infty}\cdot C_{f_1}\infty}^{\text{indeterminate}}$$

$$+ \underbrace{\frac{L_{f_1}}{R^{(1)}} C_{f_2} R^{(3)}_{(1)} C_{f_1} R^{(4)}_{(31)}}_{0} + \underbrace{\frac{L_{f_2}}{R^{(2)}} C_{f_2} R^{(3)}_{(2)} C_{f_1} R^{(4)}_{(32)}}_{0}$$

The indeterminacy can be removed by changing the order in which the ports are taken

$$\underbrace{\frac{L_{f_1}}{R^{(1)}}}_{1}\ \underbrace{\frac{L_{f_2}}{R^{(2)}_{(1)}}}_{2}\cdot \underbrace{C_{f_1} R^{(4)}_{(21)}}_{4} = \underbrace{\frac{L_{f_1}}{R^{(1)}}}_{1}\cdot \underbrace{C_{f_1} R^{(4)}_{(1)}}_{4}\cdot \underbrace{\frac{L_{f_2}}{R^{(2)}_{(41)}}}_{2}$$



$R^{(2)}_{(41)}$ is determined from Fig. (a) above which can be transformed to Fig. (b) by reflecting R through $(D'/D)^2$. It follows from Fig. (b) that

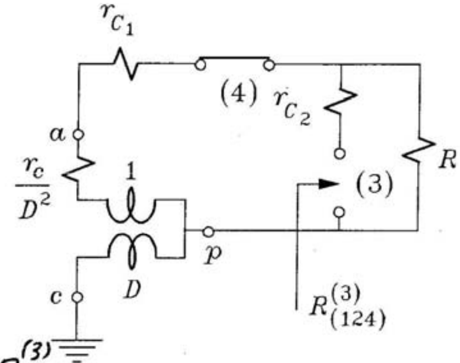
$$R^{(2)}_{(41)} = r_{L_2} + D^2\left[r_{C_1} + \frac{r_c}{D^2} + R\left(\frac{D'}{D}\right)^2 \right]$$

$$= r_{L_2} + r_c + D^2 r_{C_1} + RD'^2 \simeq RD'^2$$

$$\therefore \quad a_3 \simeq \frac{L_{f_1}}{\left(\frac{D'}{D}\right)^2 R} \cdot C_{f_1} R\left(\frac{D'}{D}\right)^2 \cdot \frac{L_{f_2}}{RD'^2}$$

$$a_3 \simeq L_{f_1} L_{f_2} C_{f_1} \frac{1}{D'^2 R}$$

Determination of a_4



Since from a_3 we alredy have the expression of the term in which the ports are taken in the order 1 4 2, we will write a_4 as

$$a_4 = \underbrace{\frac{L_{f_1}}{R^{(1)}}}_{1} \cdot \underbrace{C_{f_1} R^{(4)}_{(1)}}_{4} \cdot \underbrace{\frac{L_{f_2}}{R^{(2)}_{(41)}}}_{2} \cdot \underbrace{C_{f_2} R^{(3)}_{(124)}}_{3}$$

$$a_4 \simeq a_3 C_{f_2} R^{(3)}_{(124)} \quad \text{and} \quad R^{(3)}_{(124)} = r_{C_2} + R \simeq R$$

$$\therefore \quad a_4 = \frac{L_{f_1} L_{f_2} C_{f_1} C_{f_2}}{D'^2}$$

Since $a_4 = 1/\omega_{01}^2 \omega_{02}^2$, we have

$$\omega_{02}^2 = \frac{D'^2}{L_{f_1} L_{f_2} C_{f_1} C_{f_2}} \cdot \left[L_{f_1} \left(C_{f_2} \left(\frac{D}{D'}\right)^2 + C_{f_1} \right) + L_{f_2} \left(C_{f_1} + C_{f_2} \right) \right]$$

$$\omega_{02} = \sqrt{\frac{1}{L_{f_2} \cdot \frac{C_{f_1}}{D^2} \| \frac{C_{f_2}}{D'^2}} + \frac{1}{L_{f_1} C_{f_1} \| C_{f_2}}}$$

Determination of Q_1

$$a_1 \simeq \frac{1}{\omega_{01} Q_1} \Rightarrow Q_1 = \frac{1}{a_1 \omega_{01}}$$

$$a_1 \simeq \frac{L_{f_1}}{R}\left(\frac{D}{D'}\right)^2 + \frac{L_{f_2}}{R}$$

$$\therefore \quad Q_1 = \frac{R}{\omega_{01}\left(L_{f_1}\left(\frac{D}{D'}\right)^2 + L_{f_2}\right)}$$

Determination of Q_2

$$a_3 \simeq \frac{1}{\omega_{01} Q_1 \omega_{02}^2} + \frac{1}{\omega_{02} Q_2 \omega_{01}^2}$$

$$= \frac{1}{\omega_{01}^2 \omega_{02}^2}\left[\frac{\omega_{01}}{Q_1} + \frac{\omega_{02}}{Q_2}\right]$$

$$= a_4\left(\frac{\omega_{01}}{Q_1} + \frac{\omega_{02}}{Q_2}\right)$$

$$\therefore \quad \frac{\omega_{02}}{Q_2} = \frac{a_3}{a_4} - \frac{\omega_{01}}{Q_1} = \frac{1}{C_{f_2} R} - \frac{\omega_{01}^2\left(L_{f_2} + L_{f_1}\left(\frac{D}{D'}\right)^2\right)}{R}$$

$$= \frac{\omega_{01}^2}{R}\left(\frac{1}{C_{f_2}\omega_{01}^2} - L_{f_2} - L_{f_1}\left(\frac{D}{D'}\right)^2\right)$$

$$= \frac{\omega_{01}^2}{R}\left(L_{f_1}\left(\frac{D}{D'}\right)^2 + L_{f_2} + \frac{L_{f_1}}{C_{f_2}} C_{f_1} + \frac{L_{f_2}}{C_{f_2}} C_{f_1} - L_{f_1}\left(\frac{D}{D'}\right)^2 - L_{f_2}\right)$$

$$\frac{\omega_{o2}}{Q_2} = \frac{\omega_{o1}^2}{R} \frac{C_{f_1}}{C_{f_2}} \left(L_{f_1} + L_{f_2} \right)$$

$$Q_2 = \frac{R}{\omega_{o2} \left(L_{f_1} + L_{f_2} \right) \frac{C_{f_1}}{C_{f_2}} \left(\frac{\omega_{o1}}{\omega_{o2}} \right)^2}$$

Putting all the results together, we have:

$$D(s) = 1 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4$$

$$a_1 \simeq \left(L_{f_1} \left(\frac{D}{D'} \right)^2 + L_{f_2} \right) \frac{1}{R}$$

$$a_2 \simeq L_{f_1} \left(C_{f_1} + \left(\frac{D}{D'} \right)^2 C_{f_2} \right) + L_{f_2} \left(C_{f_1} + C_{f_2} \right)$$

$$a_3 \simeq L_{f_1} L_{f_2} C_{f_1} \frac{1}{R D'^2}$$

$$a_4 \simeq L_{f_1} L_{f_2} C_{f_1} C_{f_2} \cdot \frac{1}{D'^2}$$

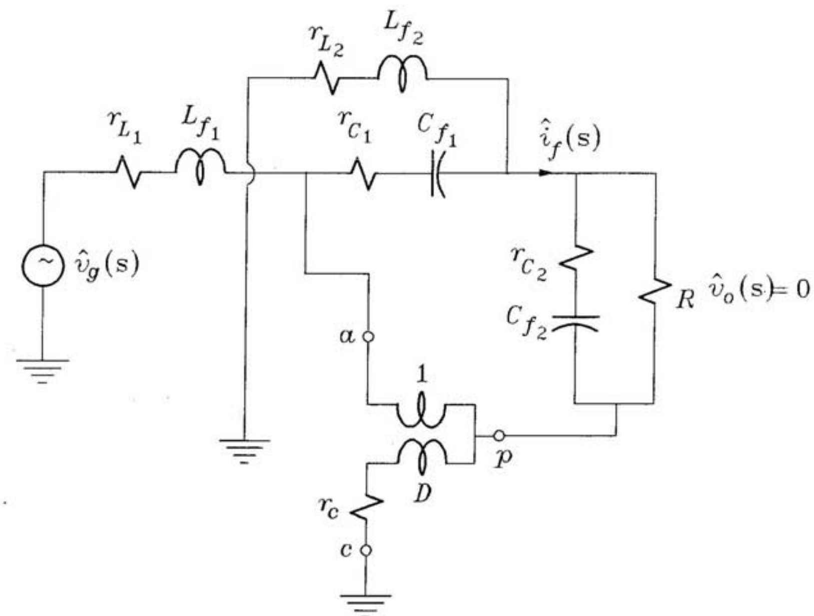
$$D(s) \simeq \left(1 + \frac{s}{\omega_{o1} Q_1} + \left(\frac{s}{\omega_{o1}} \right)^2 \right) \left(1 + \frac{s}{\omega_{o2} Q_2} + \left(\frac{s}{\omega_{o2}} \right)^2 \right)$$

$$\omega_{o1} \simeq \frac{1}{\sqrt{L_{f_1} \left(C_{f_2} \left(\frac{D}{D'} \right)^2 + C_{f_1} \right) + L_{f_2} \left(C_{f_1} + C_{f_2} \right)}}$$

$$Q_1 \simeq \frac{R}{\omega_{o1} \left(L_{f_1} \left(\frac{D}{D'} \right)^2 + L_{f_2} \right)}$$

$$\omega_{o2} \simeq \sqrt{\frac{1}{L_{f_2} \frac{C_{f_1}}{D^2} \| \frac{C_{f_2}}{D'^2}} + \frac{1}{L_{f_1} C_{f_1} \| C_{f_2}}}$$

3. Determination of the line-to-output transfer function



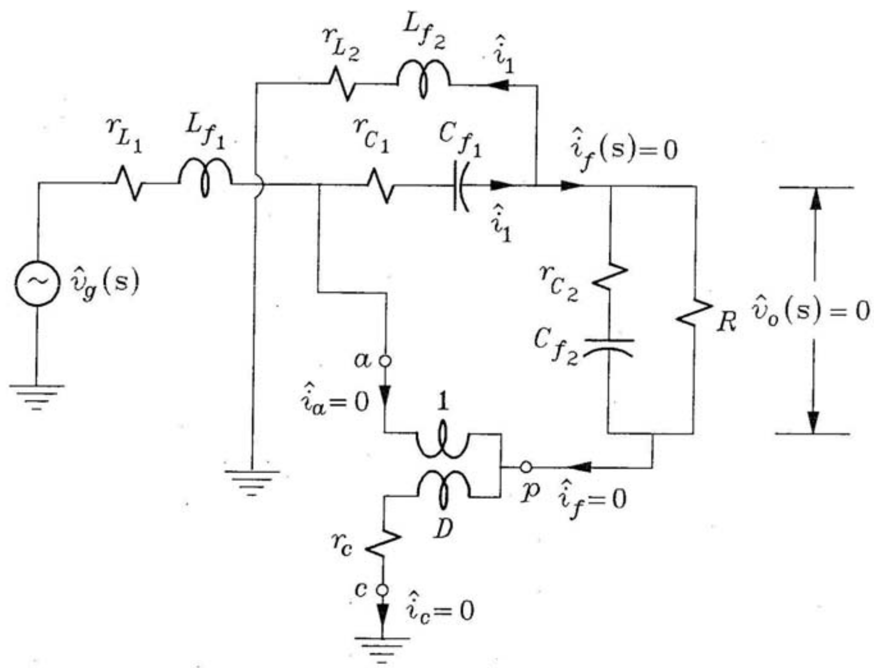
The line-to-output transfer function is of the form

$$\frac{\hat{v}_o(s)}{\hat{v}_g(s)} = M \, \frac{N_g(s)}{D(s)}$$

in which M and $D(s)$ have already been determined and $N_g(s)$ corresponds to the null conditions in the response $\hat{v}_o(s)$ to the excitation $\hat{v}_g(s)$. The first null is given by the zero of the impedance branch, $r_{C_2} + 1/sC_{f_2}$, connected across the response $\hat{v}_o(s)$ so that the first factor of $N_g(s)$ is

$$N_g(s) = (1 + s r_{C_2} C_{f_2})\, N_2(s)$$

The second null condition, $N_2(s)$, is given by $i_f(s) = 0$ which we will investigate next.



With $\hat{i}_f = 0$, KCL at node "p" yields $\hat{i}_a = \hat{i}_c = 0$. With $\hat{v}_o(s) = 0$, we see that the voltage drop across the impedance branch $r_{C_1} + 1/sC_{f_1}$ is $\hat{v}_{ap}$

$$\hat{v}_{ap} = \hat{i}_1 (r_{C_1} + 1/sC_{f_1})$$

Since $\hat{i}_f = 0$, $\hat{i}_1$ also flows through the impedance branch $r_{L_2} + sL_{f_2}$. With $\hat{v}_o(s) = 0$ and the voltage drop across r_c being zero, the voltage drop across the branch $r_{L_2} + sL_{f_2}$ is $\hat{v}_{cp}$

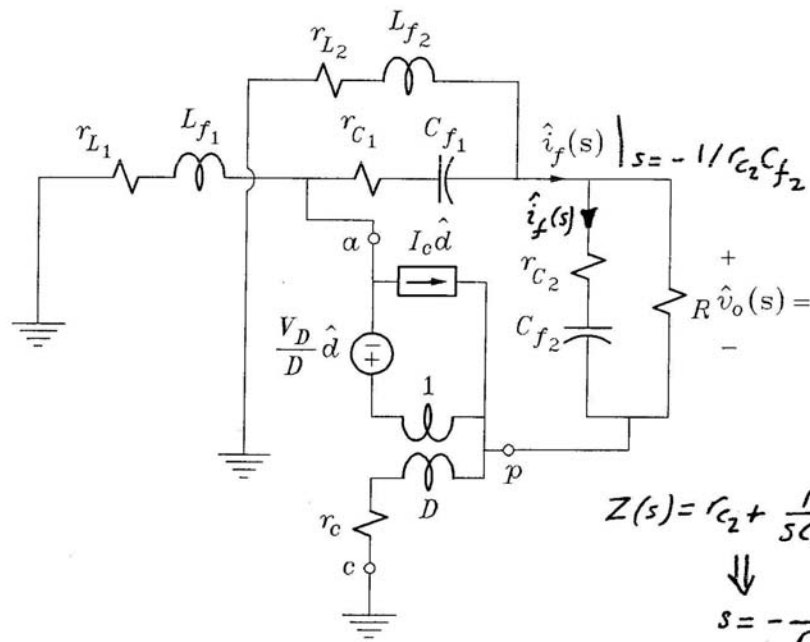
$$\hat{v}_{cp} = -\hat{i}_1 (r_{L_2} + sL_{f_2})$$

Since $\hat{v}_{cp} = D\,\hat{v}_{ap}$, we have

$$\hat{i}_1 (r_{C_1} + 1/sC_{f_1}) D = -(r_{L_2} + sL_{f_2})\hat{i}_1 \Rightarrow 1 + sC_{f_1}(Dr_{C_1} + r_{L_2}) + s^2 L_{f_2} C_{f_1} = 0$$

$$\therefore \quad N_2(s) = 1 + sC_{f_1}(Dr_{C_1} + r_{L_2}) + s^2 L_{f_2} C_{f_1}$$

4. Determination of the control-to-output transfer function



$$Z(s) = r_{C_2} + \frac{1}{sC_2} = 0$$

$$\Downarrow$$

$$s = -\frac{1}{r_{C_2}C_{f_2}}$$

The control-to-output transfer function is given by

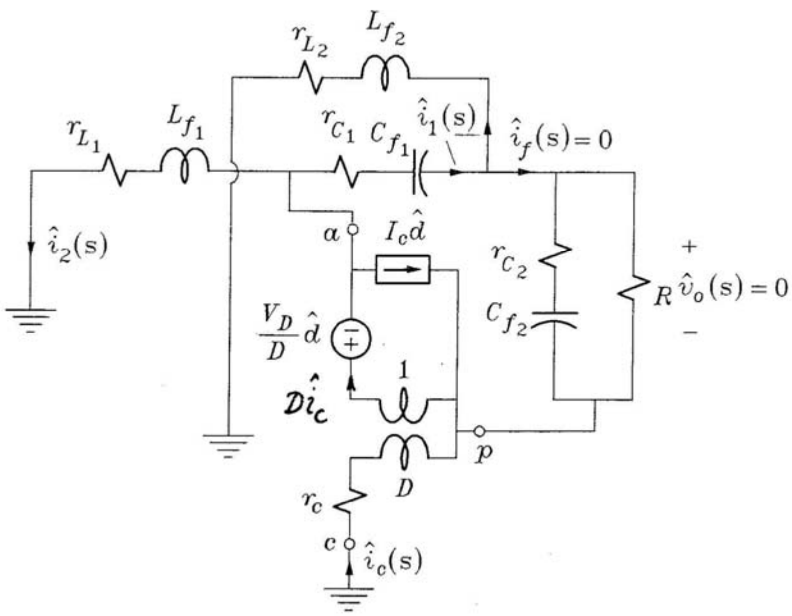
$$\frac{\hat{v}_o(s)}{\hat{d}(s)} = K_D \, \frac{N_d(s)}{D(s)}$$

Although K_D can be determined from the circuit above by letting $s \to 0$ (inductors short and capacitors open), we can derive it from the expression of the conversion ratio

$$V_o = M(D) \cdot V_g \Rightarrow K_D = \frac{dV_o}{dD} = V_g \frac{dM}{dD} = V_g \frac{d}{dD}\left(\frac{D}{1-D}\right)$$

$$\therefore K_D = \frac{1}{D'^2}$$

$N_d(s)$ corresponds to the null conditions in the response, $\hat{v}_o(s)$, to the excitation sources $(V_D/D)\hat{d}$ and $I_c\hat{d}$. The first



The second null condition in the control-to-output transfer function

null is given by the zero of the impedance connected across $\hat{v}_0(s)$. This impedance consists of $r_{C_2} + 1/sC_{f_2}$ so that the first factor in $N_d(s)$ is $(1 + s r_{C_2} C_{f_2})$

$$N_d(s) = (1 + s r_{C_2} C_{f_2}) N_2(s)$$

In which $N_2(s)$ corresponds to the second null condition shown in the circuit above given by the condition $\hat{i}_f(s) = 0$.

Referring to the circuit above, we have at ground node "c", according to KCL,

$$\hat{i}_c(s) = \hat{i}_1 + \hat{i}_2 \qquad (1)$$

With $\hat{i}_f(s)=0$ and KCL at node "p", we have

$$I_c\hat{d} = D\hat{i}_c - \hat{i}_c = -D'\hat{i}_c \qquad (2)$$

Equations (1) and (2) given

$$\hat{i}_1 + \hat{i}_2 + \frac{I_C}{D'}\hat{d} = 0 \qquad (\mathrm{I})$$

Next, we will assume $r_{L_1} = r_{L_2} = r_{C_2} = r_C = 0$. The voltage across L_{f_1} is the same as the voltage drop across C_{f_1} and L_{f_2} which, with $\hat{i}_f = 0$, can be written as:

$$\hat{i}_2 sL_{f_1} = \hat{i}_1\left(sL_{f_2} + \frac{1}{sC_{f_1}}\right) \qquad (3)$$

$$\hat{i}_2 s^2 L_{f_1} C_{f_1} - \hat{i}_1\left(1 + s^2 L_{f_2} C_{f_1}\right) = 0 \qquad (\mathrm{II})$$

Finally, with $\hat{v}_0(s) = 0$, the voltage across port "a-p" of the PWM switch is

$$\frac{V_D}{D}\hat{d} + \hat{i}_1\frac{1}{sC_{f_1}} = \hat{v}_{ap} \qquad (4)$$

and the voltage across port "c-p" is

$$\hat{v}_{cp} = -\hat{i}_1 sL_{f_2} \qquad (5)$$

Since $\hat{v}_{cp} = D\hat{v}_{ap}$, we have

$$\left[\frac{V_D}{D}\hat{d} + \hat{i}_1\frac{1}{sC_{f_1}}\right]D = -\hat{i}_1 sL_{f_2} \qquad (6)$$

$$\therefore \quad sC_{f_1}V_{ap}\hat{d} + \hat{i}_1\left(D + s^2 L_{f_2} C_{f_1}\right) = 0 \qquad \mathrm{III}$$

Putting equations (I)-(III) together ($V_D = V_{ap}$)

$$\hat{i}_1 + \hat{i}_2 + \frac{I_c}{D}\hat{d} = 0 \qquad (I)$$

$$-\hat{i}_1(1+s^2L_{f_2}C_{f_1}) + \hat{i}_2 s^2 L_{f_1}C_{f_1} = 0 \qquad (II)$$

$$\hat{i}_1(D+s^2L_{f_2}C_{f_1}) + sC_{f_1}V_{ap}\hat{d} = 0 \qquad (III)$$

With $\hat{i}_1$, $\hat{i}_2$ and $\hat{d}$ different from zero, the only way the above can be satisfied is to have their determinant vanish:

$$\begin{vmatrix} 1 & 1 & I_c/D' \\ -(1+s^2L_{f_2}C_{f_1}) & s^2L_{f_1}C_{f_1} & 0 \\ D+s^2L_{f_2}C_{f_1} & 0 & sC_{f_1}V_{ap} \end{vmatrix} = 0 \qquad (7)$$

OR

$$s^3L_{f_2}L_{f_1}C_{f_1} - \frac{V_{ap}}{I_c}D'\left[L_{f_1}C_{f_1} + L_{f_2}C_{f_1}\right]s^2 + sL_{f_1}D - \frac{V_{ap}}{I_c}D' = 0 \qquad (8)$$

$$\frac{V_{ap}}{I_c} = \frac{V_g+V_o}{I_o/D'} = D'\frac{V_o}{I_o}\left(1+\frac{V_g}{V_o}\right) = D'R\left(1+\frac{1}{M}\right)$$

$$\therefore \quad \frac{V_{ap}}{I_c} = \frac{D'^2}{D}\cdot R \qquad (9)$$



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Substitution of Eq. (9) in (8) yields the desired result, $N_2(s)$:

$$N_2(s) = 1 - sL_{f_1}\left(\frac{D}{D'}\right)^2\frac{1}{R} + s^2C_{f_1}(L_{f_1} + L_{f_2}) - s^3\frac{L_{f_2}L_{f_1}C_{f_1}}{R}\left(\frac{D}{D'}\right)^2 \quad (10)$$

$N_2(s)$ factors as follows:

i) Normal load conditions

$$N_2(s) \simeq \left(1 - sL_{f_1}\left(\frac{D^2}{D'^2}\right)\frac{1}{R}\right)\left(1 - s\frac{C_{f_1}(L_{f_1}+L_{f_2})R}{L_{f_1}}\left(\frac{D'}{D}\right)^2 + s^2\frac{L_{f_2}C_{f_1}}{D}\right) \quad (11)$$

The first factor corresponds to the expected RHP zero due to the pulsating output filter current during $D'T_S$

ii) No load (Synchronous rectification)

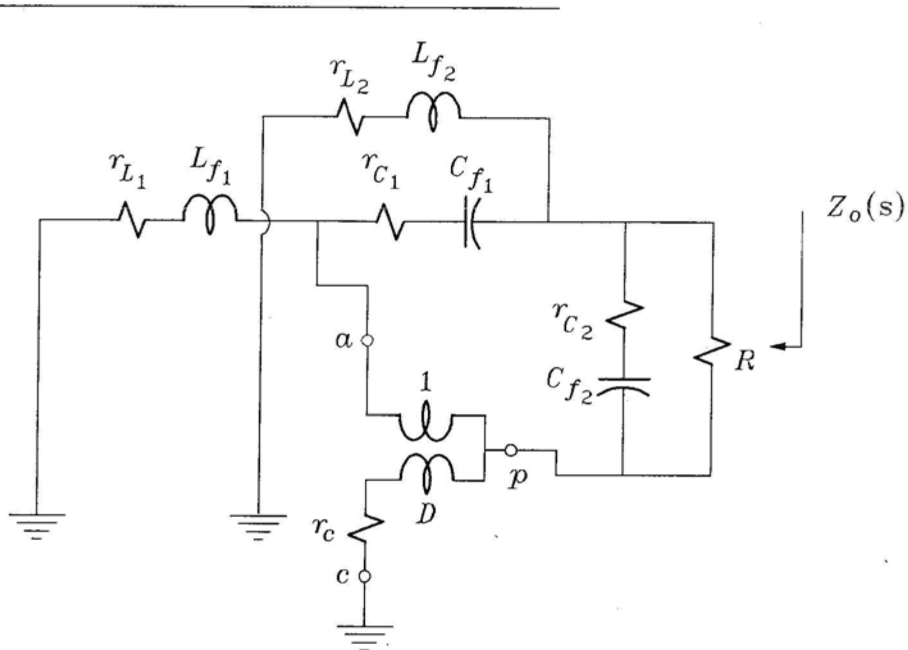
$$N_2(s) \simeq 1 + s^2C_{f_1}(L_{f_1} + L_{f_2}) \quad (12)$$

In this case the RHP zero behavior disappears as expected.

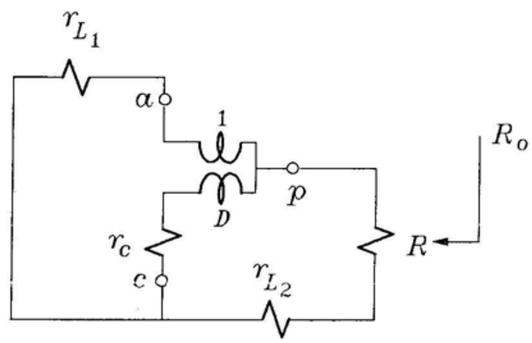
iii) Very light load (Synchronous rectification)

$$N_2(s) \simeq \left[1 - sL_{f_1}\left(\frac{D}{D'}\right)^2\frac{1}{R} + s^2C_{f_1}(L_{f_1} + L_{f_2})\right]\left(1 - s\frac{L_{f_1}\|L_{f_2}}{R}\frac{D}{D'^2}\right) \quad (13)$$

5. Determination of the open-loop output impedance

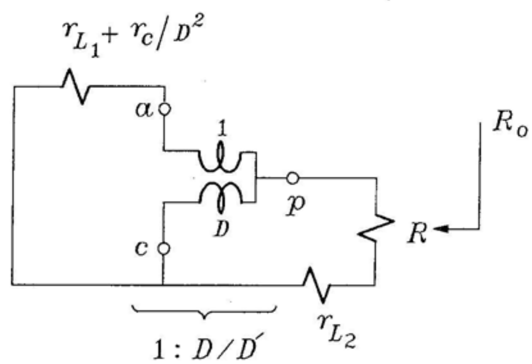


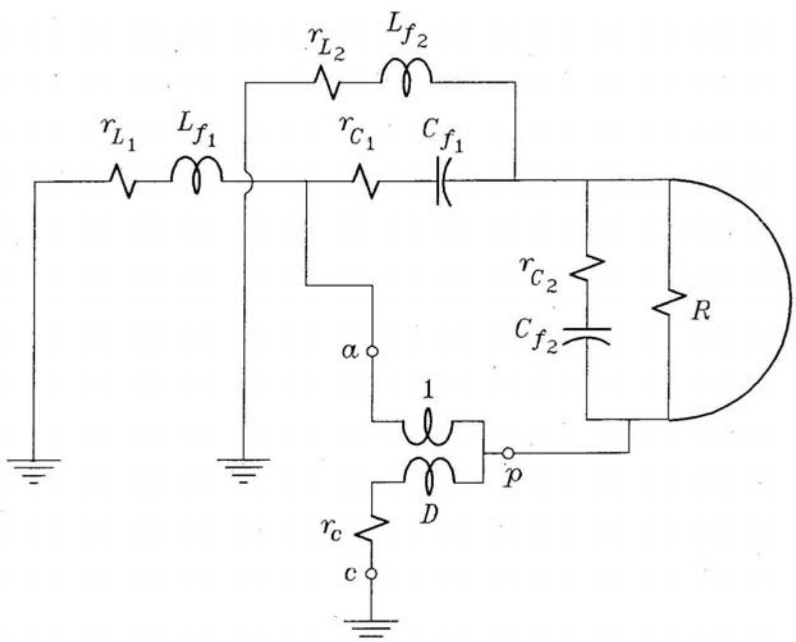
$$Z_o(s) = R_o \frac{N_o(s)}{D(s)}$$



where $D(s)$ is the same as before and R_o is determined from the adjacent circuits

$$R_o = R \,\|\, \left(r_{L_2} + \left(\frac{D}{D'}\right)^2 \left(r_{L_1} + \frac{r_c}{D^2}\right) \right)$$





Circuit for the determination of the denominator of the output admitance or the numerator of the output impedance

Since the output impedance is the reciprocal of the output admitance we have

$$Z_o(s) = R_o \frac{N_o(s)}{D(s)} \qquad Y_o(s) = \frac{1}{Z_o(s)} = G_o \frac{D(s)}{N_o(s)}$$

whence we see that the denominator of $Y_o(s)$ is $N_o(s)$ which can be determined by setting the excitation of $Y_o(s)$ to zero. The excitation of $Y_o(s)$ is a test voltage source connected at the output which upon setting equal to zero results in the circuit shown above. It follows immediately that

$$N_o(s) = \lim_{R \to 0} D(s)$$

Before taking the limit, we realize that one of the factors of $N_0(s)$ must be $1+sr_{c_2}C_{f_2}$ because this is the zero of an impedance branch connected directly across the output:

$$N_0(s) = \left(1 + s r_{c_2} C_{f_2}\right) N_0'(s)$$

$$N_0'(s) = \lim_{\substack{R \to 0 \\ r_{c_2} \to 0 \\ C_{f_2} \to 0}} D(s)$$

In taking this new limit, we will first ignore all the parasitic elements and obtain

$$N_0'(s) \simeq b_1 s + b_3 s^3 = b_1 s\left(1 + \frac{b_3}{b_1} s^2\right)$$

which implies that $N_0'(s)$ has a zero at the origin and an undamped resonance at $\omega_{oz} = \sqrt{b_1/b_3}$.

The zero at the origin is consistent with the fact that the low-frequency asymtote of $Z_0(s)$, R_0, goes to zero when all the parasitic elements go to zero.

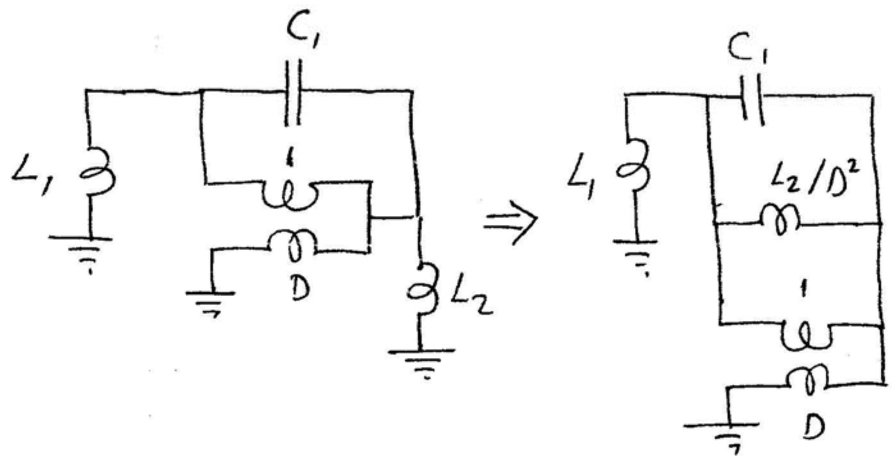
Next we determine ω_{oz}:

$$\omega_{oz}^2 = \frac{b_1}{b_3} = \lim_{\substack{C_{f_2} \to 0 \\ r_{c_2} \to 0}} \left(\frac{a_1}{a_3}\right) = \frac{D'^2}{L_{f_1} L_{f_2} C_{f_1}} \left(L_{f_2} + \left(\frac{D}{D'}\right)^2 L_{f_1}\right)$$

It follows that

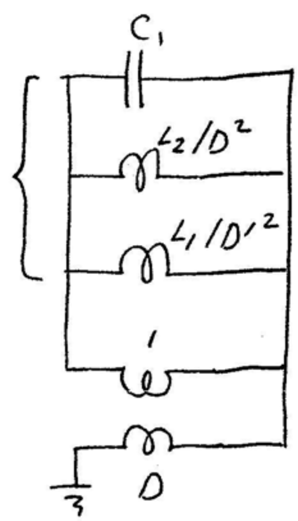
$$\omega_{oz} = \sqrt{\frac{1}{C_{f_1}\ \frac{L_{f_2}}{D^2} \Big\| \frac{L_{f_1}}{D'^2}}}$$

This is very consistent with the physical picture, without parasitic elements shown below



$\Downarrow$

$$\omega_{oz} = \sqrt{\frac{1}{C_1\ \frac{L_2}{D^2} \Big\| \frac{L_1}{D'^2}}}$$



Finally, we include the parasitic elements to determine the zero at very low frequencies and the damping of the resonance at ω_{oz}. Now we have

$$
\begin{aligned}
N_0'(s) &= 1 + b_1 s + b_2 s^2 + b_3 s^3 \\
&\simeq (1 + b_1 s)\left(1 + \frac{b_2}{b_1} s + \frac{b_3}{b_1} s^2\right) \\
&\simeq \left(1 + \frac{s}{s_{z_1}}\right)\left(1 + \frac{s}{\omega_{oz} Q_z} + \frac{s^2}{\omega_{oz}^2}\right)
\end{aligned}
$$

$$
b_1 = \lim_{\substack{R \to 0 \\ r_{c_2} \to 0 \\ C_{f_2} \to 0}} a_1 \simeq \frac{L_{f_1}}{r_{L_1} + \frac{r_c}{D^2} + r_{L_2}\left(\frac{D'}{D}\right)^2} + \frac{L_{f_2}}{r_{L_2} + r_{L_1}\left(\frac{D}{D'}\right)^2 + \frac{r_c}{D'^2}} + C_{f_1}\left[r_{c_1} + \underbrace{(r_{L_1} + r_{L_2})\frac{r_c + D^2 r_{L_1} + D'^2 r_{L_2}}{r_c + r_{L_1} \| r_{L_2}}}_{\to 0}\right]
$$

$$
\therefore \quad b_1 \simeq \frac{L_{f_1} + \left(\frac{D'}{D}\right)^2 L_{f_2}}{r_{L_1} + \frac{r_c}{D^2} + \left(\frac{D'}{D}\right)^2 r_{L_2}}
$$

$$
s_{z_1} = \frac{1}{b_1}
$$

$$b_2 = \lim_{C_{f_2} \to 0} a_2 = C_{f_1}\left(L_{f_1} + L_{f_2}\right)$$

$$\frac{1}{\omega_{oz} Q_z} = \frac{b_2}{b_1} \Rightarrow Q_z = \omega_{oz} \frac{b_1}{b_2 \omega_{oz}^2}$$

making the necessary substitutions, we get

$$Q_z = \frac{\omega_{oz} L_{f_1} \| L_{f_2}}{r_C + r_{L_1} D^2 + r_{L_2} D'^2}$$

Putting all the results together, we have:

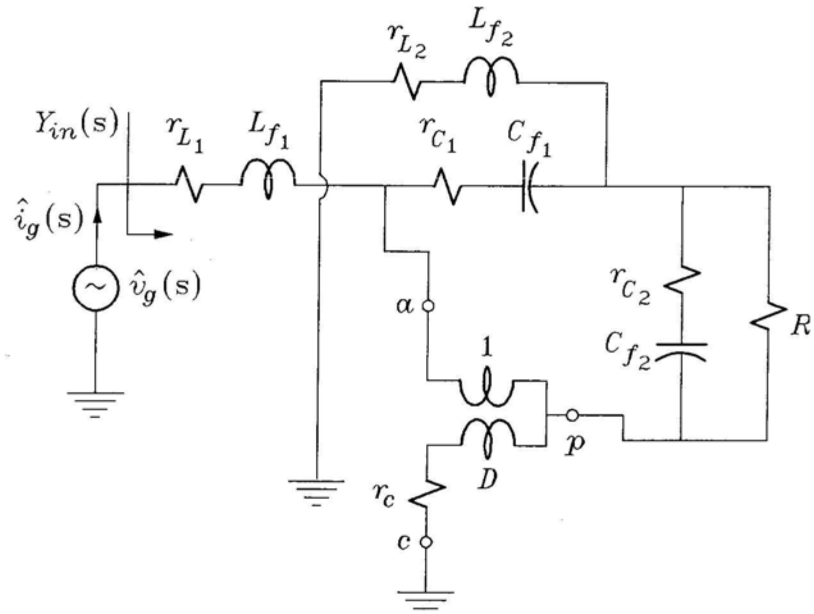
$$N_o(s) = \left(1 + \frac{s}{s_{z_1}}\right)\left(1 + \frac{s}{s_{z_2}}\right)\left(1 + \frac{s}{\omega_{oz} Q_z} + \frac{s^2}{\omega_{oz}^2}\right)$$

$$s_{z_1} = \frac{r_{L_1} + r_C/D^2 + r_{L_2}(D'/D)^2}{L_{f_1} + \left(\frac{D'}{D}\right)^2 L_{f_2}}$$

$$s_{z_2} = \frac{1}{r_{C_2} C_{f_2}}$$

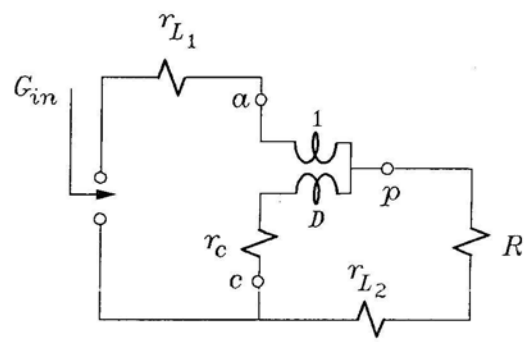
$$\omega_{oz} = \sqrt{\frac{1}{C_{f_1} \frac{L_{f_2}}{D^2} \| \frac{L_{f_1}}{D'^2}}}$$

6. Determination of the open-loop input admitance



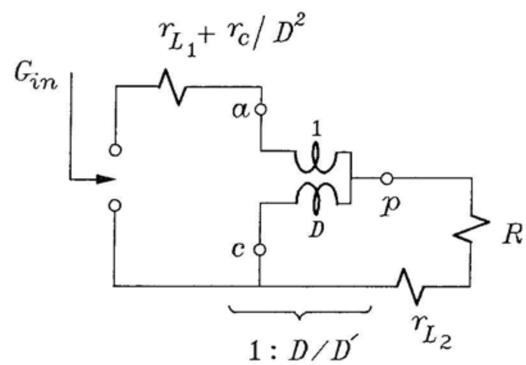
The input admitance is of the form

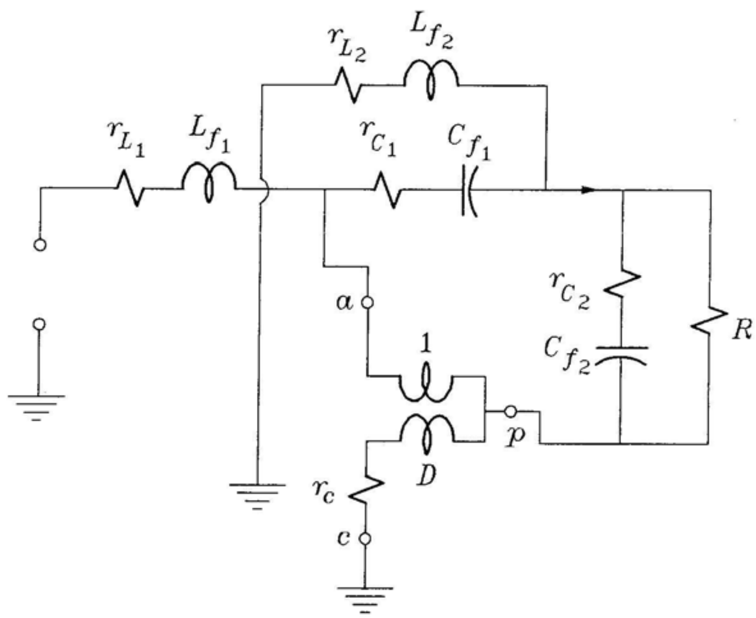
$$Y_{in}(s) = G_{in}\,\frac{N_i(s)}{D(s)}$$



D(s) is the same as before while G_{in} is determined from the adjacent circuit

$$G_{in} = \frac{1}{r_{L_1} + r_c/D^2 + \left(\frac{D'}{D}\right)^2 (R + r_{L_2})}$$





Circuit for the determination of the denominator of the input impedance or the numerator of the input admitance

Since the input admitance is the reciprocal of the input impedance, we have:

$$Y_{in}(s) = G_{in}\frac{N_i(s)}{D(s)} \qquad Z_{in}(s) = R_{in}\frac{D(s)}{N_i(s)}$$

whence we see that $N_i(s)$ is the denominator of $Z_{in}(s)$ which can be determined from the circuit by reducing the excitation of $Z_{in}(s)$ to zero. The excitation of $Z_{in}(s)$ is a test current source connected at the input which upon reducing to zero results in the circuit shown above whence it follows that

$$N_i(s) = \lim_{r_{L_1}\to\infty} D(s)$$

Next we determine the coefficients a_i in the limit $r_{L_1} \to \infty$

$\lim_{r_{L_1}\to\infty} a_1$

$$\lim_{r_{L_1}\to\infty} R^{(1)} \to \infty$$

$$\text{"} \quad R^{(2)} \to \infty$$

$$\text{"} \quad R^{(3)} \to r_{C_2} + R$$

$$\text{"} \quad R^{(4)} \to r_{C_1} + \lim_{r_{L_1}\to\infty} (r_{L_1} + r_{L_2}) \frac{1 + \frac{r_{L_1} \| r_{L_2}}{[\ \ ]}}{1 + \frac{r_{L_1} + \left(\frac{D'}{D}\right)^2 r_{L_2}}{[\ \ ]}}$$

$$= r_{C_1} + r_{L_1} \frac{1 + \frac{r_{L_2}}{[\ \ ]}}{\frac{r_{L_1}}{[\ \ ]}}$$

$$= r_{C_1} + [\ \ ] + r_{L_2}$$

$$\therefore \lim_{r_{L_1}\to\infty} R^{(4)} = r_{C_1} + \frac{r_C + r_{L_2} + D'^2 R}{D^2}$$

$$\therefore \lim_{r_{L_1}\to\infty} a_1 = C_2 (r_{C_2} + R) + C_{f_1}\left(r_{C_1} + \frac{r_C + r_{L_2} + D'^2 R}{D^2}\right)$$

$$\simeq \left[C_{f_2} + C_{f_1} \left(\frac{D'}{D}\right)^2 \right] R$$

$\lim\limits_{r_{L_1}\to\infty} a_2$

In this limit only the following two terms survive

$$\lim_{r_{L_1}\to\infty} a_2 = \lim_{r_{L_1}\to\infty}\left[\frac{L_{f_2}}{R^{(2)}} C_{f_1} R^{(4)}_{(2)}\right]$$

$$+\lim_{r_{L_1}\to\infty}\left[C_{f_1} C_{f_2} R^{(3)} R^{(4)}_{(3)}\right]$$

$$= L_{f_2}C_{f_1}/D^2 + C_{f_1}C_{f_2}(r_{c_2}+R)\left(r_{c_1} + \frac{r_c + r_{L_2} + D'^2 R \| r_{c_2}}{D^2}\right)$$

$$\simeq \frac{L_{f_2}C_{f_1}}{D^2}$$

$\lim\limits_{r_{L_1}\to\infty} a_3$

In this limit only one term servives

$$\lim_{r_{L_1}\to\infty} a_3 = \lim_{r_{L_1}\to\infty}\left[\frac{L_{f_2}}{R^{(2)}} \cdot C_{f_2} R^{(3)}_{(2)} \cdot C_{f_1} R^{(4)}_{(32)}\right]$$

$$\simeq L_{f_2}C_{f_2}C_{f_1}(r_{c_2}+R)\lim_{r_{L_1}\to\infty}\left[\frac{R^{(4)}_{(32)}}{R^{(2)}}\right]$$

$$= L_{f_2}C_{f_2}C_{f_1} R \frac{r_{L_1}/D'^2}{r_{L_1}\ (D/D')^2}$$

$$= L_{f_2}C_{f_2}C_{f_1}\frac{R}{D^2}$$

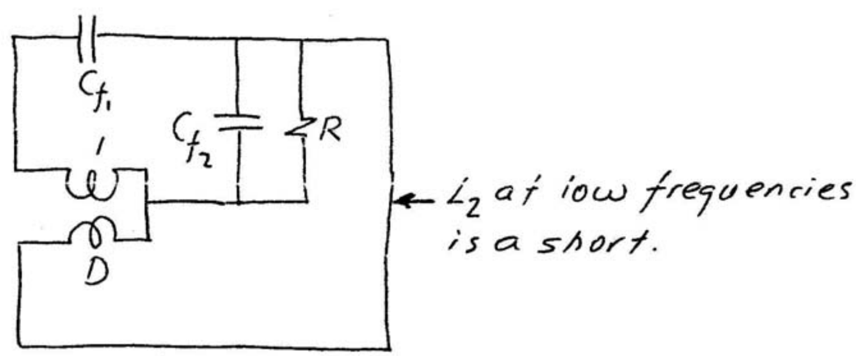
$$\lim_{r_{L_1}\to\infty} a_4 = 0$$

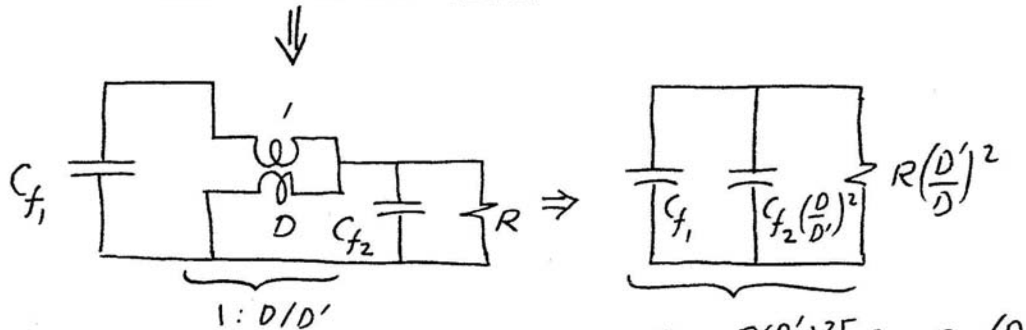
Putting all the results together we get

$$N_i(s) = 1 + sR\left(C_{f_2} + C_{f_1}\left(\frac{D'}{D}\right)^2\right) + s^2\frac{L_{f_2}C_{f_1}}{D^2} + s^3\frac{L_{f_2}C_{f_1}C_{f_2}R}{D^2}$$

In order to determine the approximate factors of $N_i(s)$ analytically, we study the circuit at low and high frequencies.

At low frequencies we have:





$$\tau = R\left(\frac{D'}{D}\right)^2\left[C_{f_1} + C_{f_2}\left(\frac{D}{D'}\right)^2\right]$$

$$= R\left(C_{f_2} + C_{f_1}\left(\frac{D'}{D}\right)^2\right)$$

Hence, having identified the low-frequency factor, we investigate the possibility of factoring $N_i(s)$ as follows:

$$N_i(s) \simeq \left(1 + \frac{s}{s_{z_i}}\right)\left(1 + \frac{s}{\omega_i Q_i} + \frac{s^2}{\omega_i^2}\right)$$

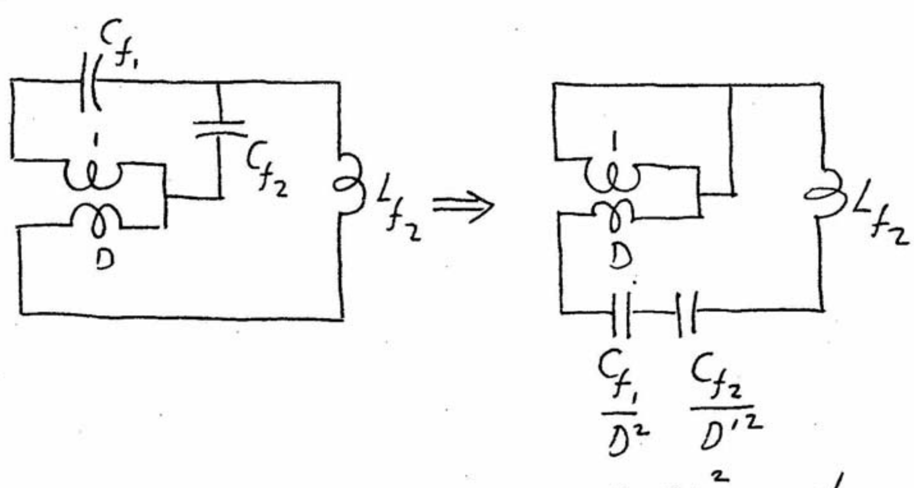
where

$$s_{z_i} = \frac{1}{R\left(C_{f_2} + C_{f_1}\left(\frac{D'}{D}\right)^2\right)}$$

$$\omega_i^2 = \frac{C_{f_2} + C_{f_1}(D'/D)^2}{L_{f_2} C_{f_1} C_{f_2} / D^2} = \frac{1}{L_{f_2} \; \frac{C_{f_1}}{D^2} \Big\| \frac{C_{f_2}}{D'^2}}$$

$$Q_i = \frac{R}{\omega_i} \frac{C_{f_2} + C_{f_1}(D'/D)^2}{C_{f_1} L_{f_2}} D^2 = \omega_i R C_{f_2}$$

This resonance can be verfied by examining the circuit without any damping as shown below



$$\therefore \; \omega_i^2 = \frac{1}{\left[\frac{C_{f_1}}{D^2} \Big\| \frac{C_{f_2}}{D'^2}\right] L_{f_2}}$$

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*SEPIC CONVERTER WITH PWM SWITCH MODEL
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*SEPIC CONVERTER FROM POWER SYSTEMS DESIGN EUROPE NOVEMBER 2006 ISSUE
*Dr. Ray Ridley's Design Tips

    Vin   1    100                    AC   0
    RL1   1    2                           0.001
    L1    2    4                           10E-05
    RL2   4    3                           0.001
    L2    3    0                           10E-05
    RC1   100  7                           0.003000
    C1    7    0                           6.80E-04
    RC2   5    6                           0.001000
    C2    6    0                           2.20E-03
    R     5    0                           1
    Vc    11   0                      AC   1
    Rvc   11   0                           10MEG
    X1    100  5    4    11                PWMCCM

.AC      DEC     100  10Hz      10000Hz
.PRINT   AC           VDB(5)  VP(5)
.PROBE

*PWM SWITCH MODEL
*SWITCH MODEL PARAMETER VALUES: Vap=-Vo/D  Ic = -Io/D'
.SUBCKT PWMCCM 1     2    3    4

    E2    7    1    4    0                 -41.667
    G1    1    2    4    0                 -37.5
    Fxf   7    2    Vxf                    0.6
    Exf   9    2    7    2                 0.6
    Vxf   9    3                           0
    Rvc   4    0                           10MEG

.ENDS
.END
```