Ramp Speed Analysis for Triangle Wave

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Introduction

For applications that want to generate a ramping waveform, a common question that comes up is how wide the loop bandwidth needs to be to not distort the modulation. For the purpose of discussion, let's assume that this waveform is as shown in Figure 1.

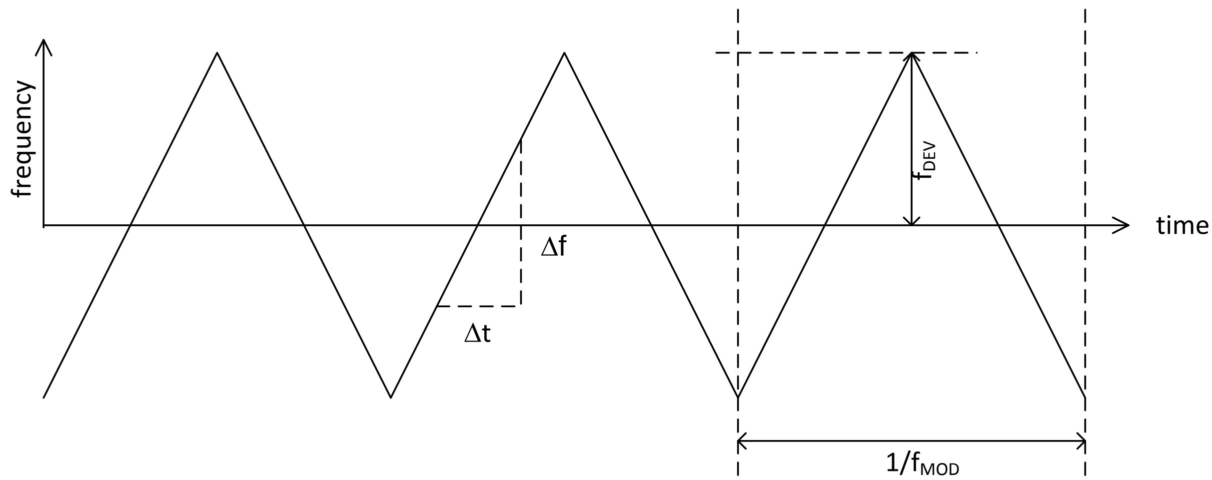


Figure 1 *Triangle Ramping Waveform*

In Figure 1, we can derive the slope along the linear waveform as:

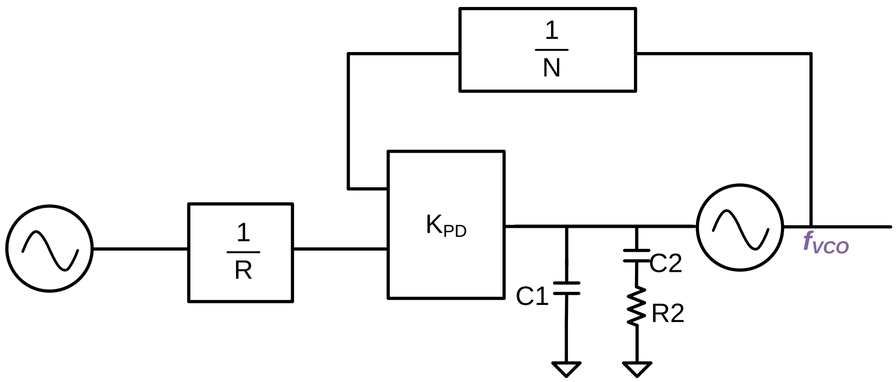
$$m = \frac{\Delta f}{\Delta t} = \frac{2 \cdot f_{DEV}}{\left(\frac{1}{f_{MOD}}\right)/2} = 4 \cdot f_{DEV} \cdot f_{MOD} \tag{1}$$

In general, we want the loop bandwidth wide enough to allow the following:

1. Rule #1: Loop Bandwidth needs to be wide enough to allow the PLL to slew fast enough, ignoring the discrete sampling effects of the charge pump.
2. Rule #2: Loop Bandwidth needs to be wide enough so that the discrete sampling effects of the charge pump do not make the slew rate of the loop too slow.
3. Rule #3: Loop Bandwidth needs to be wide enough such that near the direction changes, the loop can respond fast enough.

Rule #1: **Determine Bandwidth necessary for sufficient slew rate**

Consider a simple 2nd order loop filter.



Now solve for components (formulas from my PLL book).

$$A0 = \frac{C1 \cdot T2}{T1} = \frac{K_{PD} \cdot K_{VCO}}{N \cdot \omega c^2} \cdot \sqrt{\frac{1 + \omega^2 \cdot T2^2}{1 + \omega^2 \cdot T1^2}} \tag{2}$$

$$Z(s) = \frac{1 + s \cdot T2}{s \cdot A0 \cdot (1 + s \cdot T1)} \tag{3}$$

$$\phi = 180 + arctan(\omega c \cdot T2) - arctan(\omega c \cdot T1) \tag{4}$$

$$T2 = \frac{\gamma}{\omega c^2 \cdot T1} \tag{5}$$

$$T1 = \frac{\sqrt{(1+\gamma)^2 \cdot tan^2\phi + 4 \cdot \gamma} - (1+\gamma) \cdot \tan\phi}{2 \cdot \omega c} \tag{6}$$

$$A0 = \frac{K_{PD} \cdot K_{VCO}}{N \cdot \omega c^2} \cdot \sqrt{\frac{1 + \omega c^2 \cdot T2^2}{1 + \omega c^2 \cdot T1^2}} \tag{7}$$

From (6), restate this as follows and realize that this is purely a function of gamma and phase margin.

$$\rho \equiv \omega c \cdot T1 = \frac{\sqrt{(1+\gamma)^2 \cdot tan^2\phi + 4 \cdot \gamma} - (1+\gamma) \cdot \tan\phi}{2} \tag{8}$$

Also introduce another constant:

$$\kappa^2 \cdot \pi^2 = \frac{1}{\rho} \cdot \sqrt{\frac{\gamma^2 + \rho^2}{1 + \rho^2}} \tag{9}$$

Combining (5), (6), and (7) yield the following result:

$$A0 = \frac{K_{PD} \cdot K_{VCO}}{N \cdot \omega c^2} \cdot \kappa^2 \cdot \pi^2 \tag{10}$$

Now A0 is the total capacitance, and from this the slew rate can be calculated:

$$\frac{df}{dt} = \frac{K_{VCO} \cdot K_{PD}}{A0} = \frac{N \cdot \omega c^2}{\kappa^2 \cdot \pi^2} \tag{11}$$

Now (11) represents the slew rate that the loop filter is capable of, but this needs to be at least as fast as the slew rate as calculated by (1). Therefore, combining (1) and (11) yields the following:

$$\frac{N \cdot \omega c^2}{\kappa^2 \cdot \pi^2} > 4 \cdot f_{DEV} \cdot f_{MOD} \tag{12}$$

$$BW > \kappa \sqrt{\frac{f_{DEV} \cdot f_{MOD}}{N}} \tag{13}$$

For phase margin of 45 degrees and $\gamma = 1$ ($\kappa = 0.495$, but round to 0.5), this can be approximated as

$$BW > \sqrt{\frac{f_{DEV} \cdot f_{MOD}}{4 \cdot N}} \tag{13}$$

Now for the parameter, κ, the following table might be helpful for quick calculations:

ϕ	γ	ρ	$\kappa^2 \cdot \pi^2$	κ
30	0.5	0.396	1.497	0.389
	1	0.577	1.732	0.419
	2	0.792	2.128	0.464
45	0.5	0.281	1.966	0.446
	1	0.414	2.414	0.495
	2	0.562	3.226	0.572
60	0.5	0.180	2.906	0.543
	1	0.268	3.732	0.615
	2	0.360	5.312	0.734
75	0.5	0.088	5.751	0.763
	1	0.132	7.596	0.877
	2	0.176	11.243	1.067

Table 1 *Calculation of κ from phase margin and gamma*

Rule #2: **How Wide to avoid cycle slipping?**

Recall this rule from (29.13) in my PLL book to avoid cycle slipping

$$\frac{f_{PD}}{BW} < \frac{5}{\left|1 - \frac{f2}{f1}\right|}$$

$$\frac{1}{BW} < \frac{N}{|f1 - f2|}$$

$$BW > \frac{|f1 - f2|}{5 \cdot N}$$

$$f2 = f1 + \frac{df}{dt}\Delta t$$

$$\frac{BW}{\Delta t} > \frac{\left|\frac{df}{dt}\right|}{5}$$

$$BW > \frac{f_{DEV} \cdot f_{MOD} \cdot \Delta t}{5 \cdot N}$$

$$BW > \frac{f_{DEV}}{5 \cdot N}$$

Rule #3: **How wide of a bandwidth to not distort corners too much?**

Now what about abruptly changing direction at the peak. Recall the Fourier series for a triangle wave is:

$$f(t) = \frac{8}{\pi^2} \cdot \left(sin(\omega t) - \frac{1}{3^2} sin(3\omega t) + \cdots \right)$$

Remember Rule #1, let's think about 2X loop bandwidth to at least allow the fundamental. In this case, I think that we can e

$$f_{DEV \cdot} f_{MOD} < \sim 6.8 \cdot N \cdot (2f_{MOD})^2$$

$$\frac{f_{DEV \cdot}}{f_{MOD}} < \ 1.7 \cdot N$$

So now we compare a sine wave to a square wave. The slew rate is equal to the derivative. So let's compare the slew rate at various phases to the max phase at 0. Note that the phase goes from -45 to +45 degrees.