Ramp Speed Analysis for Triangle Wave

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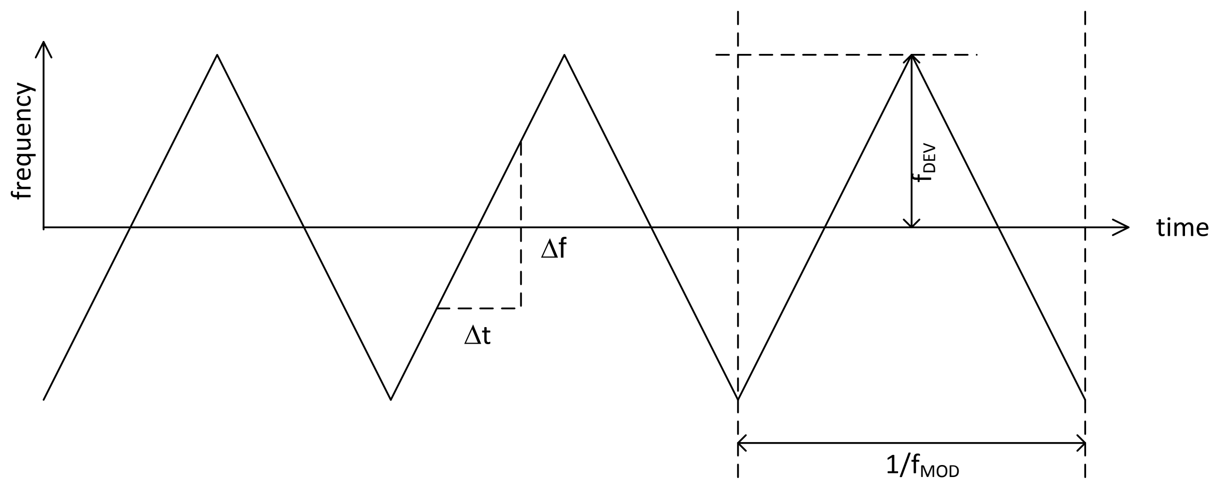
Introduction

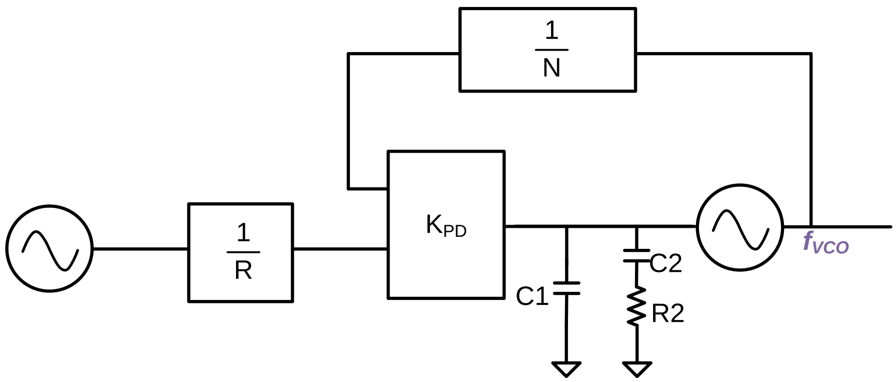
Let's derive 3 rules to try to come up with a loop bandwidth. First consider a triangle wave as shown below. What loop characteristics are necessary to prevent distortion.



$$m = \frac{\Delta f}{\Delta t} = \frac{df}{dt} = 4 \cdot f_{DEV} \cdot f_{MOD}$$

Rule #1: How wide a bandwidth to allow a max slew rate

Consider a simple 2nd order loop filter.



Now solve for components (formulas from my PLL book).

$$A0 = \frac{C1 \cdot T2}{T1} = \frac{K_{PD} \cdot K_{VCO}}{N \cdot \omega c^2} \cdot \sqrt{\frac{1 + \omega^2 \cdot T2^2}{1 + \omega^2 \cdot T1^2}}$$

$$Z(s) = \frac{1 + s \cdot T2}{s \cdot A0 \cdot (1 + s \cdot T1)}$$

$$\phi = 180 + arctan(\omega c \cdot T2) - arctan(\omega c \cdot T1)$$

$$T2 = \frac{\gamma}{\omega c^2 \cdot T1}$$

$$T1 = \frac{\sqrt{(1+\gamma)^2 \cdot tan^2\phi + 4 \cdot \gamma} - (1+\gamma) \cdot \tan\phi}{2 \cdot \omega c}$$

Without (much) loss of generality and to simplify the math, we can assume that the gamma optimization factor is 1 and the phase margin is 45 degrees. This simplifies to:

$$T1 = \frac{\sec\phi - \tan\phi}{\omega c}$$

$$T2 = \frac{1}{\omega c \cdot (\sec\phi - \tan\phi)}$$

$$A0 = \frac{K_{PD} \cdot K_{VCO}}{N \cdot \omega c^2} \cdot \sqrt{\frac{1 + \omega c^2 \cdot T2^2}{1 + \omega c^2 \cdot T1^2}} = \frac{K_{PD} \cdot K_{VCO}}{N \cdot \omega c^2} \cdot (\sec\phi - \tan\phi)$$

Now A0 is the total capacitance, $i = C \cdot \frac{dV}{dt} = A0 \cdot \frac{dV}{dt}$

$$\frac{df}{dt} = \frac{K_{VCO} \cdot K_{PD}}{A0} = N \cdot \omega c^2 \cdot (\sec\phi - \tan\phi)$$

For an easier rule of thumb, lets express the loop bandwidth in radians and assume that phase margin is about 45 degrees for optimal lock time. Also round off numbers to make them convenient to get the max slew rate

$$\frac{df}{dt} = \frac{K_{VCO} \cdot K_{PD}}{A0} = N \cdot (2\pi \cdot BW)^2 \cdot \left(\sqrt{2} - 1\right) \sim 0.4 \cdot N \cdot BW^2$$

$$4 \cdot f_{DEV} \cdot f_{MOD} < 0.4 \cdot N \cdot BW^2$$

$$BW > \sqrt{\frac{1}{10 \cdot N} \cdot \left(\frac{f_{DEV \cdot}}{f_{MOD \cdot}}\right)}$$

Rule #2: How Wide to avoid cycle slipping?

Recall this rule from (29.13) in my PLL book to avoid cycle slipping

$$\frac{f_{PD}}{BW} < \frac{5}{\left|1 - \frac{f2}{f1}\right|}$$

$$\frac{1}{BW} < \frac{N}{|f1 - f2|}$$

$$BW > \frac{|f1 - f2|}{5 \cdot N}$$

$$f2 = f1 + \frac{df}{dt}\Delta t$$

$$\frac{BW}{\Delta t} > \frac{\left|\frac{df}{dt}\right|}{5}$$

$$BW > \frac{f_{DEV} \cdot f_{MOD \cdot}\Delta t}{5 \cdot N}$$

$$BW > \frac{f_{DEV}}{5 \cdot N}$$

Rule #3: **How wide of a bandwidth to not distort corners too much?**

Now what about abruptly changing direction at the peak. Recall the Fourier series for a triangle wave is:

$$f(t) = \frac{8}{\pi^2} \cdot \left(sin(\omega t) - \frac{1}{3^2} sin(3\omega t) + \cdots \right)$$

Remember Rule #1, let's think about 2X loop bandwidth to at least allow the fundamental. In this case, I think that we can e

$$f_{DEV} \cdot f_{MOD} < \sim 6.8 \cdot N \cdot (2 f_{MOD})^2$$

$$\frac{f_{DEV} \cdot}{f_{MOD}} < \ 1.7 \cdot N$$

So now we compare a sine wave to a square wave. The slew rate is equal to the derivative. So let's compare the slew rate at various phases to the max phase at 0. Note that the phase goes from -45 to +45 degrees.