

Measurement of RMS values of non-coherently sampled signals

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Abstract If the RMS value is gained by digital processing of sequence of signal samples, both uncertainty and bias of the measured value depend on the algorithm used. Since in practice signal sampling is usually non-coherent, leakage occurs in signal DFT spectrum and definition of the RMS of periodic signals in time domain is violated. The paper compares three different DSP algorithms of RMS measurement by non-coherent sampling from the point of view of measurement bias and uncertainty for various leakage levels and data window used. Results of simulations and example of measurement are evaluated for monofrequency signals. For non-coherent signal sampling a new and effective approach to RMS value measurement in time domain, a new method of finding exact signal frequency (different from those of DFT grid) based on two DFT phase spectra computation, and a method of automatic RMS value bias correction in frequency domain are presented. The reported results can easily be extended to multifrequency signals.

I. Introduction

Measurement of RMS values of voltage and current is a common everyday task performed by DSP in measurement systems using PC DAQ plug-in boards and also some high accuracy digital voltmeters. The RMS value can be found by processing the samples of signal either in time domain or in the frequency domain. Measurement bias and uncertainty depends (besides influencing quantities) on the ADC and on the DSP algorithm. Evaluating bias and uncertainty of RMS measurements of monofrequency signals for non-coherent sampling using different DSP algorithms and different data windows is the goal of this contribution. The present paper is in a certain sense continuation of [1] (uncertainty of measurement [2] is evaluated and additional methods and windows are analyzed). Measurement of RMS in frequency domain is closely connected to DFT amplitude spectrum measurement using different windows [3]. Either interpolation in frequency domain is used after signal windowing (e.g. [4 – 8]), or processing several frequency lines surrounding local maxima of the spectrum is used [9, 10].

II. Measurement of the RMS values in time domain

A. Classical RMS computation in time domain

RMS value of analog signal $x(t)$ with period T_{sig} or discrete signal $x(n)$ with N samples per period is

$$X_{RMS} = \sqrt{\frac{1}{T_M} \int_0^{T_M} x^2(t) dt} = \sqrt{\frac{1}{N} \sum_{n=0}^{N-1} x^2(n)} \quad (1),$$

where the second square root is used for processing of sequence of signal samples, N being (integer) number of samples per signal period (coherent sampling). If the T_{sig} in (1) is replaced by the time of measurement (of sampling) of the analog signal T_M (or N is replaced by N' , positive integer number), and if the signal $x(t)$ (or $x(n)$) is a sinusoidal signal with zero phase (the RMS value is independent of phase), then

$$X'_{RMS} = \sqrt{\frac{1}{T_M} \int_0^{T_M} (X_m \sin(\omega_{\text{sig}} t))^2 dt} = \sqrt{\frac{1}{N'} \sum_{n=0}^{N'-1} (X_m \sin(\omega_{\text{sig}} n))^2} \quad (2).$$

The (continuous) measurement time can be expressed as

$$T_M = (M + \lambda)T_{sig} \quad (3)$$

where M is number of integer periods sampled and λ is decimal part of the last period sampled ($0 \leq \lambda < 1$). There is $X'_{RMS} = X_{RMS}$ for $\lambda = 0$ (coherent sampling), and $X'_{RMS} \neq X_{RMS}$ for $\lambda \neq 0$. The difference between X'_{RMS} and X_{RMS} is the bias of RMS measurement in time domain caused by non-coherent sampling. The relative bias could be expressed as

$$\delta_{RMS} = \frac{X'_{RMS} - X_{RMS}}{X_{RMS}} 100\% = \left(\sqrt{1 - \frac{\sin 4\pi(M + \lambda)}{4\pi(M + \lambda)}} - 1 \right) \cdot 100\% \quad (4)$$

The dependence of δ_{RMS} found by processing signal in time domain on measurement time is shown in Fig.1, and the discrete-time waveform computed from the right-hand part of (2) is practically identical with it in sampling instants.

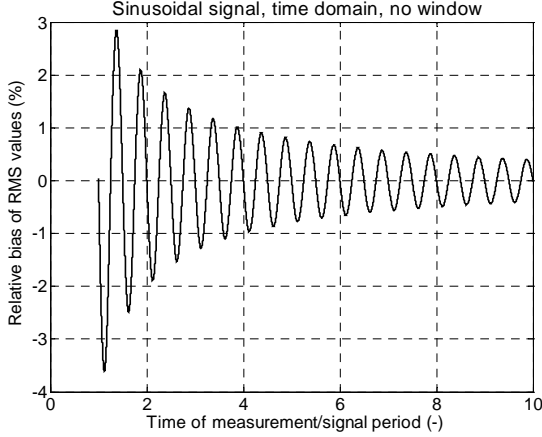


Fig.1 RMS value bias, classical algorithm (see (2))

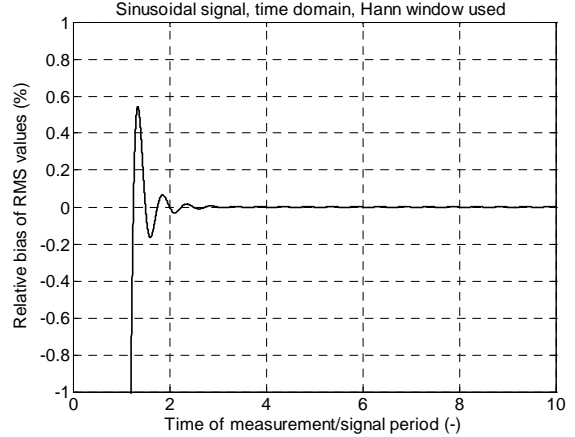


Fig.2 RMS value bias, new approach (see (7))

Since δ_{RMS} from (4) approaches zero for $M \rightarrow \infty$, (2) is an unbiased estimate of (1). The measurement in time domain can therefore even by non-coherent sampling have low bias for high sampling times. (The envelopes of the curve - one connecting local maxima, and the other one connecting local minima - represent bands from which the component of measurement uncertainty caused by non-coherent sampling can be found as a function of time.) Eq. (4) is derived for single frequency signal, it could however easily be modified for multifrequency signal (with known magnitudes of all harmonic components). If multifrequency signal is measured, application of (4) to all harmonic signal components and taking into account that RMS of signal is found as geometrical sum of signal harmonic components RMS values, the resulting bias of the RMS value of signal can be found.

Uncertainty of the basic algorithm (1) was for coherent sampling analyzed in [11]. RMS uncertainty component due to signal quantization or another external noise with variance equal to u_n^2 can be found as

$$u^2(X_{RMS}) = \frac{u_n^2}{N} \quad (5)$$

B. New DSP algorithm for RMS computation in time domain

Bias of RMS value shown in Fig.1 is caused by part λT_{sig} of T_m in (3). If we multiply the (sampled part of) signal by a tapering window of the length identical with signal length (as is often done before processing signal in frequency domain), the weight of the part λT_{sig} at the very end of the sampled signal will be substantially reduced. The most popular windows are the cosine windows defined as

$$w_p(n) = \sum_{r=0}^P V_r \cos\left(\frac{2\pi nr}{N}\right), \quad n = 0, 1, 2, \dots, N-1 \quad (6)$$

where N is window length, V_r are window coefficients and P is window order. The higher is the window order, the lower are window spectrum side lobes and the broader is window spectrum main lobe. The RMS value of the windowed (using a cosine window) and coherently sampled signal is, if more than $2P+1$ signal periods are sampled, is

$$X_{RMS}(w(n) \cdot x(n)) = \sqrt{\frac{1}{N} \sum_{n=0}^{N-1} (w(n) \cdot x(n))^2} = \sqrt{nnpg} \cdot X_{RMS}(x(n)) \quad (7),$$

where $nnpg = \frac{1}{N} \sum_{n=0}^{N-1} (w(n))^2$ is normalized noise power gain [3]. RMS bias by non-coherent sampling is substantially reduced as compared to (4) because of windowing. The efficiency of this method (as compared to classical RMS computation described in par. A) can be seen from Figs.1 and 2 and Tab.1. This method is simple and can be fast, since finding local maxima in FFT spectrum is not necessary, and even FFT computation is not required. Moreover, RMS value from (7) corresponds to RMS found in frequency domain using the complete spectrum, not only the main lobe, and its bias should be therefore lower than that of the method described in [9, 10] and in par.B2 here. It does not allow however to find only the RMS values of selected components. Uncertainty of RMS value computed using (7) caused by noise can be approximately found as that from found from (5) divided by \sqrt{nnpg} (and with the exception of rectangular window, there is $nnpg < 1$).

Tab.1 Comparison of RMS value bias for different windows, first local extreme for $T_M > M$ from (3)

M	Rectangular (P=0) (no windowing)	Hann (P=1)	Hamming (P=1)	Blackman (P=2)	5 Term Blackman-Harris (P=4)
2P+1	-3.61 %	$-3.85 \cdot 10^{-3}$ %	$-1.7 \cdot 10^{-2}$ %	$-3.6 \cdot 10^{-5}$ %	$-1.2 \cdot 10^{-9}$ %
10	-0.39 %	$-9.45 \cdot 10^{-6}$ %	$-6.2 \cdot 10^{-3}$ %	$-1.4 \cdot 10^{-6}$ %	$-9.2 \cdot 10^{-10}$ %

III. Measurement of the RMS values in frequency domain

A. Sinusoidal or periodical signal, coherent sampling

If the width of the frequency bin Δf equal to the (fundamental) signal frequency is chosen and rectangular window is used, then after applying the DFT (in practice FFT) algorithm of the length equal to the signal samples frame N and dividing the DFT spectrum by N , the so gained DFT spectrum components are identical with two-sided Fourier spectrum of the periodic signal. For *harmonic signal*, two spectral lines corresponding to the exponential components for frequencies $-f_{sig}$ and $+f_{sig}$ are found. For ideally coherent sampling (i.e. exactly integer number of signal periods sampled) the accuracy of the found RMS values is given mainly by quantization error (and other errors) of the ADC used. Uncertainties of amplitude DFT spectrum for coherent sampling and no data window used are analyzed in [11].

B. Sinusoidal signal, non-coherent sampling

If the signal sampling is non-coherent, leakage occurs, i.e. the signal power is spread to other frequency bins. In this case e.g. signal windowing and interpolation in frequency domain (WIFD) (see e.g. [5-8]) can help to reduce leakage and so reduce the bias of spectrum measurement. Since the purpose of this paper is not spectrum analysis but estimation of the RMS value of signal, processing of the signal in the presence of leakage should be inspected from this point of view. There are two basic possibilities how to find the RMS value by processing non-coherently sampled signal in frequency domain – either using windowing and interpolation based on known spectral window shape (in frequency domain), or processing directly chosen parts of the windowed frequency spectrum without using any interpolation. Application of correction factor in the second case is described below. To the most frequently used windows belong classical cosine windows (Hann, Hamming, Blackman) or higher order windows optimized from the various point-of-view (e.g. Blackman-Harris windows).

B1. Using signal windowing and interpolation in frequency domain (WIFD), the classical and the new approach

If frequency-domain interpolation (using some simplification in formulae for spectrum values based on the spectral window shape) or finding correction factor for decreasing the bias of the measured RMS value based on true (not interpolated) window shape should be applied, it is important firstly to find the decimal part δ (frequency displacement) in signal frequency expressed in bins. The frequency of local amplitude spectrum's maximum can be expressed as

$$f_{\delta} = (k + \delta) \times \Delta f, \quad \Delta f = 1/(NT_s) \quad (8)$$

where Δf is the frequency bin (the distance between neighboring DFT spectrum lines in Hz) and $k + \delta$ is rational number expressing frequency in frequency bins. Finding the value of decimal frequency bin δ ($|\delta| \leq 0.5$) is a task of basic importance for processing of the spectrum influenced by leakage. In the interpolation in frequency domain based on Rife-Vincent windows of the first class (RV1) [5,6] the finding of the δ is based on ratios of lengths of the local maximum value of the amplitude spectrum to the length of the neighboring spectrum component, and depend on the RV1 window order (RV1 windows are subset of cosine windows for concrete coefficients values, Hann window being the RV1 window of the first order, but Hamming and Blackman windows do not belong the RV1 windows [5]). *The new and more general method of finding δ* is based on using the phase frequency spectrum, more exactly on using linear shape of the phase spectrum of the used windows. Fourier spectrum of real and even windows is also real and even. Their phase spectrum is therefore zero or stepwise (0 or $\pm\pi$). Even windows are non-causal. Causal windows used in DFT analysis can be gained by shifting originally even windows in time so that they start in time origin. Such a shift corresponds in frequency domain multiplication by complex exponential, and leads to linear phase spectrum $\varphi = -(N-1) \cdot \pi \cdot f / f_s$. The slope of the (linear) phase spectrum therefore depends only on the magnitude of the shift, proportional to the window length N . Ratio of two slopes α_1 a α_2 for window lengths N_1 a N_2 is $\alpha_1/\alpha_2 = (N_1-1)/(N_2-1)$. This is valid for continuous DTFT (or FTD) spectrum. By means of DFT we gain the DTFT spectrum samples on the DFT grid. (DTFT spectrum of windowed signal corresponds to the convolution of signal and window spectra.) We can therefore get the phase value ψ in the closest point of the DFT grid f_{bin} to the actual frequency f_{sig} $\psi = \varphi + \alpha(f_{sig} - f_{bin})$, where φ is the (true) value of phase. Finding so phase values ψ_1 and ψ_2 for DFT (and window) lengths N_1 and N_2 , their difference can be found to be

$$\psi_2 - \psi_1 = \varphi + \alpha_2(f_{sig} - f_{bin}) - \varphi - \alpha_1(f_{sig} - f_{bin}) = -\frac{N_2 - N_1}{f_{vz}} \pi (f_{sig} - f_{bin}) = -\frac{N_2 - N_1}{N_1} \pi \delta \quad (9)$$

A numerical correction should be applied if δ found from (9) falls outside interval $(-0.5, 0.5)$. This might be caused by different distances of (true) frequency from the nearest frequency bin for N_1 a N_2 , and it results in shifting estimate of δ by an integer number. Deviation from the (true) value is caused by the influence of side lobes of other spectrum components, including the mirrored ones.

Finding δ using this new approach allows to apply interpolation to any selected data window which is a time shifted originally real and even windows, frequency spectrum of which can be found. The interpolation method described in [5-7] is applicable for class 1 Rife-Vincent windows only. As can be seen from Fig.3, influence of spectrum side lobes decreases with increasing number of sampled periods, since in this case distance of side lobes from the mirrored spectrum component expressed in bins increases as well. Finding δ from (9) is more exact that using ratio of adjacent DFT amplitude spectrum lines and simplified interpolation formulae from [6].

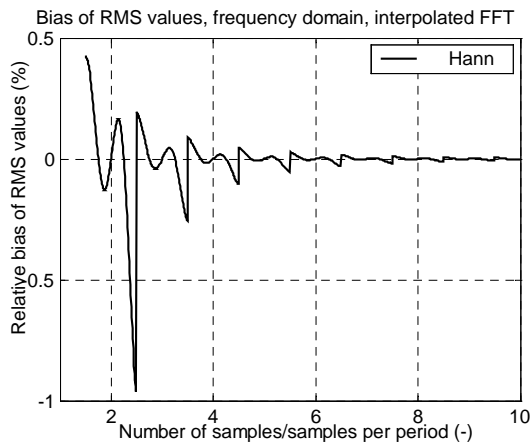


Fig.3 RMS bias by interpolation in frequency domain

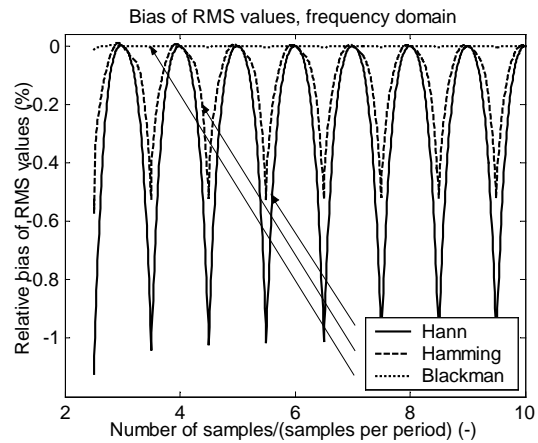


Fig.4 RMS bias found using (9)

B2. Using RMS computation from the main lobe of window spectrum

One leakage suppression method mentioned above is based on windowing. Windowing results in concentration of energy of any harmonic component to smaller number of DFT lines, and therefore decreasing influencing of more distant spectrum components. By coherent sampling there is the whole energy concentrated in the main lobe of windowed signal spectrum, since zeroes of window spectrum fall in points of DFT grid. By non-coherent sampling a part of energy is in side lobes and it this part is smaller if higher-order windows are used. The estimate of RMS value of a harmonic spectrum component is found using (10) (see [9, 10]).

$$X_{RMS} = \sqrt{\frac{1}{N^2 \cdot nmpg} \left(\sum_{f=f_1}^{f_2} |M(f)|^2 + \sum_{f=-f_2}^{-f_1} |M(f)|^2 \right)} \quad (10)$$

Here f_1 and f_2 are frequencies defining the main lobe of the window spectrum shifted to the signal harmonic component and containing $2P+1$ DFT amplitude spectrum lines. $M(f)$ are components of amplitude spectrum and $nmpg$ is normalized noise power gain [3], see (7).

Bias of RMS estimate for some windows is depicted in Fig. 2, from which it follows that the bias does practically not depend on number of sampled signal periods. Influence of noise on coherently sampled non-windowed signal was analyzed in [11]. For windowed signal the (square of) RMS uncertainty caused by noise can be found as

$$u^2(X_{RMS}) = u_n^2 \frac{1}{N^2 \cdot nmpg^2} \sum_{n=0}^{N-1} w^4(n) \quad (11)$$

if at least $2P+1$ period are sampled. Since higher-order windows have lower $nmpg$ (< 1), they are more sensitive to noise.

B3. Applying correction factor to increase accuracy of the RMS value measurement described in B2

Bias of RMS value found from the main lobe (see par. B2) stems from ignoring the part of signal energy in window spectrum side lobes. The estimated RMS value is therefore smaller than the (true) value – see Fig.4. Bias is influenced mainly by main lobe shape and value of δ . Relative bias can be found using the window shape, and can be subsequently corrected. Multiplication correction factor can be found by applying (10) to window spectrum for given δ . Fig. 5 shows the RMS bias after this correction for Hann window. Contrary to non-corrected method (par. B2) bias here decreases rapidly with increasing number of signal periods sampled.

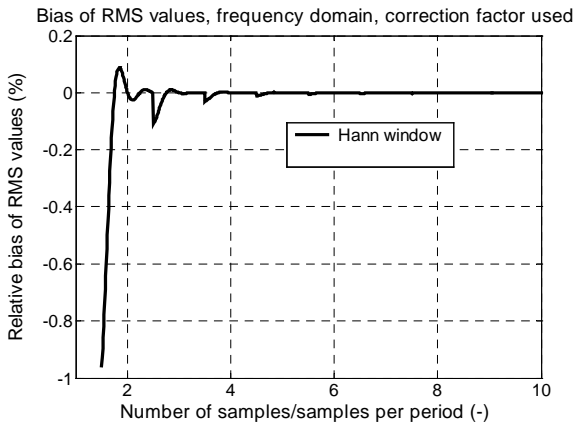


Fig 5 RMS bias using correction for Hann window

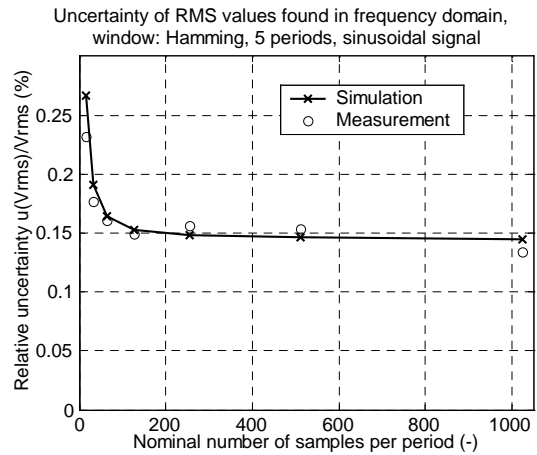


Fig.6 Uncertainty of RMS for sinusoidal signal

Tab.2 Bias of described methods of RMS measurement in frequency domain for Hann window used

Method	3.5 periods sampled	10.5 periods sampled	50.5 periods sampled	120.5 periods sampled
WIFD -par. B1	$-9 \cdot 10^{-2} \%$	$-6 \cdot 10^{-3} \%$	$-5 \cdot 10^{-5} \%$	$-1 \cdot 10^{-6} \%$
Main lobe-par.B2	-1 %	-1 %	-1 %	-1 %
Main lobe corrected-par.B3	$-3 \cdot 10^{-2} \%$	$-6 \cdot 10^{-4} \%$	$-4 \cdot 10^{-6} \%$	$-1 \cdot 10^{-7} \%$

C. Multifrequency signal, non-coherent sampling

The described methods can be used also for multifrequency signals. The only limitation is requirement to sample so many signal periods that main lobes of adjacent windowed signal spectrum components do not overlap. Our figures are plotted for sinusoidal signal, where bias is caused by side lobes of mirrored frequency component. In multifunction signal contribution of all spectrum components (including the mirrored ones) has to be considered.

D. Measurement results

Example of measurement as compared to simulation is shown in Fig. 6. Uncertainty was found for using slowly swept sinusoidal signal, ensuring slow changes of δ from (8). Uncertainty here corresponds to standard deviation of biased RMS values (see Fig. 4) for uniformly changing δ , $-0.5 < \delta < +0.5$. Both simulations and measurement were performed for 12-bit ADC resolution, Hamming window and five signal periods (for $\delta = 0$) sampled. RMS value was found using method described in par. B2, i.e. by processing DFT amplitude spectrum lines within window spectrum main lobe (see (10)). As can be seen from Fig.6, uncertainty does depend only slightly on number of samples per period if this number is larger than 128.

IV. Conclusion

A new and effective approach to RMS value measurement in time domain was presented, requiring much shorter time for reaching small RMS bias than the classical algorithm. In frequency domain a new method of finding decimal frequency bin δ was briefly introduced, allowing much broader window selection than class 1 Rife-Vincent windows to be used for interpolations in this domain. Application of correction factor to decrease RMS bias when processing spectrum components within the window main lobe was also described, and its efficiency was demonstrated graphically for Hann window. Influence of noise on uncertainty of RMS value was also briefly mentioned. The limited paper range did not allow a more detailed analysis of RMS measurement.

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