

# Baseband Harmonic Distortions in Single Sideband Transmitter and Receiver System

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## Abstract:

Telecommunications industry has widely adopted single sideband (SSB or complex quadrature) transmitter and receiver system, and one popular implementation for SSB system is to achieve image rejection through quadrature component image cancellation. Typically, during the SSB system characterization, the baseband fundamental tone and also the harmonic distortion products are important parameters besides image and LO feedthrough leakage. To ensure accurate characterization, the actual frequency locations of the harmonic distortion products are critical. While system designers may be tempted to assume that the harmonic distortion products are simply up-converted in the same fashion as the baseband fundamental frequency component, the actual distortion products may have surprising results and show up on the different side of spectrum. This paper discusses the theory of SSB system and the actual location of the baseband harmonic distortion products.

## Introduction

Communications engineers have utilized SSB transmitter and receiver system because it offers better bandwidth utilization than double sideband (DSB) transmitter system. The primary cause of bandwidth overhead for the double sideband system is due to the image component during the mixing process. Given data transmission bandwidth of  $B$ , the former requires minimum bandwidth of  $B$  whereas the latter requires minimum bandwidth of  $2B$ . While the filtering of the image component is one type of SSB implementation, another type of SSB system is to create a quadrature component of the signal and ideally cancels out the image through phase cancellation.

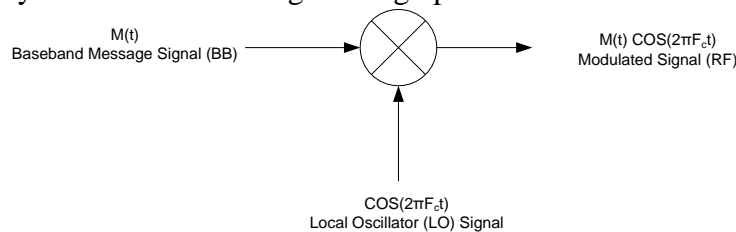
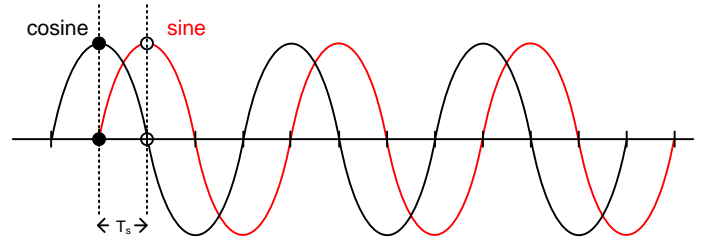


Figure 1. Double Sideband Transmitter and Associated RF Spectrum

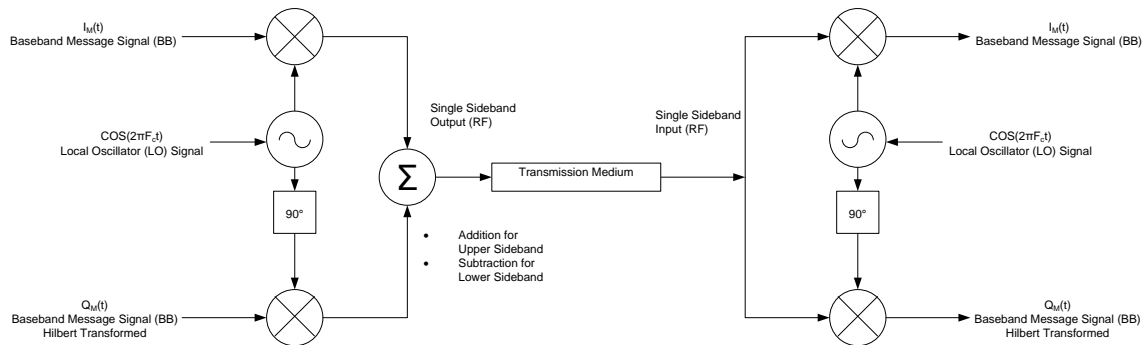
The method of image cancellation through quadrature component gains popularity because the image rejection through filtering requires infinite amount of filter roll-off, which is not possible to realize in actual circuits. Another alternative is vestigial sideband transmitter with practical filter roll-off through baseband frequency planning, although the filtering may still require fairly large order roll-off.

The most popular and practical method is to take advantage of orthogonality of cosine and sine waves, which are 90 degrees out of phase from each other and have zero interfere with each other at each sampling point. As shown in Figure 2, the cosine's peak value is sine wave's zero crossing point at each ideal sampling point, and vice versa. Due to this nature two separate information signals could theoretically be up-converted to RF signals that are 90 degrees out of phase. Both signals could occupy the same time and same frequency spectrum, and will not be able to interfere with each other at the exact sampling point. (Note: assuming ideal sampling) The details of upconversion can be summarized in Figure 3.



Orthogonality of cosine and sine wave. Given ideal  $T_s$  sampling, the two signals will not interfere with each other.

**Figure 2. Orthogonality of Cosine and Sine.**



**Figure 3. Single Sideband Modulation and Associated Spectrum**

With ideal circuit components with perfect gain and phase balance, the output spectrum should have only the actual transmission bandwidth and not the image. However, due to gain and phase imbalance, image rejection may not be perfect, and hence the output spectrum may show some image leakage and also LO frequency component leakage.

Another non-ideal factor is the finite non-linear behavior from single sideband system circuit components. Figure 4 shows the entire transmitter and receiver chain for single sideband system. The transmitting signal are first processed in the baseband processing unit such as DSP or FPGA, and then transferred to digital-to-analog converter (DAC) and the RF modulator circuit for upconversion. The transmitted signal is RF or microwave signal that go through some sort of transmission medium such as air or transmission line. The received signal would first go through RF demodulator and then to analog-to-digital

converter (ADC). The final digital signal would then go to DSP or FPGA for further signal processing.

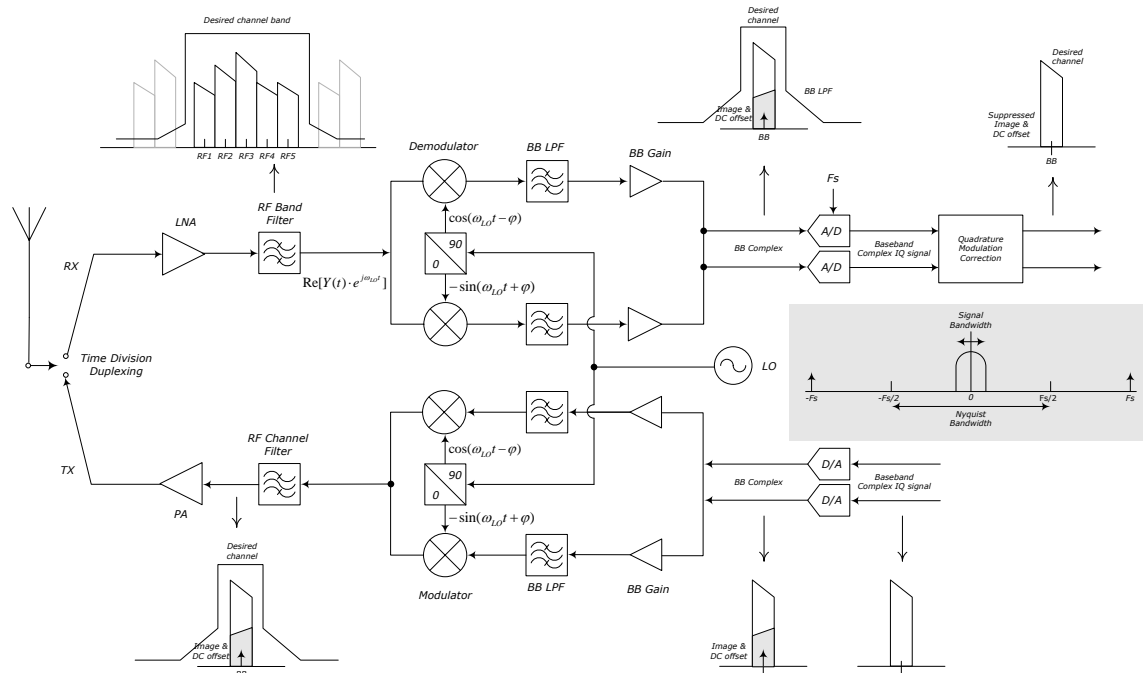


Figure 4. Single Sideband Transmitter and Receiver Chain

Even though all the analog and mixed-signal components such as DAC, modulator, demodulator, and ADC should ideally be purely linear components, all of them have some finite amount of non-linear gain. Circuit components such as transistors have higher order relationship terms such as second order gain and third order gain. As the result, the output may contain harmonic distortion terms such as second and third harmonic distortions. These terms may be expressed from Taylor series expansion in the form of  $y = a_1 * x + a_2 * x^2 + a_3 * x^3 + \dots$ . All terms except  $a_1$  are expected to be small since the design intention of the circuit components is to achieve linear gain.

In practice, the second order and third order effects are more significant to the overall system performance budget. The higher order terms that greater than third order are typically less significant, and if the system design requires the budgeting of these higher order terms, the same series expansion and derivation can be applied. This practice is aligned with the general Taylor series expansion principle: the more accurate the model, the higher the expansion orders are needed. Note that the gain of the higher order terms may be less than the lower order terms, and for most systems, these higher order terms may be considered as insignificant for the system budget.

These distortion products results in harmonic spectrums that are multiples of the fundamental tone. In single sideband system, these distortion products can be in various unexpected frequency locations. Figure 5 below shows the actual measurement of the TRF3705 output spectrum with fundamental tone of 10MHz with LO of 1840MHz. The second and third harmonic of the baseband are expected to be 20MHz and 30MHz,

respectively. However, +20MHz, -20MHz, +30MHz, and -30MHz spurious tones are observed around the LO frequency, and the amplitude of +30MHz and -30MHz tones are not balanced.

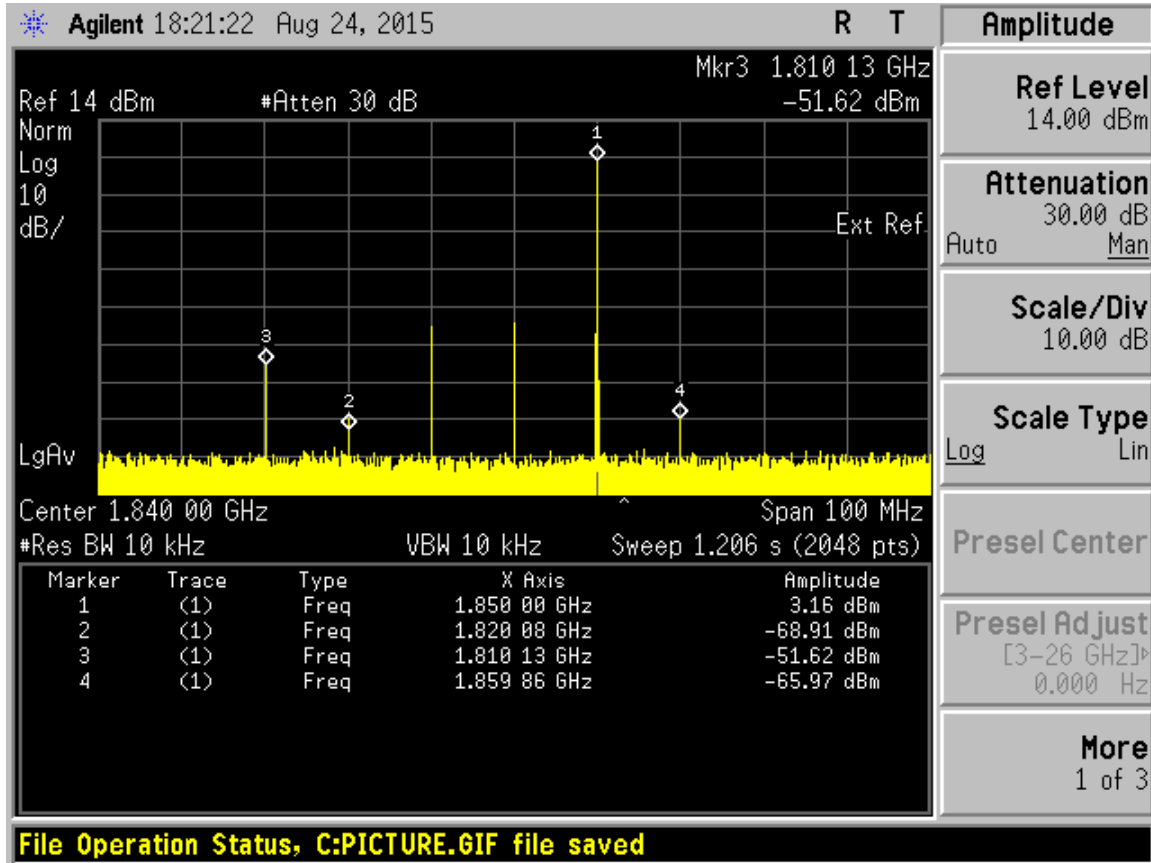


Figure 5. TRF3705 RF Output

The primary effects of second order and third order baseband harmonics are modeled in Matlab to demonstrate the difference between the ideal SSB systems versus SSB system with baseband harmonic distortion. In ideal SSB system without any baseband harmonic components, the fundamental signal present at the input of the SSB transmitter will be demodulated perfectly at the output of the SSB receiver. As shown in Figure 6, an 100Hz baseband tone is transmitted and received correctly throughout the system. However, with baseband harmonic components introduced to the model, various distortion products show up on the spectrum in a similar fashion as the measurement spectrum. Figure 7 shows the overall effect with baseband harmonic component introduced, and additional tones at -200Hz, 200Hz, and -300Hz are now present in the system. The following sections model the behavior and discuss the root cause of these tone allocations.

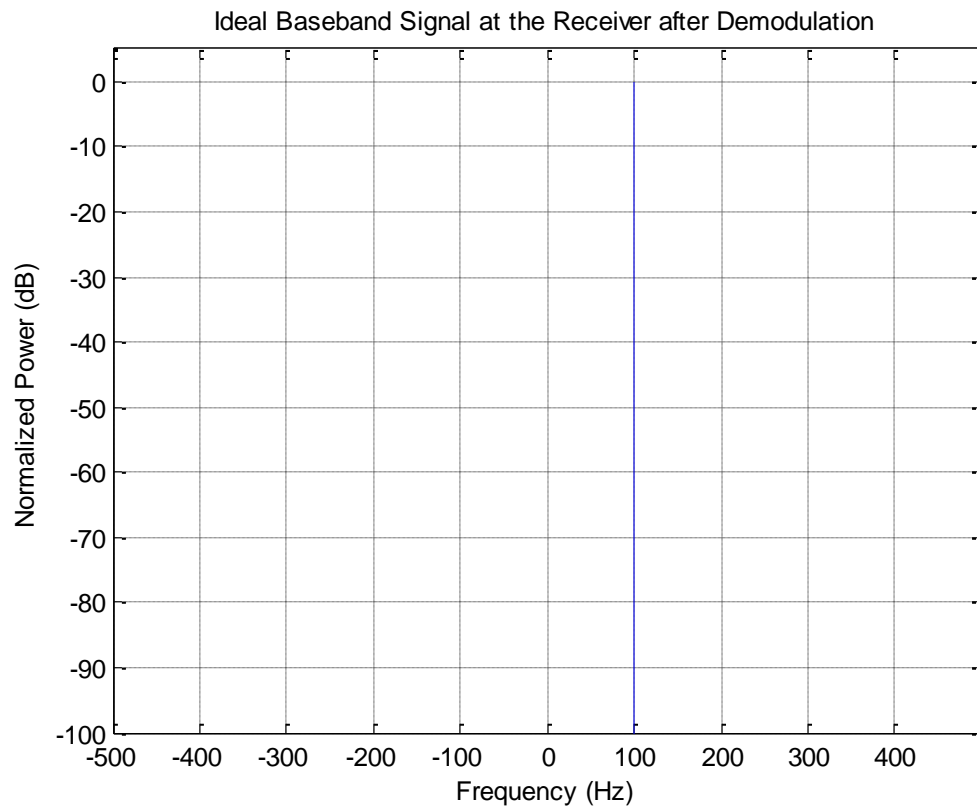


Figure 6. Ideal Baseband Signal at the Receiver after Demodulation

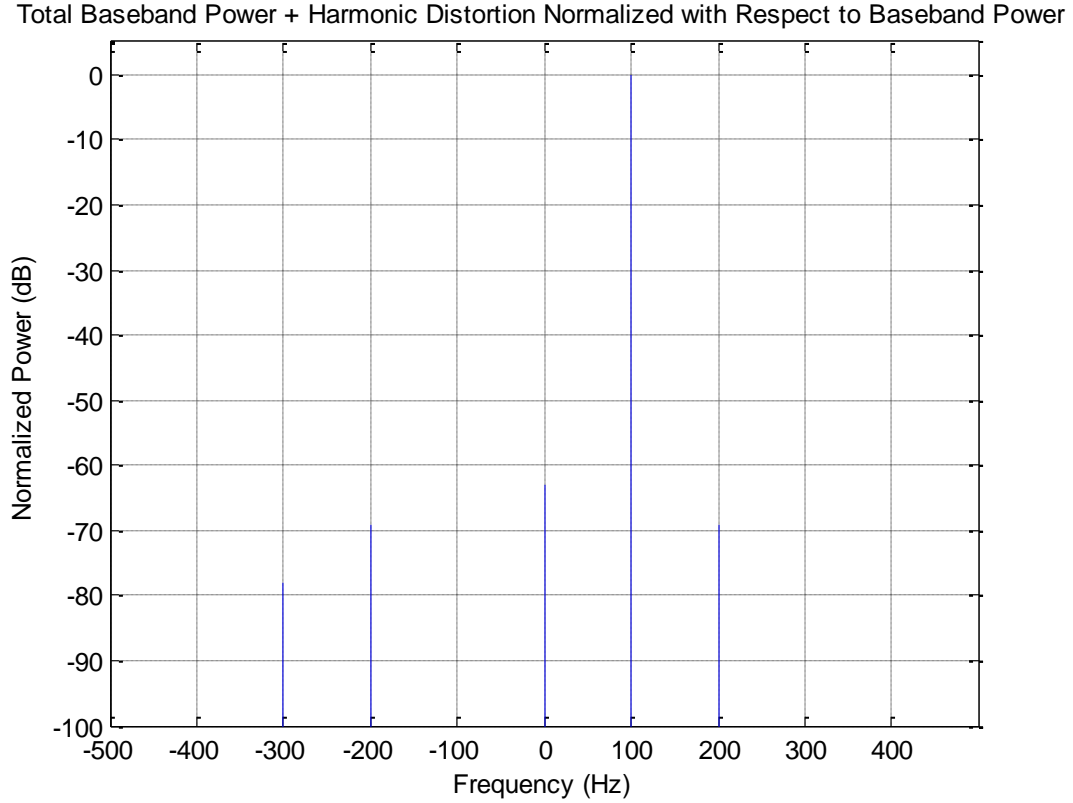


Figure 7. Simulated Result with Baseband Harmonics

### Location of Second Order Baseband Harmonic

The second order baseband harmonic can be modeled as squared terms of cosine and sine. As shown in the equation, the squared terms of cosine and sine contain both a DC term and also a cosine term with twice the frequency as the fundamental baseband tone.

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

When the baseband input contains an effective DC term, the DC term will multiply with the LO tone through the mixer portion of the modulator. Therefore, the effective DC term shows up as an LO leakage component in the RF spectrum. Moreover, the baseband I and Q inputs presents the same equivalent  $\cos(2\pi \cdot 2f_{bb})$  terms, and the SSB system can no longer suppress these terms since they contain the same phase. As a result, the RF output spectrum will show two tones locating at  $+2f_{bb}$  and  $-2f_{bb}$  with the same amplitude. The overall effect of the second order baseband harmonic is shown in Figure 8.

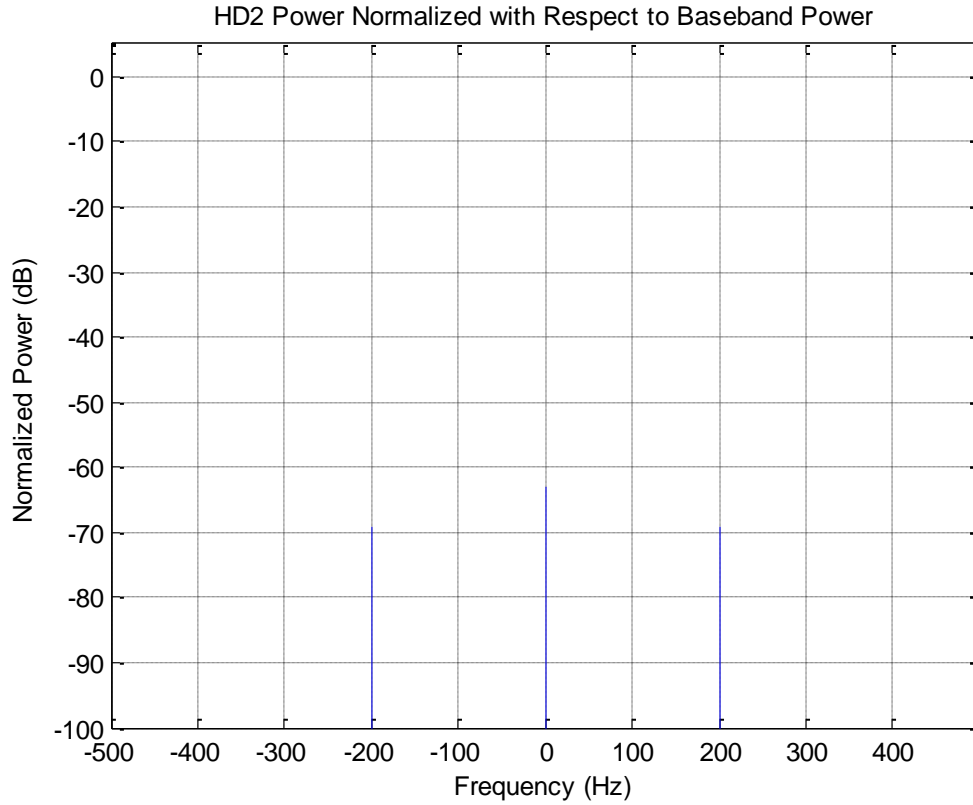
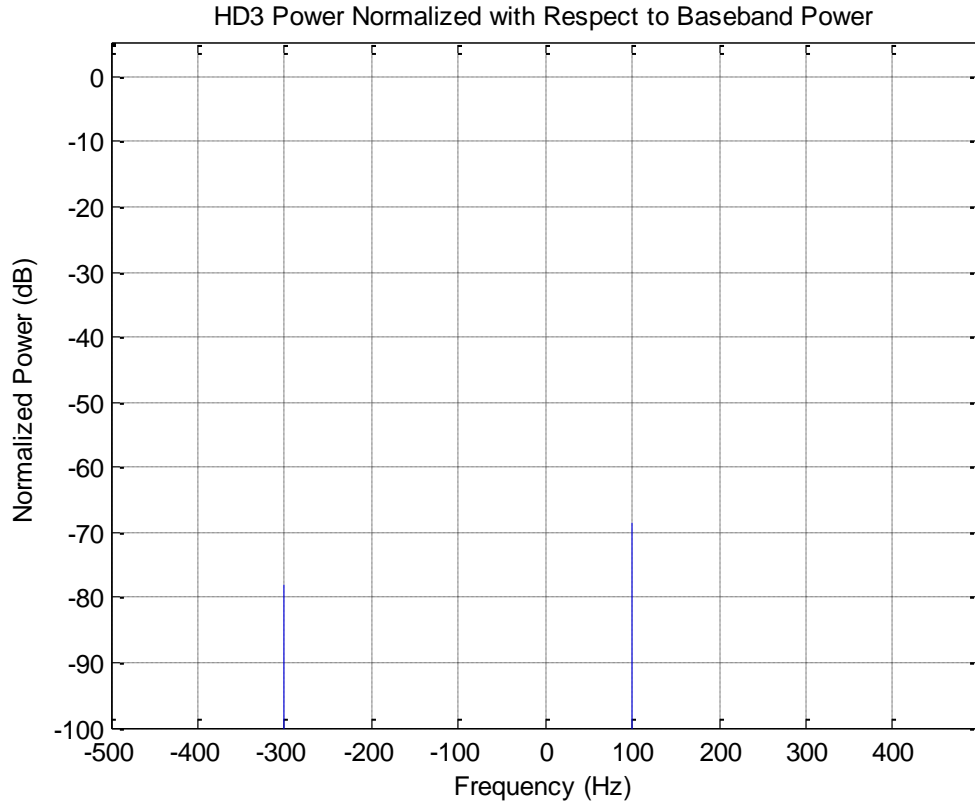


Figure 8. Simulated HD2 Spectrum

### Location of Third Order Baseband Harmonic

The third order baseband harmonic can be modeled as cubed term of cosine and sine. As shown in the equation, the cubed term of cosine and sine contain both cosine and sine of fundamental baseband frequency term and also sine and cosine of three times the frequency as the fundamental baseband tone.

$$\sin^3 \theta = \frac{3 \sin \theta - \sin 3\theta}{4} \quad \cos^3 \theta = \frac{3 \cos \theta + \cos 3\theta}{4}$$



**Figure 9. Simulated HD3 Spectrum**

The equivalent cosine and sine term with fundamental frequency has a factor of  $\frac{3}{4}$  by default, and the third order baseband harmonic gain, which should be much smaller than fundamental gain by design, should also lower the overall power contribution to the primary baseband signal.

More importantly, the sine term with three times the fundamental frequency has a sign reversal, and the up-conversion process with the cosine term with three times the fundamental frequency will result the third order tone showing up on the image side instead of the fundamental side (i.e.  $-3f_{bb}$ ).

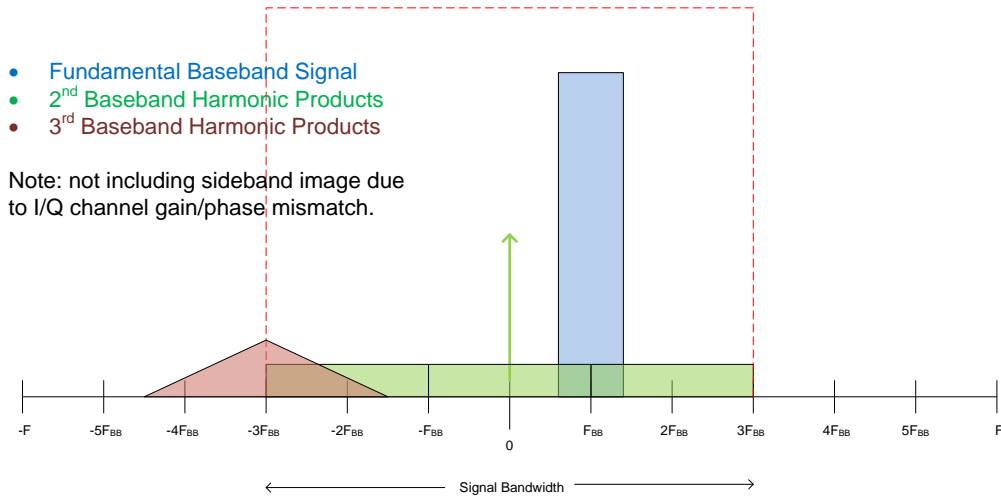
## Consequences and Frequency Plan Mitigation

As shown in Figure 10, both the second and third order artifacts will impact applications requiring direct baseband to RF up-conversion. The direct baseband to RF up-conversion usually have information location in both the primary and the image side. The second order artifacts show up as both DC component and symmetrically located  $\pm 2f_{bb}$ . The signal bandwidth of these artifacts is two times the signal bandwidth due to the second order effect. .

While basic instinct may tempt communication engineers to think that the third order artifacts may be located out of band due to the three times the fundamental frequency (i.e.



+3fbb), the actual artifact is located on the -3fbb side and may overlap with the fundamental information band. The signal bandwidth of these artifacts is three times the signal bandwidth due to the third order effect. Besides understanding the actual frequency location of the distortion, the bandwidth of these distortion products also adds complexity to the overall RF planning. Engineers need to plan with caution, and also consider that harmonic distortions become more significant as baseband frequency increases.



**Figure 10. Effect of Baseband Harmonics on Modulated Communication Signals.**

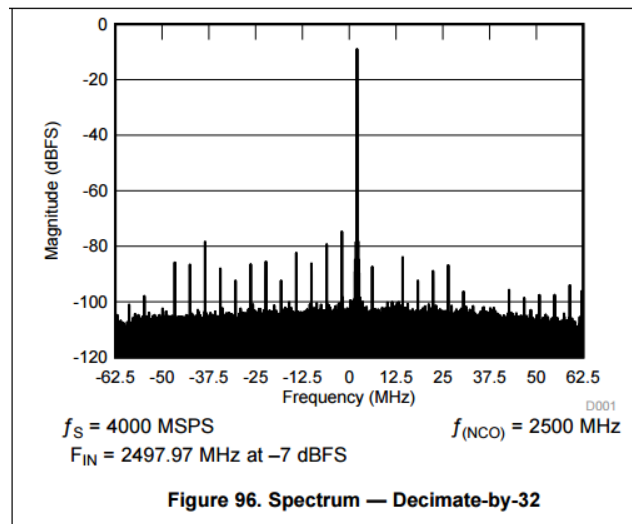
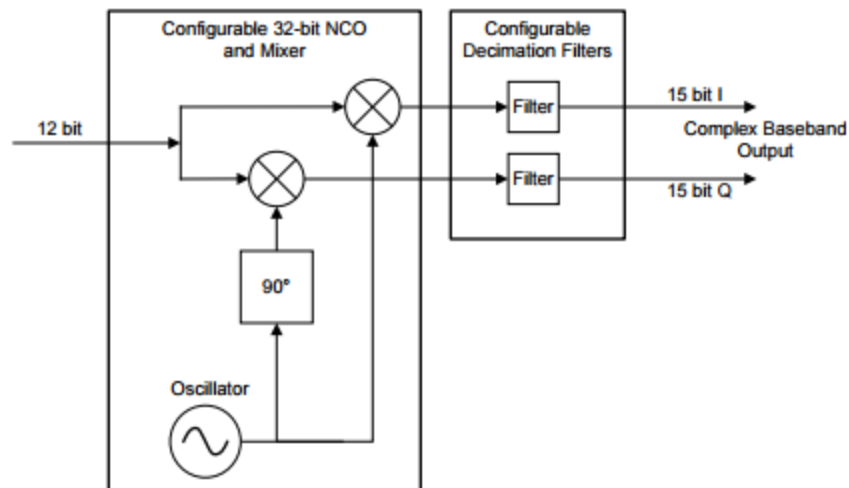


Figure 11. DDC Circuitry from ADC12J4000

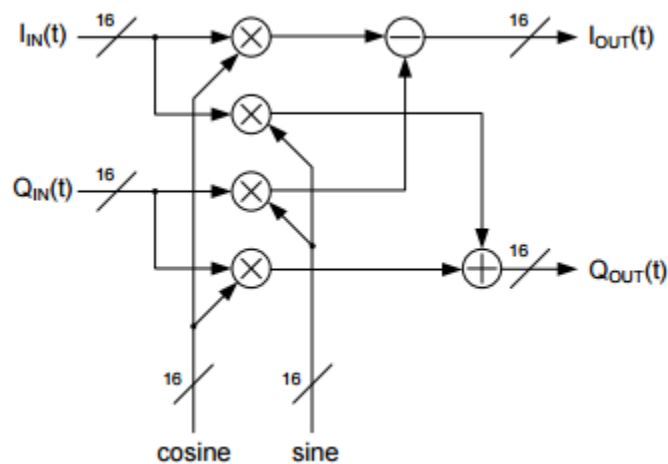


Figure 12. Complex Mixer + NCO from DAC38J84

## Appendix

### Matlab Code

```
close all;
clear all;

%% definitions
% the following section defines the sampling frequency and fft size for
the
% entire transceiver design.
% define sampling frequency
fs = 1000;
% define FFT size
N = 1000;
% define number of samples
n = 1:N;
% complex bin size = n/fs instead of n/(2fs)
fft_bin_size = fs/N
% fft x-axis
bin = -fs/2:fft_bin_size:fs/2-fft_bin_size;

%% circuit gain
% these are the circuit gains introduced by the baseband analog
components.
% The example values are derived based on the TRF3750 HD components at
1840MHz
% ideal first order gain
a1 = 1;
% 2nd order gain. Based on TRF3705 typical baseband HD2 performance at
% 1840MHz
a2 = 0.001; %0.1% THD
% 3rd order gain. Based on TRF3705 typical baseband HD2 performance at
% 1840MHz
a3 = 0.0005; % 0.05% THD

%% baseband model.
% defines the baseband frequency and introduce baseband analog higher
order
% distortion gain in both quadrature and real paths.
fb = 100; %define baseband frequency
flo = 300; %define LO frequency
i_in = cos(2*pi*fb/fs*n);
q_in = sin(2*pi*fb/fs*n);
bb_in = a1*i_in + a1*j*q_in;
%calculate the energy of baseband signal for normalization.
energy_bb_in = bb_in*bb_in';

% convert time sample into frequency samples via Matlab's FFT function
BB_IN = fft(bb_in);
% plotting the ideal baseband signal after modulation and demodulation
% note: the operation of BB_IN/sqrt(N) is to ensure energy calculation
of
% time domain and frequency domain are conserved. I.e. Parseval
identity.
% 10*log10(energy_bb_in) is used to normalize the result to 0dB.
```

```

figure(1);
plot(bin, 20*log10(abs(fftshift(BB_IN/sqrt(N))))-
10*log10(energy_bb_in));
title('Ideal Baseband Signal at the Receiver after Demodulation')
xlabel('Frequency (Hz)');
ylabel('Normalized Power (dB)');
axis([min(bin) max(bin) -100 5]);
grid on;

%% Calculating HD2 Power
% summing HD2 distortion products in both I and Q path
hd2 = a2*i_in.^2 + j*a2*q_in.^2;
% calculate the energy of HD2 products
energy_hd2 = hd2*hd2';
% perform FFT of HD2
HD2 = fft(hd2);
figure(2)
% plot HD2 power. This is normalized to baseband power
plot(bin, 20*log10(abs(fftshift(HD2/sqrt(N))))-10*log10(energy_bb_in));
title('HD2 Power Normalized with Respect to Baseband Power')
xlabel('Frequency (Hz)');
ylabel('Normalized Power (dB)');
axis([min(bin) max(bin) -100 5]);
grid on;

%% Calculating HD3 Power
% summing HD3 distortion products in both I and Q path
hd3 = a3*i_in.^3 + j*a3*q_in.^3;
% calculate the energy of HD3 products
energy_hd3 = hd3*hd3';
% perform FFT of HD3
HD3 = fft(hd3);
figure(3)
% plot HD3 power. This is normalized to baseband power
plot(bin, 20*log10(abs(fftshift(HD3/sqrt(N))))-10*log10(energy_bb_in));
title('HD3 Power Normalized with Respect to Baseband Power')
xlabel('Frequency (Hz)');
ylabel('Normalized Power (dB)');
axis([min(bin) max(bin) -100 5]);
grid on;

%% total baseband power + harmonic distortion power
% sum the baseband signal + distortions
bb_dist = bb_in + hd2 + hd3;
% perform FFT
BB_DIST = fft(bb_dist);
figure(4)
% plot total power. This is normalized to baseband power
plot(bin, 20*log10(abs(fftshift(BB_DIST/sqrt(N))))-
10*log10(energy_bb_in));
title('Total Baseband Power + Harmonic Distortion Normalized with
Respect to Baseband Power')
xlabel('Frequency (Hz)');
ylabel('Normalized Power (dB)');
axis([min(bin) max(bin) -100 5]);
grid on;

```