**f-Squared Noise by Arthur Kay Jan 9, 2007**

The Op-Amp current noise spectral density curve may contain a region at high frequency where the voltage noise increases with frequency. The increase in current noise is directly proportionate to frequency (i.e. if frequency increases by a factor of 10 current noise will increase by a factor of 10). This noise is called f-squared noise because the power spectral density is proportionate to f-squared (remember $P = i^2R$). Figure 1 shows how current noise increases by a factor of 10 per decade of frequency. Note that 1/f noise will typically decrease by the square root of 10.
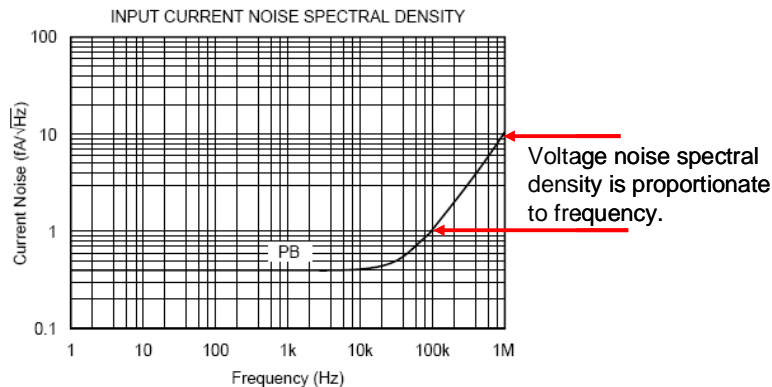


Figure 1:  Current Noise Spectral Density Diagram Showing f-Squared Noise

The f-squared noise region is typically the result of voltage noise interacting with the capacitive reactance of the input capacitance. For this reason, some engineers do not consider f-squared noise as current noise. The important thing to keep in mind when analyzing this effect is that this noise will be generated automatically if voltage noise and the input capacitance are properly modeled. So, if you are including the input capacitance in your op-amp model, you should not use a current noise spectral density curve that contains the f-squared region.

Figure 2 and Figure 3 summarize the two possible ways to analyze a noise problem that contains f-squared noise. In figure 2, the f-squared noise is not included in the noise source and so, the input capacitance is included. This is the recommended way of analyzing f-squared noise because it models the actual circuit condition. Note that the actual effect of the input capacitance will very depending on the circuit configuration. For example, C2 in figure 2 is shorted out by Vg1. In a different configuration, C2 may play an important roll. In figure 3, the noise source models the f-squared region and so the input capacitance is not included in the model. This method of noise analysis may produce accurate results; however, the f-squared effect is the same regardless of circuit configuration. Be careful not to count the effect of f-square noise twice as is shown in figure 4.

Unfortunately, data sheets are not all consistent regarding f-squared noise. Some data sheets show the f-squared region and others assume the effect will be accounted for by the proper modeling on the input capacitance.
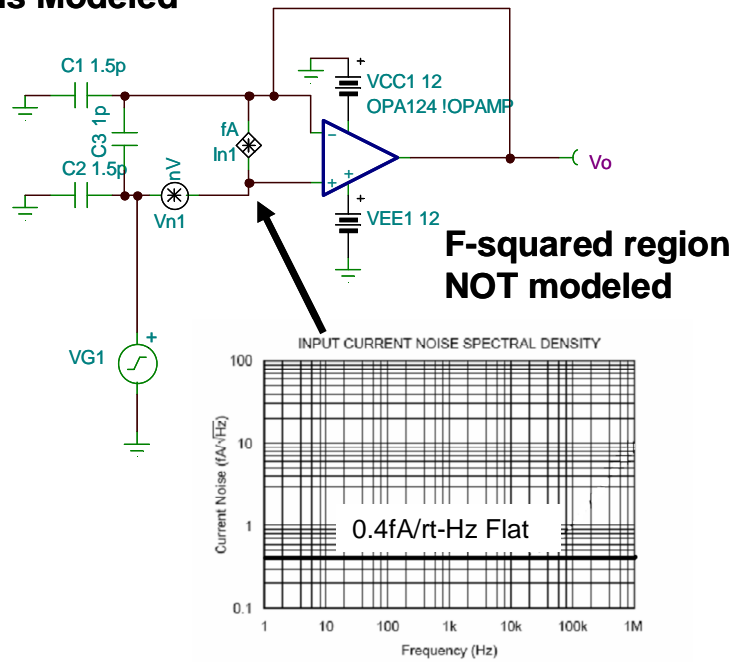
**Input Capacitance is Modeled**

C1 1.5p
C3 1p
C2 1.5p
fA In1
nV
Vn1
VG1
VCC1 12
OPA124 !OPAMP
Vo
VEE1 12

**F-squared region NOT modeled**

INPUT CURRENT NOISE SPECTRAL DENSITY

100

Current Noise (fA/√Hz)

10

1    0.4fA/rt-Hz Flat

0.1

1   10   100   1k   10k   100k   1M
Frequency (Hz)

Figure 2:   f-Squared region generated by Noise Voltage Over Input Capacitance

**Input Capacitance is NOT Modeled**

fA In1
nV
Vn1
VG1
VCC1 12
OPA124 !OPAMP
Vo
VEE1 12

**F-squared region modeled**

INPUT CURRENT NOISE SPECTRAL DENSITY

100

Current Noise (fA/√Hz)

10

1       PB

0.1

1   10   100   1k   10k   100k   1M
Frequency (Hz)
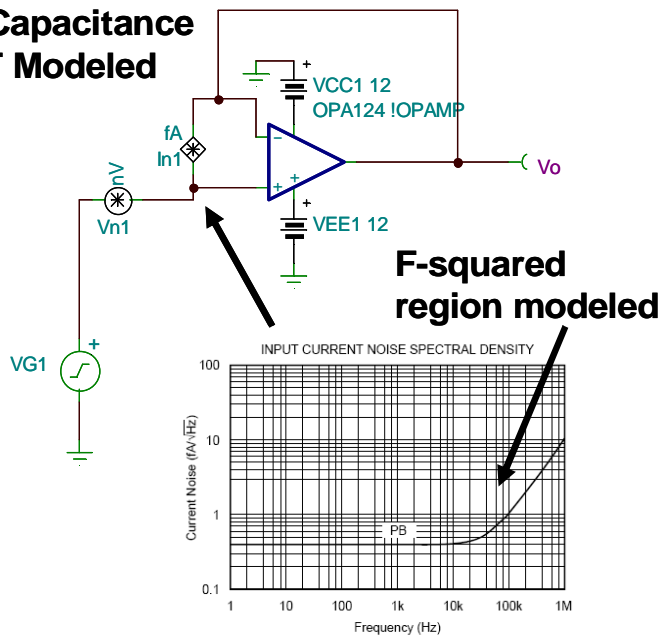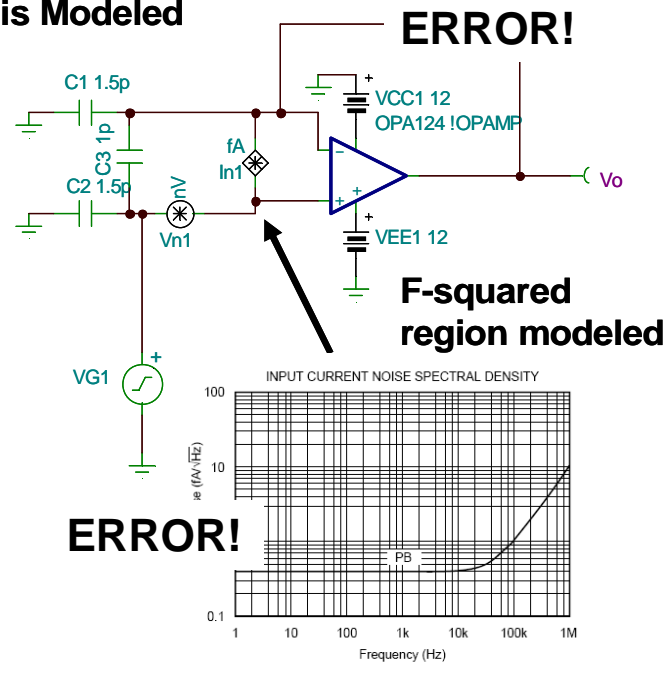
Figure 3:   f-Squared region included in Current Source

Figure 4: Wrong way to analyze f-squared noise.

## Exception to the f-Squared Rule:

Up to this point we have assumed that f-squared noise is the interaction of the voltage noise source and the input capacitance. While in general this is true, in some cases there is an additional source of noise. Figure 5 illustrates the differential input stage of an op-amp including parasitic capacitance and noise sources. The voltage noise sources Vn1 and Vn2 are correlated and so, the voltage noise is rejected by the op-amps common mode rejection. However, these sources will interact with the parasitic capacitance to generate f-squared current noise. This f-squared noise is in addition to the f-squared noise that was discussed previously. Further complicating the matter is the fact that most data sheets will not describe this effect. It must be stressed, however, that this effect is rarely a significant factor in the overall noise.
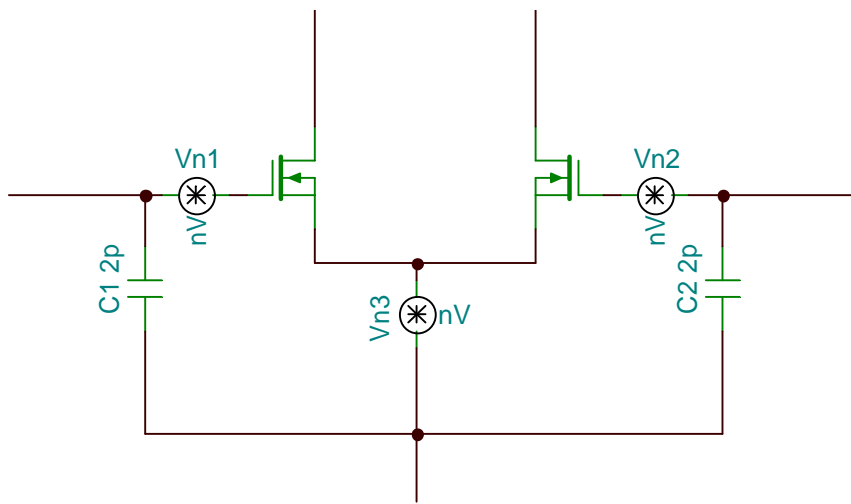


Figure 5: Internal Op-Amp Noise configuration Leading to Additional f-Squared Noise

**Hand calculations of f-squared noise:**

Assuming that the f-squared noise it results from an interaction of the voltage noise source and the reactance of the parasitic input capacitance, then the f-squared noise spectral density curve can be predicted using simple hand calculations. The first part of the calculation is to derive a formula for the f-squared portion of the current noise spectral density curve (see figure 6). The second part of the calculation is to integrate the current spectral density curve squared (power spectral density). The square root of this quantity is taken to convert the result to current (see figure 7). Finally, the current from the f-squared region is combined with current from the broad band region (see figure 8). Note that in this example the f-squared noise dominates.

$$X_c = \frac{1}{2 \cdot \pi \cdot f \cdot C_{dif}}$$

$$i_{spec}(f) = \frac{e_n}{X_c} = e_n \cdot \left(2 \cdot \pi \cdot f \cdot C_{dif}\right)$$

**For this example:**

$$e_n = 5 \cdot 10^{-9} \qquad C_{dif} = 1 \cdot 10^{-12}$$

$$i_{spec}(f) = 3.142 \, 10^{-20} \cdot f$$

Figure 6:  Derive the Current Spectral Density Function for f-Squared Region

$$i_{n\_f\_squ} = \sqrt{\int_{f_1}^{f_2} i_{spec}(f)^2 \, df}$$

**For this example:**

$$i_{n\_f\_squ} = \sqrt{\int_{1}^{10^6} \left(3.142 \, 10^{-20} \cdot f\right)^2 df} = 18.14 \times 10^{-12} \quad \text{A rms}$$

Figure 7:   Compute Noise Current From f-Squared Region

$$i_{nBB} = i_{BB} \cdot \sqrt{BW_n}$$

$$i_{nBB} = \left(0.5 \cdot 10^{-15}\right) \cdot \sqrt{\left(10^6 - 1\right)} = 500 \times 10^{-15}$$

$$i_{n\_total} = \sqrt{i_{n\_f\_squ}^2 + i_{nBB}^2}$$

**For this example:**

$$i_{n\_total} = \sqrt{\left(18.14 \times 10^{-12}\right)^2 + \left(500 \times 10^{-15}\right)^2} = 18.147 \times 10^{-12} \quad \text{A rms}$$

Figure 8:  Combine f-Squared Region with Broadband Region

**Tina Spice Analysis Including the f-Squared Region:**

SPICE Noise Analysis of circuits containing the f-squared region follows the same procedures outlined in Part IV of this series.  So the first step to doing any noise analysis is to test the model.  Proving that the f-squared region of the SPICE model is correct is complicated by the fact that data sheets may not show this region in the current spectral density curve.  Also, some amplifiers do not have a 1/f region.  The hand calculation is one way to determine if f-squared noise is present.

Figure 9 shows how to build a Tina SPICE model that includes the f-squared effect.  The procedure outlined in Part IV is used to configure the noise sources and the generic op-amp.  The only additional step is to include the common mode and differential input capacitance.  The input capacitance specification is listed in the data sheet as shown in Figure 10.  Figure 11, 12 and 13 show the Tina SPICE listing for the noise sources and the generic op-amp.  Figure 14 shows the current noise spectral density curve as measured by the current to voltage converter CCV1 and the data sheet current noise curve.  Note that the f-squared region from the Tina Spice Simulation closely follows the data sheet curve.
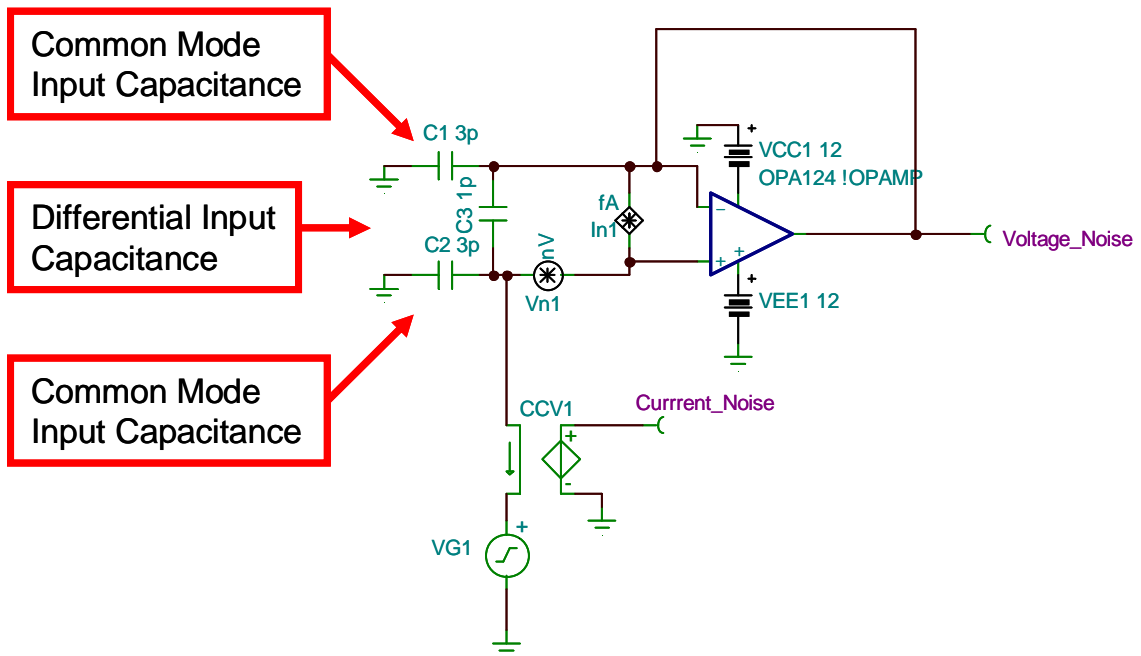
Figure 9: Build a Tina Spice Model



| Input Offset Current | $V_{CM} = 0VDC$ | | | $\pm 1$ | $\pm 5$ | pA |
|---|---|---|---|---|---|---|
| IMPEDANCE Differential Common-Mode | | | | $10^{13} \parallel 1$ $10^{14} \parallel 3$ | | $\Omega \parallel pF$ $\Omega \parallel pF$ |
| VOLTAGE RANGE | | | | | | |

Figure 10: Input Capacitance from Specification for OPA124
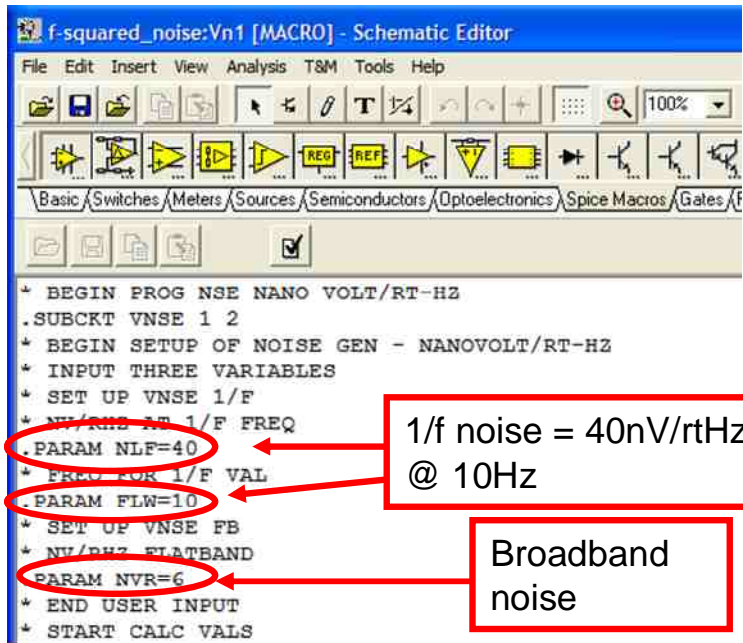
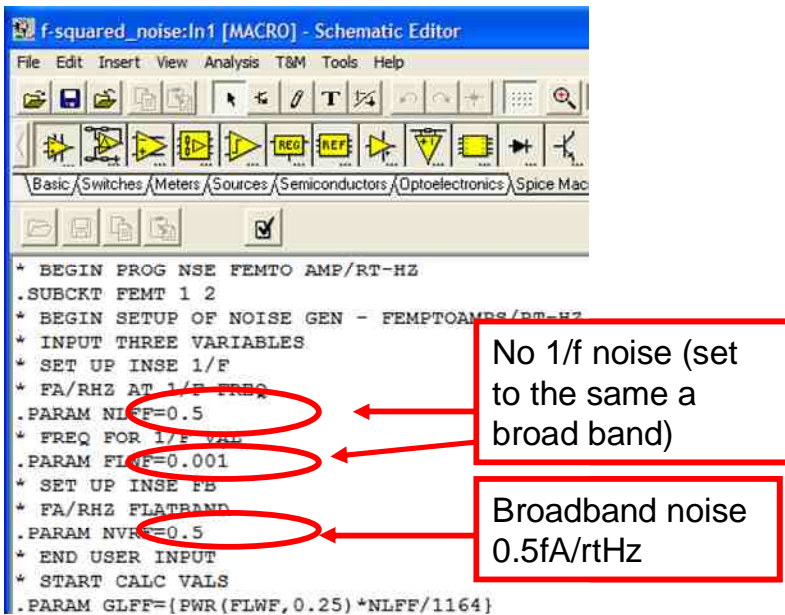Figure 11:  Voltage Noise Source Configuration



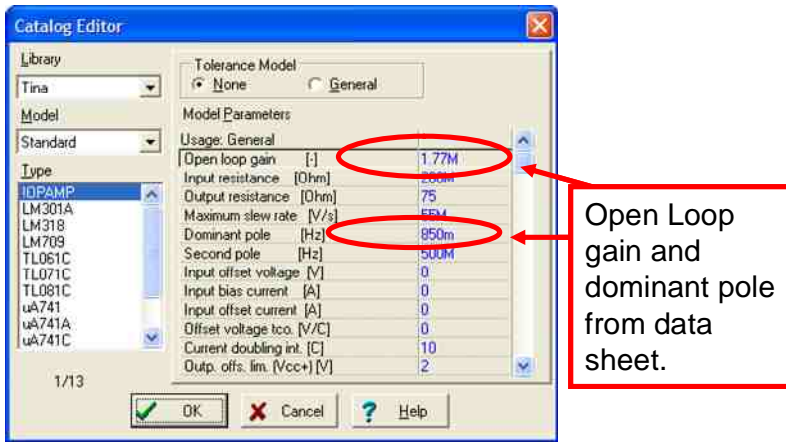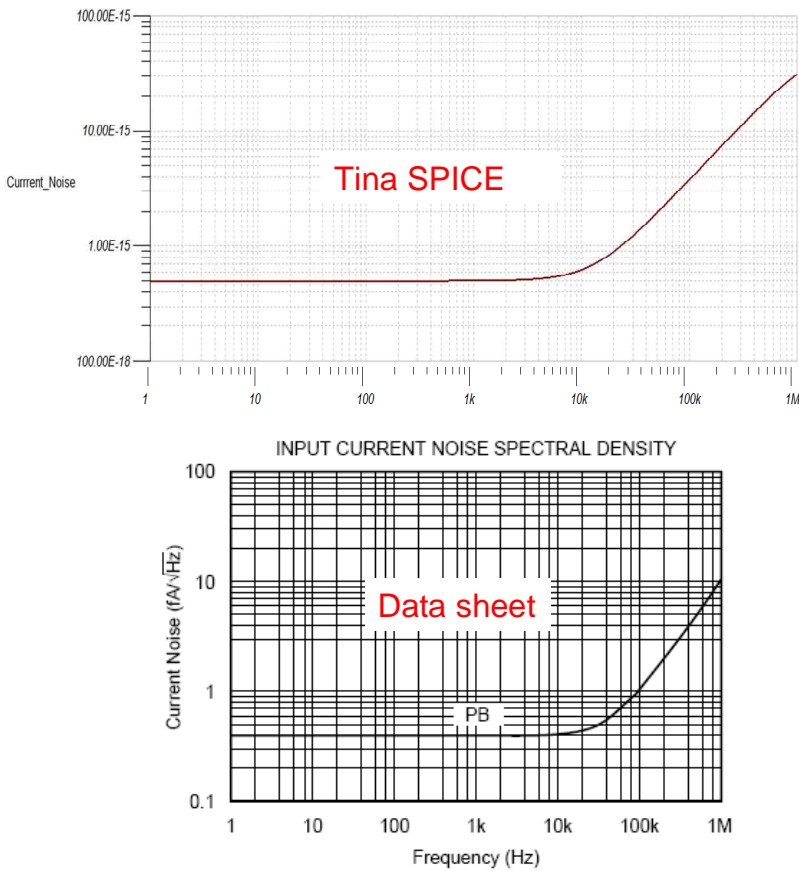Figure 12:  Current Noise Source Configuration

Figure 13: Generic Op-Amp Configuration



Figure 14: Tina Spice Results Compared with Data Sheet